Recent Advances in reduced-order modeling of complex interconnects

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Abstract: This paper addresses some important issues related to reduced-order modeling of complex interconnects. Two different but complementary directions are investigated. On one hand, multiport interconnect structures with possibly complex geometry are analyzed by means of model order reduction from transient scattering responses. On the other hand, some recent advances on stable and robust treatment of transmission lines with arbitrary frequency-dependent parameters are illustrated. Both modeling strategies lead to the automatic generation of a SPICE-ready equivalent circuit for system-level simulation.

1 Introduction

The continuous increase of clock rates in digital applications results in unwanted parasitic electromagnetic effects that can seriously affect the performance of electrical interconnects. Therefore, the generation of accurate models for the representation of all relevant signal degradation effects has become an unavoidable step for modern technologies. In this paper, we concentrate on reduced-order modeling of complex interconnects, with special attention to two challenging modeling problems described below.

A first problem is the extraction of a suitable circuit model for interconnect structures with high structural complexity, like, *e.g.*, packages with a large number of ports. In such cases, the presence of a complex geometry, with discontinuities and several adjacent conductors, calls for either a full-wave characterization or a direct measurement. However, the extraction of a simple yet accurate model from the output of the electromagnetic simulation or the measurement process must be performed. In Section 2 we describe an algorithm producing reduced-order models from transient scattering waves defined at the ports of the structure.

A second modeling problem we investigate in this paper involves the characterization of interconnects with controlled geometry, *e.g.*, uniform transmission lines, described in terms of their per-unit-length parameters. The inclusion of a possibly complex frequency behavior of such parameters due to skin effect and dielectric losses may be difficult. Some recent advances on generation of reduced-order models for such structures are detailed in Section 3.

2 Model order reduction from transient scattering waves

In this section, we describe an algorithm for the generation of lumped equivalents of complex linear junctions from their measured or simulated transient scattering responses. The algorithm is based on the Block Complex Frequency Hopping (BCFH) technique [1]. This method identifies an approximate lumped multiport through a set of rational transfer functions characterized in terms of poles and residues through partial fraction expansion. The poles are obtained via moment matching with the actual transient response data, while the residues are computed by least squares fitting the function samples. The models obtained with the BCFH method have a solid physical foundation, because their poles are approximations of the actual poles of the modeled multiport element. This guarantees the stability of estimated models and helps the control of their passivity.

We tested the BCFH-scattering approach on both ideal multiport elements, composed of transmission lines and lumped parts, and on actual package structures, whose transient responses are computed by full-wave methods [2]. The ideal test structures are devised to validate both the poles and the responses of the estimated models, as their actual poles can be computed. On the other hand, test structures characterized by full-wave simulation are examples of really complex junctions. For both types of test structures, we obtain good models reproducing the scattering behavior of the original junctions over several GHz wide bandwidths. We also find that possible interconnect delays comparable to the rise/fall time of waveforms do not reduce significantly the ability of the BCFH algorithm to estimate the set of poles.

In order to demonstrate the modeling process, we apply it to the two-port element of Fig. 1, that is composed of ideal transmission lines and lumped parts. The structure is simulated in time domain with SPICE, recording its transient scattering responses for gaussian input pulses (Fig. 2, left panel).

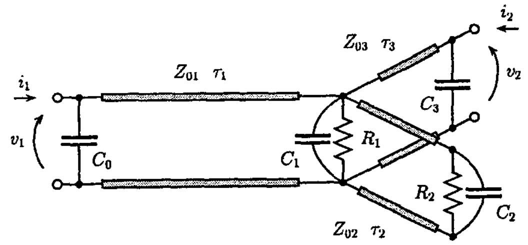

Figure 1: Two-port element composed of transmission lines and lumped elements for the modeling example of Sec. 2. The parameter values are: $C_0 = C_1 = C_2 = C_3 = 10\,\mathrm{pF}$, $R_1 = 500\,\Omega$, $R_2 = 200\,\Omega$, $Z_{01} = 100\,\Omega$, $Z_{02} = 80\,\Omega$, $Z_{03} = 120\,\Omega$, $\tau_1 = 1\,\mathrm{ns}$, $\tau_2 = 0.694675\,\mathrm{ns}$, $\tau_3 = 0.43\,\mathrm{ns}$. For both ports, the value of the reference impedance defining the wave variables is $50\,\Omega$.

Such data are then used to compute the moments of the scattering matrix elements via time-domain integration and to estimate the set of poles via BCFH. For a given decay rate of the natural response and length of the time window, the expansion centers minimizing the error on the moments are used. Furthermore, time and frequency normalization of responses as well as moment scaling are exploited to generate well-conditioned matrices of moments. The right panel of Fig. 2 shows the obtained reduced-order model compared to the actual frequency-domain scattering matrix elements computed with FFT. The model and the reference curves are in good agreement, with only small deviations. This example shows the capability of the proposed algorithm to process transient input/output scattering waves and extract an accurate equivalent model.

3 Model order reduction of frequency-dependent transmission lines

This section deals with the generation of accurate, stable, and robust models for transmission lines with frequency-dependent parameters. The models are intended to be used within standard circuit solvers (*e.g.*, SPICE) in order to perform system-level simulations. Therefore, the main task is here to translate a possibly complex frequency-dependence for the line parameters into a simple lumped equivalent circuit. In the following we give some details for scalar transmission lines, while the generalization to multiconductor transmission lines will be the subject of a forthcoming report.

Frequency-dependent line parameters $R(j\omega)$, $L(j\omega)$, $C(j\omega)$, $G(j\omega)$ evaluated at some discrete and finite frequency points constitute the input data set for the model generation. The line equations in frequency domain read

$$-\frac{d}{dz}V(z,j\omega) = [R(j\omega) + j\omega L(j\omega)]\, I(z,j\omega),$$

$$-\frac{d}{dz}I(z,j\omega) = [G(j\omega) + j\omega C(j\omega)]\, V(z,j\omega),$$

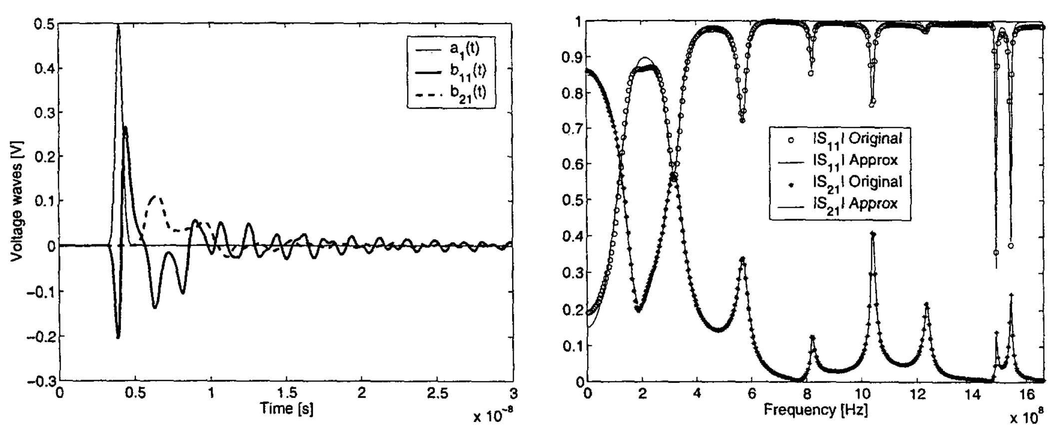

Figure 2: Left panel: transient voltage waves used for the indentification of the reduced-order model. Right panel: selected elements of the frequency-domain scattering matrix illustrating the accuracy of the proposed reduction algorithm.

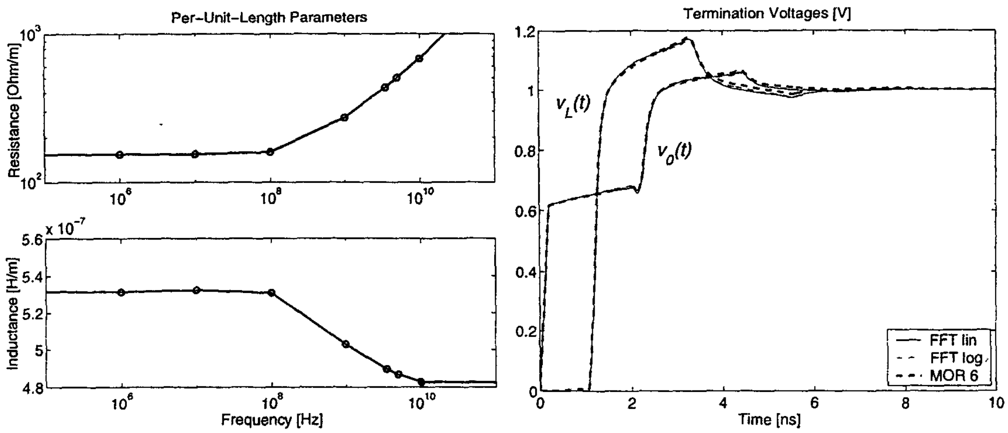

Figure 3: Reduced-order modeling of a scalar transmission line. The frequency-dependent per-unit-length parameters are depicted in the left panels (dots represent measured data; the solid line is a linear interpolation not used in the approximation algorithm), while the right panel reports the termination voltages.

where the line is assumed of length $\mathcal{L}$. Many approaches can be found in literature for the extraction of a reduced-order model from the above equations(see, *e.g.,* [3] and references therein). The common background is the choice of a suitable frequency-domain function to be approximated with a simpler and known model (usually rational for ease of implementation). This approximation step has been performed with several different techniques (Padé, least squares, etc.). Regardless of the fitting method, the success of the approximation stage depends on the suitability of the model with respect to the specific target function to be approximated. If the model has significantly different features from the target function, convergence will be slow and sometimes even impossible over the required bandwidth.

In order to guarantee the suitability of the model to the target function, we propose in this paper a rational model with asymptotic constraints. The target functions to be approximated are the characteristic admittance $Y_c(s)$ and the delayless propagation operator $H(s)$, where s is the Laplace variable. A rational function approximation is therefore sought for as

$$
\begin{aligned}
Y_c(s) &= \sqrt{\frac{G(s)+sC(s)}{R(s)+sL(s)}} \simeq \sum_k \frac{R_k^y}{s-p_k} + Y_\infty, \\
H(s) &= \exp\{sT + \mathcal{L}\sqrt{(G(s)+sC(s))(R(s)+sL(s))}\} \simeq \sum_k \frac{R_k^p}{s-p_k} + H_\infty
\end{aligned}
$$

through a least-squares fit. The line delay T is extracted from the asymptotic values of the line parameters at $s = \infty$. The success of the least squares fit is insured by the explicit enforcement of their asymptotic values at low and high frequency, which are easily determined by the asymptotic values of the per-unit-length parameters. It should be noted that these asymptotic values are finite in any case. Therefore, convergence is insured, for a sufficiently high model order, over an extended bandwidth. This procedure insures also a correct identification of steady states (matching at $s = 0$) and causality of the line responses due to the specific form of the model. The extraction of asymptotic behavior would be impossible if the rational model were fitted directly to the per-unit-length impedance and admittance, since their high frequency behavior does not settle to a finite value due to skin effect and dielectric losses. Note also that we do not make any hypothesis on the functional form of line parameters. There is no restriction to the common model $A + B\sqrt{s}$ for internal impedance of conductors, since the asymptotic behavior is determined case by case.

We illustrate the performance of the proposed technique on two transmission lines of practical interest. The first line is 10 cm long, with frequency-dependent per-unit-length resistance and inductance, depicted in Fig. 3, and constant capacitance $C = 2.2446$ pF/cm. The line is excited by a voltage source (unitary step with 200 ps rise time) with internal resistance $R_S = 30\Omega$, and terminated with a 1.5 pF capacitance. The performance of the proposed reduced-order model (order 6) is compared to the transient response obtained

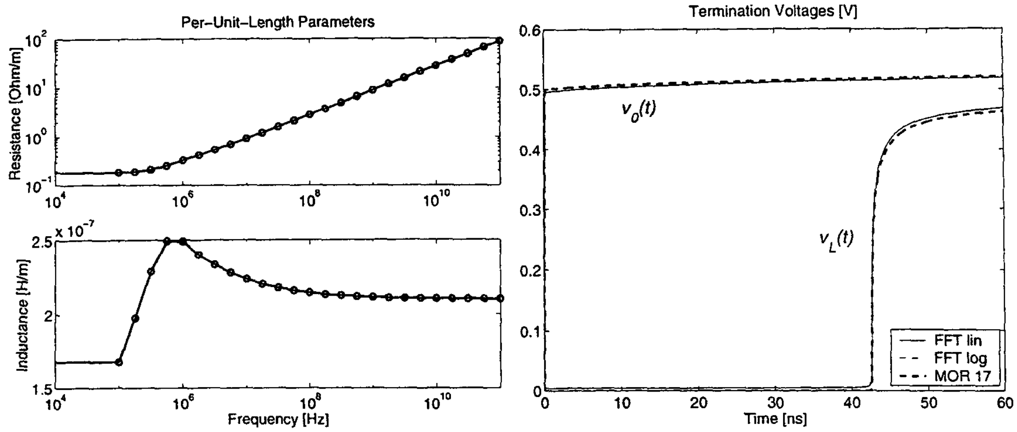

Figure 4: As in Fig. 3, for another transmission line case.

through inverse FFT of the frequency-domain solution over a very large number of frequency points (due to the time scales involved). It should be noted that the frequency-domain solution requires some interpolation between input frequency-domain samples of line parameters in order to get sufficient data for inverse FFT. Straightforward interpolation schemes (linear or log-linear) can be easily shown not to satisfy the physical causality conditions. Therefore, the curves obtained through inverse FFT are necessarily unphysical and are dependent on the adopted interpolation scheme. The reduced-order model satisfies the causality conditions by definition.

The second example refers to a 10 m line with frequency-dependent per-unit-length resistance and inductance (Fig. 4), constant capacitance $C = 0.858841$ pF/cm, and dielectric losses with $\tan\delta = 0.00075$. The line is terminated by 50 Ω loads and excited by a unit step with 100 ps rise time. The termination voltages are reported in the right panel of Fig. 4. Apart from the evident causality violation of the inverse FFT solution, the two methods lead to consistent results.

4 Conclusions

We have shown the application of model-order reduction techniques to two interconnect modeling problems. The first consists of the representation of possibly complex interconnect networks characterized by means of measured or simulated transient responses at their accessible ports. The second problem involves modeling of transmission lines characterized by arbitrary frequency-dependent parameters. Both modeling tasks can be successfully accomplished by simple algorithms, giving rise to lumped equivalent circuits readily available within standard simulation environments.

5 Acknowlegments

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