A Plane Wave Monte Carlo Simulation Method for Reverberation Chambers

A PLANE WAVE MONTE CARLO SIMULATION METHOD FOR REVERBERATION CHAMBERS

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Abstract - A Monte Carlo simulation is proposed to represent the electromagnetic environment of mode stirred reverberation chambers. The proposed method is based on a plane wave model and allows us to simulate both the point statistics and the spatial correlation of the electromagnetic distribution inside an ideal reverberation chamber. By means of this approach, it is possible to predict the electromagnetic coupling to electrical devices. The application of the proposed method to predict the electromagnetic coupling to a transmission line inside a reverberation chamber is proposed, and prediction results are compared with measurements.

I. INTRODUCTION

Different approaches have been used to simulate the reverberation chambers (RC) electromagnetic (EM) environment. Deterministic approaches allowing the simulation of EM fields distribution inside a RC have been investigated by a Ray Tracing method [1], by the FDTD method [2], and by the Finite Element method [3], for a two-dimensional case. On the other hand, a statistical approach has been proposed in [4], describing how to generate stochastic fields by using a random numbers generator. This approach was applied in [5] to predict the coupling of fields to an electrical monopole by using the method of moments. By this statistical approach, random fields are generated for each spatial point following ideal RC probabilistic distributions, and a spatial correlation is imposed on the random generator, in order to satisfy the spatial correlation existing in a RC. Mean values of EM quantities over one stirrer rotation are found as mean values over a set of random fields superposition.

A different approach is investigated in this work, which combines the plane waves integral model for RC and a Monte Carlo (MC) statistical approach. Drawing on Hill's plane wave model for a RC [6], the RC EM environment is recreated in this work by a superposition of a finite number of random plane waves. The statistical properties attributed to the fields by the stirrer rotation are taken into account by suitably modelling the probabilistic distribution of plane waves parameters: a random number generator is adopted to produce such parameters. Mean values for EM quantities are then found as mean values over a set of random plane waves superposition. This novel approach allows a simple simulation of the EM conditions of the ideal RC environment. It will be shown that resulting fields point statistics satisfy ideal RC statistics, and that fields spatial correlation intrinsic to this approach naturally reproduce RC spatial correlation. The interest of this approach lies in the application of the method to the prediction of EM quantities coupled to electrical devices. It will be shown how to predict mean values of EM coupled quantities inside a RC by disposing of a set of responses of the device to offending random plane waves. The applicability of the method is thus relied to the ability of predicting the response of the device to offending plane waves. For distributed devices, this method can be readily applied, since the theory of plane waves coupling to transmission lines is well established [7]. As an example, the case of induced current into the terminal load of a single wire over a ground plane will be investigated in this work.

The work is organised as follows. Section II proposes the fundamentals of the method: plane waves random properties assumptions and resulting fields probabilistic description. Section III is focused on the application of the method to predict coupled EM quantities into electrical devices, and discusses a "complete" and three "fast" MC methods. Finally, in Section IV such methods are applied to the prediction of the induced current flowing in the terminal load of a single wire transmission line exposed to RC fields.

II. PLANE WAVE MONTE CARLO METHOD FOR A REVERBERATION CHAMBER

Ideal properties of the working volume of a RC are modelled here by a superposition of random plane waves. A finite number of plane waves is considered and the integral representation contained in [6] is approximated by a finite sum of plane waves contributions. Repeated trials of a superposition of random contributions allow us the estimation of EM quantities mean values according to a MC approach.

The first step in performing a MC simulation of a physical process is to establish the probabilistic distributions of parameters that must be randomly generated. In this case, the parameters are the propagation direction, field polarisation, amplitude and phase of the contributing plane waves. In an isotropic environment, as in the case of the working volume of a RC, plane waves are supposed to have no preferred propagation direction and no preferred field polarisation. This means that uniform distributions are chosen for propagation direction angles and for the polarisation angle, over the solid angle and over 2π, respectively. Additionally, multiple scattering phenomena inside a RC result in the fact that the phase of plane waves has no preferred value; thus, a uniform distribution is chosen. Finally, constant amplitude is chosen for all plane waves, equal to $|E_0|$, for conveniently matching simulations with experiments. Probabilistic distributions for plane waves parameters are summarised in Table 1.

Table I – Plane waves probabilistic distributions

Parameter	*Distribution*
Propagation direction $\Omega(\theta,\varphi)$	$U[0,4\pi]$
Polarisation θ_P	$U[0,2\pi]$
Phase ϕ	$U[0,2\pi]$
Amplitude $\|E\|$	$\delta(\|E\|-\|E_0\|)$

In Table 1, *U* stands for uniform distribution.

According to the assumptions of Table 1, it is possible to analytically determine the mean value and variance for one rectangular component of the electric field resulting in one point of the space by the contribution of one random plane wave. For example, it can be shown that for the *z* component of the electric field in the origin of a Cartesian system, mean value and variance of real and imaginary parts are given by

$$mean(\mathrm{Re}\{E_{z0}\}) = mean(\mathrm{Im}\{E_{z0}\}) = 0 \qquad (1.a)$$

$$\mathrm{var}(\mathrm{Re}\{E_{z0}\}) = \mathrm{var}(\mathrm{Im}\{E_{z0}\}) = |E_0|^2/6 \qquad (1.b)$$

Then, by applying the central limit theorem (CLT), the probabilistic distribution of real and imaginary parts of the same field component given by the superposition of *n* random plane waves can be determined. The CLT states that for a great number of contributing plane waves, real and imaginary parts of the resulting field component are distributed as a normal distribution, whose mean value μ and variance σ are given in the following equation.

$$\mathrm{Re}\{E_{zRC}\}, \mathrm{Im}\{E_{zRC}\} \approx N\left(\mu = 0, \sigma^2 = n\cdot\frac{|E_0|^2}{6}\right) \qquad (2)$$

Equation (2) allows us to characterise fields in one spatial point of an ideal RC working volume. Finally, the isotropy and homogeneity properties of fields inside the RC working volume allow us to extend these results to any electric field rectangular component at any point inside the volume.

As a result, the distributions of the amplitudes of a generic field rectangular component E_i and of the total electric field E_{tot} at any point inside the chamber, can be shown to be in agreement with RC statistics contained in [6], [8] and [9]. Furthermore, as each plane wave contribution concerns the entire working volume, spatial correlation for fields is also in agreement with RC model contained in [6].

The interest of the proposed approach is that it is possible to relate mean values for fields inside a RC to the constant chosen plane waves amplitude $|E_0|$ and to the number *n* of plane waves considered. In fact, starting from equation (2) and indicating mean values in RC as $\langle\ \rangle$, it is possible to obtain:

$$\langle|E_{i,RC}|\rangle = |E_0|\sqrt{n}\sqrt{\pi/12} \qquad \langle|E_{i,RC}|^2\rangle = n|E_0|/3$$

$$\langle|E_{tot,RC}|\rangle = |E_0|\sqrt{n}\frac{15}{16}\sqrt{\frac{\pi}{3}} \qquad \langle|E_{tot,RC}|^2\rangle = n|E_0| \qquad (3)$$

Furthermore, the advantage of keeping $|E_0|$ constant is that it becomes a free parameter to be matched with measurements of the mean field level inside the chamber, thus allowing for comparisons between simulations and measurements.

III. MONTE CARLO "COMPLETE" AND "FAST" METHODS FOR EM COUPLED QUANTITIES MEAN VALUES PREDICTION

According to the approach formulated in Section II, MC predictions of EM quantities coupled to electrical devices in RC can be obtained as superposition of plane waves coupling contributions. In particular, if *x* is a generic coupled complex quantity (current or voltage), its mean value in RC can be computed as:

$$\langle x_{RC}\rangle = \mathrm{E}\left\{\sum_{i=1}^{n} x_{PW,i}\right\} \qquad (4)$$

where x_{RC} is the coupled quantity in RC, $x_{PW,i}$ is the coupled quantity corresponding to the *i-th* random plane wave, $\langle\ \rangle$ stands for mean value over a stirrer rotation, and E is the expected value. The MC "complete" method consists in estimating the expected value in (4) by taking the arithmetic mean value of several (let us say *m*) simulation trials of the sum in the right-hand term of (4). This means that $m \times n$ simulations are required. In this case, each of the *m* simulation results (i.e. each of the *m* sums of the right term of (4)) correspond to one position of a virtual stirrer. If plane waves parameters are chosen as described in Table 1, and plane waves amplitude is chosen according to equation (3), matching between simulation and measurements made in a real chamber can be obtained.

If we are able to numerically simulate the response of a device to one offending plane wave, i.e. $x_{PW,i}$, equation (4) can be used in a MC simulation method to predict mean value of the response in RC. The inconvenience of this method is in the large computing time required by the $m \times n$ simulations. The possibility of simplifying the general method of equation (4) is investigated in the following.

If we are looking for mean value of the received power, which is proportional to the squared magnitude of coupled current (voltage), (4) can be written as

$$\langle P_{RC} \rangle \propto \mathrm{E}\left\{ \left| \sum_{i=1}^{n} x_{PW,i} \right|^2 \right\} = n \cdot \mathrm{E}\left\{ \left| x_{PW,i} \right|^2 \right\} \quad (5)$$

where the expected value in the last term of (5) is taken with respect to the squared amplitude of a coupled quantity due to one random plane wave. It can be easily shown that the last equality in (5) is valid for any complex random quantity $x_{PW,i}$ that has zero mean value for the real and imaginary parts. This last assumption can be considered true for any linear electrical device.

Last equality in (5) gives thus a means to simplify the "complete" method of equation (4) by replacing the $m \times n$ simulations described above by simply *n* simulations (the expected value in the last term of (5) is estimated as mean arithmetic value over *n* plane waves contributions), and multiplying the result times the number of simulations *n*. We will call this procedure "fast1" MC method. An important physical implication of (5) is that it equals mean value in MSRC and mean values over plane wave incidence times *n*, provided that plane waves amplitude is chosen according to mean fields values inside the MSRC as in (3). The same result was obtained by a different approach (see equation (39) in [6]).

To obtain an equivalent result for mean value of induced current amplitude prediction, it is not sufficient to take the square root of (5). Two different solutions can be used to obtain an equivalent "fast" method. Such solutions are shown by (6) and their validity domains are discussed in the following.

$$\mathrm{E}\left\{ \left| \sum_{i=1}^{n} x_{PW,i} \right| \right\} = \sqrt{n} \cdot \mathrm{E}\left\{ \left| x_{PW,i} \right| \right\} \quad (6.a)$$

$$\mathrm{E}\left\{ \left| \sum_{i=1}^{n} x_{PW,i} \right| \right\} = \sqrt{n} \sqrt{\frac{\pi}{2} Var\left(\mathrm{Re}\left\{ x_{PW,i} \right\} \right)} \quad (6.b)$$

It can be shown [10] that (6.a) is valid only when real and imaginary parts of the quantity $x_{PW,i}$ are normally distributed with zero mean and equal variance, which is not true in a general case[1]. In cases where this assumption is verified, (6.a) equals mean value in RC and mean values over plane wave incidence times $\sqrt{n}$. Of more general validity is (6.b), which is valid for any random complex quantity $x_{PW,i}$ which has zero mean value and equal variance for real and imaginary parts.

In conclusion, four different MC methods are proposed. The "complete" method in (4) can be used to predict any EM quantity inside a RC (amplitude or squared amplitude); mean values over one stirrer rotation as well as values for single stirrer positions can be predicted by this method (this implies that also maximal values can be predicted). Three "fast" methods for the prediction of only mean values are also proposed. For squared amplitudes mean values predictions, "fast1" method in (5) can be used if the predicted quantity has zero mean value for real and imaginary parts over random plane waves. For amplitudes mean values predictions, "fast2" method in (6.a) can be used for quantities whose real and imaginary parts are normally distributed with zero mean and equal variance over random plane waves incidence; "fast3" method in (6.b) can be used for quantities whose real and imaginary parts have zero mean and equal variance over random plane waves incidence.

IV. MONTE CARLO PREDICTION OF THE COUPLED CURRENT FLOWING IN A TRANSMISSION LINE

The interest of the MC prediction method discussed in sections II and III lies into the ability of predicting EM coupled quantities to an electrical device excited by an offending plane wave. One case of interest is coupled current to multiconductor transmission lines, for which coupling theory is well established [7]. An application example is considered in this section, considering a single lossless transmission line over a ground plane.

The different MC methods discussed in Section III have been applied to the prediction of the induced current flowing in the terminal load of a single wire running over a ground plane when it is tested inside a RC. For

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[1] It is helpful to notice that normal distribution is required here for $x_{PW,i}$ and not for $\sum x_{PW,i}$ which is always normally distributed according to the CLT.

the simulation, each plane wave coupling contribution was computed by means of the conventional theory of fields coupling to transmission lines [7]. One further consideration must be made concerning computation of random plane waves contributions. According to physical properties of RCs, independent frequencies of excitation of the chamber correspond to independent exited modal structures, and, in a plane wave model, correspond to different plane waves patterns. This means that for a proper MC simulation, a different pattern of random plane waves should be generated for each independent frequency. However, as plane waves are random and independent, the solution of keeping the same random pattern for the entire frequency range can be adopted, thus reducing computational time.

In the following, a 50 cm long single wire running at a height of 3 cm above the chamber floor is considered both for measurement and simulation. Several experimental transmission line devices were tested in two different chambers [11], and experimental results compared with simulation results.

At first, the induced current was predicted according to the "complete" method of equation (4). The "complete" simulation was made by illuminating the line 144 times by packets of 20 random plane waves, and the mean value of one electric field rectangular component was measured inside the chamber and used to set the value for the simulated plane waves amplitude, according to equation (3). For the purpose of comparison, the mean value of the induced current in the line termination was calculated over 144 stirrer positions. Fig. 1 shows the comparison between predicted and measured mean values and maximal values of the induced current amplitude as a function of frequency; Fig. 2 shows cumulative density functions for prediction and measurement of current amplitude, both compared with an ideal χ_2 distribution.

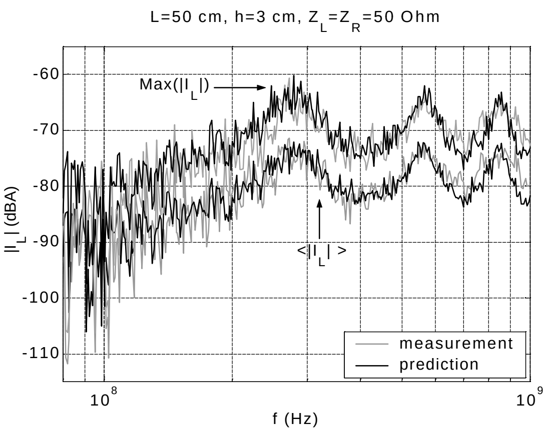

Figure 1: MC "complete" method: amplitude of induced current at the transmission line terminal load – mean current value as a function of frequency

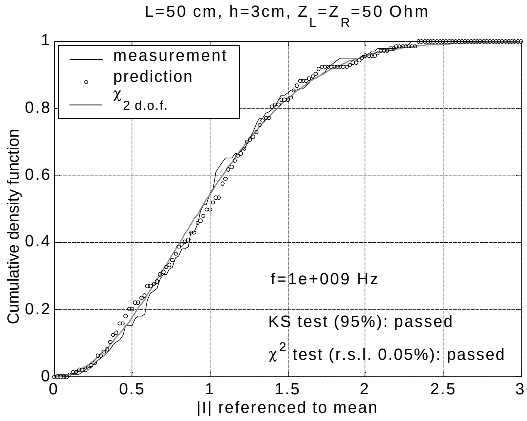

Figure 2: MC "complete" method: amplitude of induced current at the transmission line terminal load – cumulative density function at a single frequency

Results of Fig. 1 and 2 show a good agreement between prediction and measurement. Furthermore, Fig. 2 shows that the coupled current follows a χ_2 distribution, both for prediction and measurement results. Visual agreement for cumulative density function as well as statistical goodness of fit tests are passed.

Once validated the "complete" method by comparison with measurement, "fast" methods of equations (5) and (6) can be validated with respect to the "complete" method.

The same transmission line of the previous case was analysed with "fast" methods. This time, a constant plane wave amplitude was chosen as a function of frequency, since we are not attempting to compare simulation results to measurements. Plane waves amplitude was chosen equal to 1 V/m for the entire frequency range.

Results in Fig. 3 show the comparison between the "fast1" method for the prediction of mean value of squared current amplitude and the "complete" method, as in equation (5). Results for the "complete" method where obtained by estimating expected value of equation (4) as mean value over 1500 trials, where each simulation was carried out by superposing 20 plane waves contributions. Results for the "fast1" method where obtained by estimating expected value of equation (5) as mean value of 1500 trials. Simulation results obtained with the two methods are within the random simulation uncertainty and validate theory.

Results in Fig. 4 show the comparison between the "fast2" and "fast3" methods for the prediction of mean value of current amplitude and the "complete" method, as in equations (6.a) and (6.b). Simulations of 1500 times 20 plane waves contributions for the "complete" method and of 1500 plane waves contributions for the two "fast" methods were carried out.

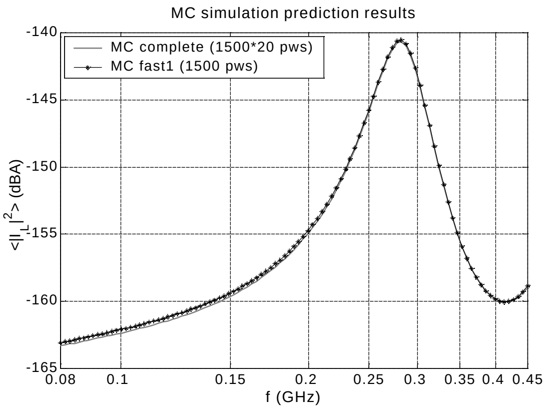

Figure 3: MC “complete” and “fast1” methods: prediction of the mean squared amplitude of coupled current

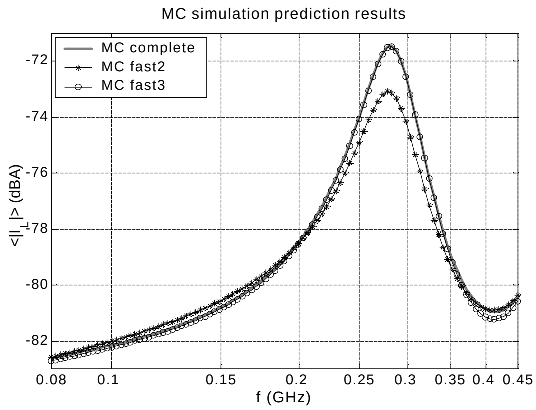

Figure 4: MC “complete”, “fast2” and “fast3” methods: prediction of the mean amplitude of coupled current. Differences between "fast2" and "fast3" methods are outlined in the text

Results show that the “fast3” method prediction according to eq. (6.b) matches the “complete” method prediction, while “fast2” method prediction according to eq. (6.a) doesn’t match the “complete” method prediction for all frequencies.

To evaluate the correctness of hypothesis discussed in Section III and laying at the basis of equations (6), induced current statistical properties, with respect to a random incident plane wave, were investigated by a numerical approach. Results of this analysis show that real and imaginary parts of coupled current have indeed zero mean value and equal variance, but the probabilistic distribution of real and imaginary parts are frequency dependent (see also [12]). Thus, “fast3” method of equation (6.b) is applicable, while “fast2” method of equation (6.a) is not generally applicable, except for frequencies where real and imaginary parts approach a Normal distribution. As shown in Fig 4, for frequencies far from resonance and anti-resonance of the line, where Normal distribution is better approached, the “fast2” results better match “complete” method results.

The uncertainty associated with simulation results was also estimated according to the classical statistical theory of inference on mean value and variance estimation. Uncertainty for simulation results of Fig. 4 are proposed in Fig. 5. Results are proposed as the total 95% confidence interval ($\Delta_{95\%}$) of mean estimated values, and must be interpreted in the following way: mean values results (Fig. 4) $\pm\Delta_{95\%}/2$ (Fig. 5) give the 95% confidence interval of simulation results.

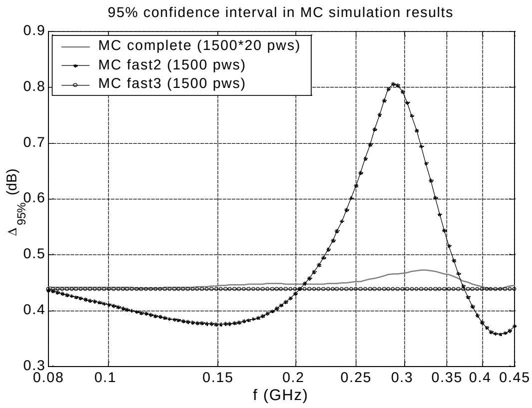

Figure 5: Uncertainty of MC “complete”, “fast2” and “fast3” methods (1500 trials of 20 plane waves superposition for “complete” method and 1500 plane waves contributions for “fast2” and “fast3”methods)

Uncertainty results in Fig. 5 are obtained with a large number of simulation trials. This was made to strongly reduce uncertainty and thus evidence as much as possible the differences in “fast2” and “fast3” method results in Fig. 4. Uncertainties for more realistic trials numbers are reported in Fig. 6, where “complete” method is applied with 50 trials of 20 plane waves contributions and “fast2” and “fast3” methods with 50 plane waves contributions.

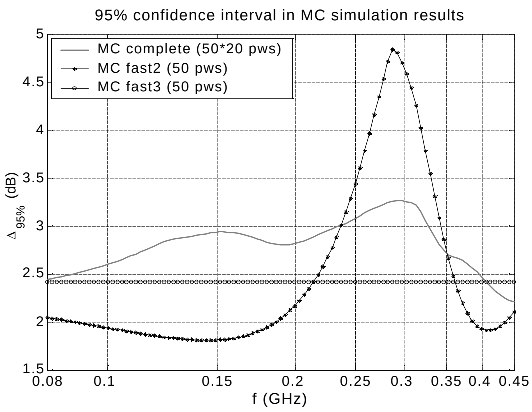

Figure 6: Uncertainty of MC “complete”, “fast2” and “fast3” methods (50 trials of 20 plane waves superposition for “complete” method and 50 plane waves contributions for “fast2” and “fast3” methods)

Results in Fig.6 evidence that acceptable uncertainty is obtained by “complete” and “fast3” methods, even for a reduced number of plane waves contributions.

V. CONCLUSIONS

A Monte Carlo simulation method based on a superposition of random plane waves has been proposed to represent reverberation chamber conditions. The generation of random plane waves parameters has been discussed and it has been shown that the resulting fields agree with the reverberation chambers fields statistics. Additionally it has been shown how to relate mean fields values measured in reverberation chamber and constant plane waves amplitude used in simulation. The proposed method can be used to predict electromagnetic coupling to electrical devices, provided that their response to an offending plane wave can be numerically computed; in particular the method is advantageous when this last computation can be obtained at a low cost. The method has been applied to predict the current induced into a single wire running over a ground plane inside a reverberation chamber. In this case transmission line theory was used to predict current induced by an offending plane wave. A good agreement was obtained between prediction and measurement results. The proposed method allows us the prediction both of mean and maximum values of coupled electromagnetic quantities over one stirrer rotation.

To reduce computational time, three faster methods have been proposed to predict only mean values over one stirrer rotation. Basing on simple statistical assumptions about real and imaginary parts of electromagnetic coupled quantities distribution, mean value for amplitude (current or voltage) and squared amplitude (power) of such quantities can be predicted by faster methods.

Uncertainty associated with Monte Carlo simulation results was also addressed in this work. Uncertainty decreases as the number of simulation trials increases, and it has been shown that, for the reasonable number of 50 plane waves contributions, the uncertainty is of the order of $\pm 1.5\,\text{dB}$, that is of the same order of measurement uncertainty inside a reverberation chamber.

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