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Infinitely Many Solutions for One-Dimensional Eigenvalue Problems for Variational Inequalities

Author's version

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Abstract. A non-symmetric perturbation of a symmetric variational inequality is considered. The existence of infinitely many solutions is proved.

1 Introduction

Let $\psi :]0,\pi[\to [0,+\infty]$ be a lower semicontinuous function and let

$$\mathbb{K} \,=\, \{u \in H^1_0(0,\pi) :\, |u| \leq \psi\}\,.$$

If $p :]0,\pi[\times\mathbb{R} \to \mathbb{R}$ is a Carathéodory function odd in the second variable, $\rho > 0$ and suitable qualitative conditions are satisfied, it is known that the problem

$$\begin{cases} (\lambda, u) \in \mathbb{R}\times\mathbb{K} \\ \int_0^\pi \,[DuD(v-u) \,+\, p(x,u)(v-u)]\,dx \,\geq\, \lambda \int_0^\pi\, u(v-u)\,dx \quad \forall v \in \mathbb{K} \\ \int_0^\pi\, u^2\,dx \,=\, \rho^2 \end{cases}$$

admits a sequence (λ_h, u_h) of solutions with $\lambda_h \to +\infty$. This has been proved, under different assumptions, in [4, 6, 13], even in the multi-dimensional case.

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In [8, 12] we have shown that, if we perturb the problem in a quite general nonsymmetric way, then the number of solutions of the perturbed problem goes to infinity, as the perturbation tends to disappear.

Here we want to treat a case in which the perturbed problem has still infinitely many solutions.

In the case of equations, results of this kind have been obtained in [1, 2, 3, 14, 15, 16]. The technique here is similar, from the topological point of view. To stress this aspect, we will work with the notion of "essential value", which is purely topological. On the contrary, from the differential point of view, the presence of the constraint $\mathbb{K}$ causes an irregularity which will be treated by the use of techniques of nonsmooth analysis. Here we will follow the approach of [7, 9].

We have restricted our attention to the one-dimensional case, because here the critical values of the unperturbed problem have a suitable growth. A similar result, in a more restrictive form, could have been proved also in two dimensions, but our argument does not work if the dimension is at least three.

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2 Essential values of continuous functionals

In this section we recall from [8, 12] some basic facts that will be useful in the following. Let X denote a metric space endowed with the metric d and $f : X \to \mathbb{R}$ a continuous function. If $b \in \overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty, +\infty\}$, let us set

$$f^b = \{u \in X : \, f(u) \leq b\} \,.$$

(2.1) Definition. Let $a, b \in \overline{\mathbb{R}}$ with $a \leq b$. The pair $\left(f^b, f^a\right)$ is said to be *trivial*, if for every neighbourhood $[\alpha', \alpha'']$ of a and $[\beta', \beta'']$ of b $\left(\alpha', \alpha'', \beta', \beta'' \in \overline{\mathbb{R}}\,, \alpha' \leq \beta'\right)$ there exists a continuous map $\mathcal{H} : f^{\beta'} \times [0,1] \to f^{\beta''}$ such that

$$\mathcal{H}(x, 0) = x \quad \forall x \in f^{\beta'},$$

$$\mathcal{H}\left(f^{\beta'} \times \{1\}\right) \subseteq f^{\alpha''},$$

$$\mathcal{H}\left(f^{\alpha'} \times [0,1]\right) \subseteq f^{\alpha''}.$$

(2.2) *Remark.* If α is a further number with $\alpha < \alpha'$, we can suppose, without loss of generality, that $\mathcal{H}(x,t) = x$ on $f^\alpha \times [0,1]$. Actually, it is sufficient to substitute $\mathcal{H}(x,t)$ with $\mathcal{H}(x, t\vartheta(x))$, where $\vartheta : f^{\beta'} \to [0,1]$ is a continuous function with $\vartheta(x) = 0$ for $f(x) \leq \alpha$ and $\vartheta(x) = 1$ for $f(x) \geq \alpha'$.

(2.3) Definition. A real number c is said to be *an essential value* of f, if for every $\varepsilon > 0$ there exist $a, b \in]c-\varepsilon, c+\varepsilon[$ with $a < b$ such that the pair $\left(f^b, f^a\right)$ is not trivial.

(2.4) Theorem. *Let $a, b \in \overline{\mathbb{R}}$ with $a < b$. Let us assume that f has no essential value in $]a, b[$.*

Then the pair $\left(f^b, f^a\right)$ is trivial.

Now let us recall a notion from [7, 9].

(2.5) Definition. For every $u \in X$ let us denote by $|df|(u)$ the supremum of the σ's in $[0, +\infty[$ such that there exist $\delta > 0$ and a continuous map $\mathcal{H} : B_\delta(u) \times [0, \delta] \to X$ with

$$d(\mathcal{H}(v,t), v) \leq t\,,$$

$$f(\mathcal{H}(v,t)) \leq f(v) - \sigma t\,.$$

The extended real number $|df|(u)$ is called *the weak slope* of f at u.

If X is a Finsler manifold of class C^1 and f a function of class C^1, it turns out that $|df|(u) = \|df(u)\|$ for every $u \in X$.

Let us point out that the above notion has been independently introduced also in [10].

(2.6) Definition. An element $u \in X$ is said to be *a critical point* of f, if $|df|(u) = 0$. A real number c is said to be *a critical value* of f, if there exists a critical point $u \in X$ of f such that $f(u) = c$. Otherwise c is said to be *a regular value* of f.

(2.7) Definition. Let c be a real number. The function f is said to satisfy *the Palais - Smale condition at level c* ($(PS)_c$ for short), if every sequence (u_h) in X with $|df|(u_h) \to 0$ and $f(u_h) \to c$ admits a subsequence (u_{h_k}) converging in X.

(2.8) Theorem. *Let c be an essential value of f. Let us assume that X is complete and that f satisfies $(PS)_c$.*

Then c is a critical value of f.

3 Infinitely many solutions for non-symmetric variational inequalities

Let $p :]0, \pi[\times \mathbb{R} \to \mathbb{R}$ be a Carathéodory function such that

$$p(x, -s) \;=\; -p(x, s)\,,$$

$$s\, p(x, s) \;\geq\; 0\,,$$

$$\forall t > 0 : \qquad \sup_{|s| \leq t} |p(x, s)| \in L^1(0, \pi)\,;$$

let $\psi :]0, \pi[\to [0, +\infty]$ be a lower semicontinuous function and let $\rho > 0$ with

$$\rho^2 \;<\; \int_0^\pi \psi^2\, dx\,.$$

We start from the nonlinear eigenvalue problem

$$\begin{cases} (\lambda, u) \in \mathbb{R} \times \mathbb{K} \\ \int_0^\pi [DuD(v-u) \,+\, p(x,u)(v-u)]\; dx \;\geq\; \lambda \int_0^\pi u(v-u)\, dx \quad \forall v \in \mathbb{K} \\ \int_0^\pi u^2\, dx \;=\; \rho^2 \end{cases} \tag{3.1}$$

where

$$\mathbb{K} = \left\{ u \in H_0^1(0, \pi) :\; -\psi(x) \leq \tilde{u}(x) \leq \psi(x) \;\forall x \in]0, \pi[\right\}$$

and $\tilde{u}$ is the continuous representative of u. It is readily seen that (3.1) possesses a symmetry. In fact, if (λ, u) is a solution of (3.1), also $(\lambda, -u)$ is a solution of (3.1).

We want to study a perturbation of (3.1) of the form

$$\begin{cases} (\lambda, u) \in \mathbb{R} \times \mathbb{K} \\ \int_0^\pi [DuD(v-u) \,+\, (p(x,u) + q(x,u))(v-u)]\; dx + \\ +\, <\mu, v-u> \geq\; \lambda \int_0^\pi u(v-u)\, dx \quad \forall v \in \mathbb{K} \\ \int_0^\pi u^2\, dx \;=\; \rho^2 \end{cases} \tag{3.2}$$

where $q :]0,\pi[\times\mathbb{R} \to \mathbb{R}$ is a Carathéodory function and $\mu \in H^{-1}(0,\pi)$. We assume that

$$|q(x,s)| \,\leq\, a_1(x) + b|s|^{\vartheta}$$

with $a_1 \in L^1(0,\pi)$, $b \in \mathbb{R}$ and $0 < \vartheta < 3$.

We want to show that problem (3.2) has infinitely many solutions.

Problems (3.1) and (3.2) have a variational structure. Let us set

$$S_\rho \,=\, \left\{ u \in L^2(0,\pi) : \int_\Omega u^2\, dx \,=\, \rho^2 \right\}$$

and let us define $f, g : \mathbb{K} \cap S_\rho \to \mathbb{R}$ by

$$f(u) \,=\, \frac{1}{2}\int_0^\pi |Du|^2\, dx \,+\, \int_0^\pi P(x,u)\, dx\,,$$

$$g(u) \,=\, \frac{1}{2}\int_0^\pi |Du|^2\, dx \,+\, \int_0^\pi P(x,u)\, dx \,+\, \int_0^\pi Q(x,u)\, dx + <\mu, u>\,,$$

where $P(x,s) = \int_0^s p(x,t)\, dt$, $Q(x,s) = \int_0^s q(x,t)\, dt$. In the following, the set $\mathbb{K} \cap S_\rho$ will be endowed with the H^1_0-metric. We will denote by $\|\cdot\|_r$ the norm of $L^r(0,\pi)$ and by $\|\cdot\|_{-1,2}$ the norm of $H^{-1}(0,\pi)$.

Let us recall a definition from [5].

(3.3) Definition. Let C be a convex subset of a Banach space X, let M be a hypersurface in X of class C^1, let $u \in C \cap M$ and let $\nu(u) \in X'$ be a unit normal vector to M at u. The sets C and M are said to be *tangent at* u, if we have either

$$<\nu(u), v - u> \leq 0 \qquad \forall v \in C$$

or

$$<\nu(u), v - u> \geq 0 \qquad \forall v \in C\,,$$

where $< \cdot, \cdot >$ is the pairing between X' and X.

The sets C and M are said to be *tangent*, if they are tangent at some point of $C \cap M$.

Let us recall a particular case of a characterization given in [8].

(3.4) Theorem. *The following facts hold:*

a) *given* $u \in \mathbb{K} \cap S_\rho$, *the sets* $\mathbb{K}$ *and* S_ρ *are tangent at* u, *if and only if*

$$\forall x \in]0, \pi[: \qquad \tilde{u}(x) \neq 0 \Longrightarrow |\tilde{u}(x)| = \psi(x) ;$$

b) *the sets* $\mathbb{K}$ *and* S_ρ *are tangent, if and only if there exists an open subset* E *of* $]0, \pi[$ *such that the function* $\psi \chi_E$ *is continuous and belongs to* $H_0^1(0, \pi) \cap S_\rho$.

(3.5) Theorem. *Let us assume that* $\mathbb{K}$ *and* S_ρ *are not tangent. For every* $h \geq 1$ *let us set*

$$c_h = \inf_{C \in \Gamma_h} \max_{u \in C} f(u) ,$$

where Γ_h *is the family of compact subsets of* $\mathbb{K} \cap S_\rho$ *of the form* $\varphi(S^{h-1})$ *with* $\varphi : S^{h-1} \to \mathbb{K} \cap S_\rho$ *continuous and odd.*

Then the following facts hold:

a) $\mathbb{K} \cap S_\rho$ *is contractible in itself;*

b) *for every* $h \geq 1$ *we have* $\Gamma_h \neq \emptyset$ *and* $c_h \geq \frac{1}{2} \rho^2 h^2$.

Proof. Property *a)* is proved in [8]. By means of [11, Lemma VI.4.5] it is then standard to deduce that $\Gamma_h \neq \emptyset$ for every $h \geq 1$. Finally, let $f_0 : H_0^1(0, \pi) \cap S_\rho \to \mathbb{R}$ be defined by

$$f_0(u) = \frac{1}{2} \int_0^\pi |Du|^2 \, dx .$$

If c_h' is defined as c_h, with $\mathbb{K} \cap S_\rho$ substituted by $H_0^1(0, \pi) \cap S_\rho$ and f substituted by f_0, it is well known (see [11]) that $c_h' = \frac{1}{2} \rho^2 h^2$. Since $\mathbb{K} \cap S_\rho \subseteq H_0^1(0, \pi) \cap S_\rho$ and $f(u) \geq f_0(u)$, it follows that $c_h \geq c_h'$, whence the assertion. ■

(3.6) Lemma. *Let* (d_h) *be a sequence of positive numbers and let* $\gamma \in]0, 1[$. *Let us assume that there exist* $h_0 \in \mathbb{N}$ *and* $c > 0$ *such that*

$$\forall h \geq h_0 : \qquad 0 \leq d_{h+1} - d_h \leq c d_h^\gamma .$$

Then there exists $h_1 \in \mathbb{N}$ *such that*

$$\forall h \geq h_1 : \qquad d_h \leq (3c)^{\frac{1}{1-\gamma}} h^{\frac{1}{1-\gamma}} .$$

Proof. Let us set $\delta_h = h^{-\frac{1}{1-\gamma}} d_h$. In [2, Lemma (5.3)] it is shown that the sequence (δ_h) is bounded. Let us consider two cases. If for every $h \geq h_0$ we have

$$\delta_{h+1} - \delta_h < -c\delta_h^\gamma h^{-1} ,$$

then (δ_h) goes to 0 and the assertion follows. Otherwise there exists $h_1 \geq h_0$ such that

$$\delta_{h_1+1} - \delta_{h_1} \geq -c\delta_{h_1}^\gamma h_1^{-1} .$$

Arguing as in the proof of [2, Lemma (5.3)], we deduce that

$$\delta_{h_1} \leq (3c)^{\frac{1}{1-\gamma}} .$$

Hence, from [2, Lemma (5.3)] it follows that for every $h \geq h_1$

$$\delta_h \leq \max\{\delta_{h_1}, [c(1-\gamma)]^{\frac{1}{1-\gamma}}\} \ \leq \ (3c)^{\frac{1}{1-\gamma}} ,$$

and the assertion follows. ∎

(3.7) Lemma. *For every $\sigma > 0$ there exists $C(\sigma) > 0$ such that*

$$\left| \int_0^\pi Q(x,u)\, dx + <\mu, u> \right| \leq \sigma \|Du\|_2 + C(\sigma)$$

whenever $u \in H_0^1(0,\pi) \cap S_\rho$.

Proof. Let $a_1 = a_{1,1} + a_{1,2}$ and $\mu = \mu_1 + \mu_2$ with $a_{1,1} \in L^1(0,\pi)$, $\|a_{1,1}\|_1 \leq \varepsilon$, $a_{1,2} \in L^2(0,\pi)$, $\mu_1 \in H^{-1}(0,\pi)$, $\|\mu_1\|_{-1,2} \leq \varepsilon$, $\mu_2 \in L^2(0,\pi)$.

Then we have

$$\left| \int_0^\pi Q(x,u)\, dx + <\mu, u> \right| \leq$$

$$\leq \|a_{1,1}\|_1 \|u\|_\infty + \|a_{1,2}\|_2 \|u\|_2 + \frac{b}{\vartheta+1} \|u\|_{\vartheta+1}^{\vartheta+1} + C_1 \|\mu_1\|_{-1,2} \|Du\|_2 + \|\mu_2\|_2 \|u\|_2 \leq$$

$$\leq C_2 \varepsilon \|Du\|_2 + (\|a_{1,2}\|_2 + \|\mu_2\|_2)\rho + C_3 \|u\|_2^{\frac{\vartheta+3}{2}} \|Du\|_2^{\frac{\vartheta-1}{2}} \leq C_4 \varepsilon \|Du\|_2 + C_5(\varepsilon) .$$

A suitable choise of ε then gives the assertion. ∎

(3.8) Theorem. *Let us assume that $\mathbb{K}$ and S_ρ are not tangent.*

Then the functional g admits a sequence (d_h) of essential values with $d_h \to +\infty$.

Proof. First of all, let us show that for every $\sigma > 0$ there exists $C(\sigma) > 0$ such that for every $u \in \mathbb{K} \cap S_\rho$ we have

$$f(u) \leq b \Longrightarrow g(u) \leq b + \sigma\sqrt{b} + C(\sigma)\,, \tag{3.9}$$

$$g(u) \leq b \Longrightarrow f(u) \leq b + \sigma\sqrt{b} + C(\sigma)\,. \tag{3.10}$$

According to the previous Lemma, there exists $C_1(\sigma) > 0$ such that

$$\forall u \in H_0^1(0,\pi) \cap S_\rho : \qquad \left| \int_0^\pi Q(x,u)\,dx + \ < \mu, u > \right| \leq \frac{\sigma}{2}\|Du\|_2 \ + C_1(\sigma)\,.$$

If $f(u) \leq b$, it follows $\|Du\|_2 \leq \sqrt{2b}$, hence

$$g(u) \leq b + \frac{\sqrt{2}}{2}\sigma\sqrt{b} + C_1(\sigma) \leq b + \sigma\sqrt{b} + C_1(\sigma)\,.$$

On the other hand, if $g(u) \leq b$, we have

$$b \geq \frac{1}{2}\|Du\|_2^2 - \frac{\sigma}{2}\|Du\|_2 - C_1(\sigma) \geq \frac{1}{4}\|Du\|_2^2 - C_2(\sigma)\,,$$

hence

$$\|Du\|_2 \leq 2\sqrt{b + C_2(\sigma)} \leq 2\sqrt{b} + 2\sqrt{C_2(\sigma)}\,.$$

It follows

$$f(u) \leq b + \sigma\sqrt{b} + \sigma\sqrt{C_2(\sigma)} + C_1(\sigma) = b + \sigma\sqrt{b} + C_3(\sigma)\,.$$

Therefore (3.9) and (3.10) are proved.

Let $\sigma > 0$ be such that $162\sigma^2 < \rho^2$ and let $c_h \in \mathbb{R}$ be defined as in Theorem (3.5) . We claim that there exists a sequence $h_k \to +\infty$ such that

$$\forall k \in \mathbb{N} : \qquad c_{h_k} + \sigma\sqrt{c_{h_k}} + C(\sigma) + \sigma\sqrt{c_{h_k} + \sigma\sqrt{c_{h_k}} + C(\sigma)} + C(\sigma) < c_{h_k+1}\,. \tag{3.11}$$

Let us argue by contradiction. Then, since $c_h \to +\infty$, there exists $h_0 \in \mathbb{N}$ such that

$$\forall h \geq h_0 : \qquad c_{h+1} - c_h \leq 3\sigma\sqrt{c_h}\,.$$

By Lemma (3.6) there exists $h_1 \in \mathbb{N}$ such that for every $h \geq h_1$ we have $c_h \leq 81\sigma^2 h^2 < \frac{1}{2}\rho^2 h^2$. This inequality contradicts Theorem (3.5) , so that (3.11) is proved.

By Theorem (2.4), it is sufficient to show that for every $M > 0$ there exists $a \geq M$ such that the pair $(\mathbb{K} \cap S_\rho, g^a)$ is not trivial. By contradiction, assume that there exists $M > 0$ such that for every $a \geq M$ the pair $(\mathbb{K} \cap S_\rho, g^a)$ is trivial. Let $h \in \mathbb{N}$ be such that

$$c_h \geq M\,, \qquad c_h + \sigma\sqrt{c_h} + C(\sigma) + \sigma\sqrt{c_h + \sigma\sqrt{c_h} + C(\sigma)} + C(\sigma) < c_{h+1}$$

and let $a \in \mathbb{R}$ be such that

$$c_h + \sigma\sqrt{c_h} + C(\sigma) < a\,, \qquad a + \sigma\sqrt{a} + C(\sigma) < c_{h+1}\,.$$

Let $d', \alpha, \alpha', \alpha'', d'' \in \mathbb{R}$ be such that $c_h < d' < \alpha < \alpha' < a < \alpha'' < d'' < c_{h+1}$ with

$$d' + \sigma\sqrt{d'} + C(\sigma) \leq \alpha\,, \qquad \alpha'' + \sigma\sqrt{\alpha''} + C(\sigma) \leq d''\,.$$

It follows that $f^{d'} \subseteq g^{\alpha}$ and $g^{\alpha''} \subseteq f^{d''}$. Since $c_h < d'$, there exists $\varphi : S^{h-1} \to \mathbb{K} \cap S_\rho$ continuous and odd with $\varphi(S^{h-1}) \subseteq f^{d'}$ and there exists a homotopy $\mathcal{H} : S^{h-1} \times [0,1] \to \mathbb{K} \cap S_\rho$ between φ and a constant map. Let $\beta' \in [\alpha', +\infty[$ be such that $\beta' \geq \max\{g(\mathcal{H}(x,t)) : \, x \in S^{h-1}, t \in [0,1]\}$. Since the pair $(\mathbb{K} \cap S_\rho, g^a)$ is trivial, there exists a continuous map $\eta : g^{\beta'} \times [0,1] \to \mathbb{K} \cap S_\rho$ such that

$$\eta(x,0) = x \quad \forall x \in g^{\beta'}\,,$$

$$\eta\left(g^{\beta'} \times \{1\}\right) \subseteq g^{\alpha''}\,,$$

$$\eta\left(g^{\alpha'} \times [0,1]\right) \subseteq g^{\alpha''}$$

and

$$\eta(x,t) = x \text{ on } g^{\alpha} \times [0,1]\,.$$

Let us define $\mathcal{K} : S^{h-1} \times [0,1] \to f^{d''}$ by $\mathcal{K}(x,t) = \eta(\mathcal{H}(x,t),1)$. Then $\mathcal{K}$ is a homotopy between $\varphi : S^{h-1} \to f^{d''}$ and a constant map. By [11, Lemma VI.4.5] there exists $\psi : S^h \to f^{d''}$ continuous and odd. This is absurd, as $d'' < c_{h+1}$. ■

(3.12) THEOREM. *Let us assume that* $\mathbb{K}$ *and* S_ρ *are not tangent. Then the following facts hold:*

a) the functional g *is continuous;*

b) *for every* $u \in \mathbb{K} \cap S_\rho$ *there exist* $\lambda \in \mathbb{R}$ *and* $\eta \in H^{-1}(0,\pi)$ *such that* $\|\eta\|_{-1,2} = |dg|(u)$ *and*

$$\int_0^\pi [DuD(v-u) + p(x,u)(v-u) + q(x,u)(v-u)]\, dx + < \mu, v-u > \geq$$

$$\geq \lambda \int_0^\pi u(v-u)\, dx + < \eta, v-u > \qquad \forall v \in \mathbb{K}\,;$$

c) *the functional* g *satisfies* $(PS)_c$ *for every* $c \in \mathbb{R}$.

Proof. Up to minor modifications, the proof is the same of [8]. ■

Now we can prove our main result.

(3.13) Theorem. *Let us assume that* $\mathbb{K}$ *and* S_ρ *are not tangent.*
Then problem (3.2) *admits a sequence of solutions* (λ_h, u_h) *with* $\lambda_h \to +\infty$.

Proof. By Theorem (3.8) the functional g admits a sequence (d_h) of essential values with $d_h \to +\infty$. By Theorem (3.12) the functional g satisfies the $(PS)_c$ condition for every $c \in \mathbb{R}$. Therefore by Theorem (2.8) each essential value d_h is a critical value of g. Hence there exists a sequence (u_h) of critical points of g with $g(u_h) \to +\infty$ and by Theorem (3.12) there exists a sequence (λ_h) such that (λ_h, u_h) is a solution of (3.2) . We have

$$\lim_h \int_0^\pi |Du_h|^2 dx \;=\; +\infty$$

and, choosing $v = 0$ in (3.2) ,

$$\lambda_h \rho^2 \geq \int_0^\pi [\,|Du_h|^2 + u_h p(x,u_h)\,]\, dx \;+\; \int_0^\pi u_h q(x,u_h)\, dx + \; < \mu, u_h > \; .$$

Since $sp(x,s) \geq 0$ and

$$\left| \int_0^\pi u_h q(x,u_h)\, dx \right| \leq \|a_1\|_1 \|u_h\|_\infty + b\|u_h\|_{\vartheta+1}^{\vartheta+1} \leq$$

$$\leq \|a_1\|_1 \|u_h\|_\infty + C\|u_h\|_2^{\frac{\vartheta+3}{2}} \|Du_h\|_2^{\frac{\vartheta-1}{2}} \;,$$

it follows that $\lambda_h \to +\infty$. ■

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