Asymptotic defectiveness of manufacturing plants: an estimate based on process learning curves

Asymptotic defectiveness of manufacturing plants: an estimate based on process learning curves

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The paper describes a method for a preliminary estimation of asymptotic defectiveness of a manufacturing plant based on the prediction of its learning curve estimated during a *p*-chart setting up. The proposed approach provides process managers with the possibility of estimating the asymptotic variability of the process and the period of revision of *p*-chart control limits. An application of the method is also provided.

1. Introduction

A manufacturing process can be described as a framework able to convert input raw material in finished or in partly finished products. Mechanisms of transformation, specific of each context, are ruled by a sequence of organized activities that involve an interaction among operators, machinery and production equipment. The combined effect of these elements together with the inner and outer influence quantities are the causes of variability.

Control charts are a proven technique to provide diagnostic information and to monitor the variability of a process over time. They are used according to two steps: control chart setting up and manufacturing process monitoring (Duncan 1986, Montgomery 1996). The phase of setting up requires the detection of 'assignable' causes, the estimation of the process natural tolerance and the process control limits.

The accepted hypothesis is that without assignable causes, the process maintains its performances over time. Assignable causes may determine an average shifting or a change of the process dispersion. They can be divided into two categories: positive and negative. Positive are those that generate an increase of process variability; negative the causes that operate on the opposite ('favourable assignable cause'; Duncan 1974). Wear and tear phenomena are typical examples of positive assignable causes. Trends or shifts can appear as positive or negative causes.

Each cause has a proper dynamic. However, the process manager observes a global combined effect, having no possibility of discriminating a single contribution. During the life of a process, we assist to a continuous 'overlap' of the two types of causes. The prevalence of positive or negative causes is detectable by means of statistical control charts.

Referring to a generic process, after removing initial out-of-control causes, we observe a gradual reduction of variability over time due to the 'learning' mechanism. The 'physiological' variability shown by a process in its early life period is not the same as that manifested after a learning period on the field: the so called 'asymptotic' variability. The learning phenomenon occurs since the operators' *knowledge* about the process flow, the production equipment and the materials become more thorough over time, allowing a more efficient allocation of production factors. In general, it is not true that the asymptotic variability or defectiveness of a manufacturing process is zero. The variability reduction depends upon the adaptability of the entire organization to changing conditions of the process (Cherrington *et al.* 1987, Dada and Marcellus 1994, Franceschini and Rossetto 1995, 1998, Li and Rajagopalan 1998). However, the mechanism has not the same intensity over time. After a preliminary phase characterized by a relative high learning, we assist to a progressive attenuation. 'It is practically certain that, given appropriate training and empowerment, quality teams can discover better ways to do things' (Box and Luceño 1997: 19).

When the process achieves its asymptotic defectiveness it cannot further improve its performance. It is in the condition of maximum 'efficiency'. Variability reduction is the main reason that drives a process manager to revise, after a certain period, the process control limits.

Referring, for example, to a generic manufacturing process, what is the asymptotic fraction of non-conforming units (defectiveness) that the process will produce in the best conditions? The problem is particularly important since the estimation of asymptotic performances can help to better address process resources. By mean of these results, for example, we could decide to redesign or strengthen some specific subsystems or parts of a process.

The paper presents a method for a preliminary estimation of asymptotic defectiveness of a manufacturing plant, based on the prediction of its learning curve, and information collected during a p-chart setting up. Practical results are finally shown on a real example taken from the literature.

2. Method

Usually, the implementation of a control chart follows two steps.

- Phase 1: control chart setting up.
- Phase 2: control limits verification and process monitoring.

The revision of control limits becomes necessary if there are margins to improve process performances. The period for a revision of process control limits is not *a priori* fixed; it is usually decided upon the trend of the process over time (Montgomery 1996). The process 'photographs' provided by the two phases of chart implementation may be used to give a preliminary estimation of the asymptotic defectiveness of a manufacturing plant.

With reference, for example, to a generic process managed by a p-chart, we propose a method for estimating the 'asymptotic fraction of non-conforming units' and the time required to achieve it.

The general assumption is that the *learning mechanism*, which can determine a process improvement, follows some evolutionary laws that are not dependent by the specific application context. Learning curves provides a means to observe and track that improvement (Adler and Klark 1991, Kantor and Zangwill 1991, Mukherjee *et al.* 1998).

The concept of learning curve has been extensively used by economists, management scientist and engineers in analysing production processes. The main area of investigation have been the empirical measurement of learning curve, the economic

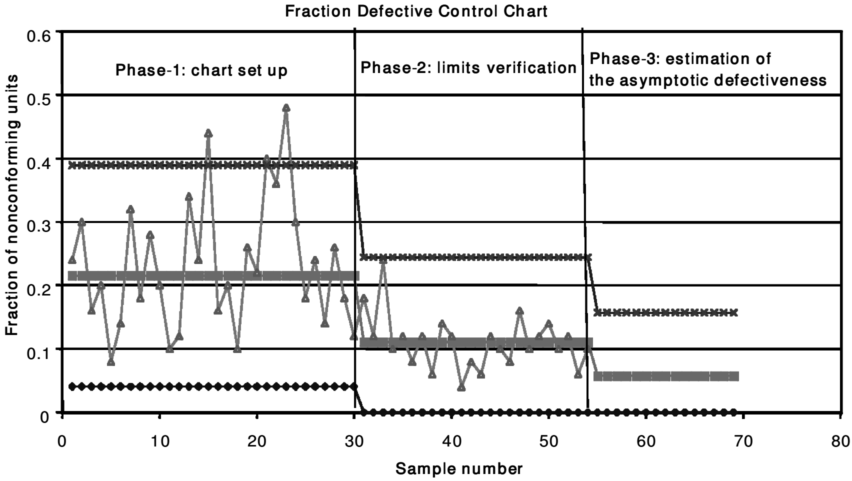

Figure 1. Fraction of non-conforming units control chart for samples of $n = 50$ elements. Control limits were calculated in the chart setting-up phase (phase 1) and control limits verification (phase 2) (Montgomery 1996). The third phase concerns the estimation of the 'asymptotic' control limits determined by the process learning curve.

implication of this phenomenon and its use in improving managerial decisions. For a detailed survey, see Venezia (1985) and Muth (1986).

A literature survey shows a wide variety of studies about particular aspects of learning curves: the effect of *prior experience* (Lippert 1976, Cherrington *et al.* 1987), the *relearning mechanism* (Bailey and McIntyre 1997), the *setting of performance standards* for productivity improvements achieved during the learning stage (Cherrington *et al.* 1987), etc.

The two most common models of learning curves over time are the following:

$$\text{Power law model: } p = \alpha t^{-\beta} + \gamma + \varepsilon \tag{1}$$

$$\text{Exponential model: } p = \gamma + (p_0 - G)e^{-t/\tau} + \varepsilon, \tag{2}$$

where p is a general average learning metric (e.g. the average fraction of non-conforming units in a manufacturing process); α is the defectiveness (fraction of non-conforming units) for the first learning cycle; β is the rate of learning; γ is the asymptotic fraction of non-conforming units; p_0 is the initial fraction of non-conforming units; τ is the learning curve time constant; and ε is the random error term ($\varepsilon = NID(0, \sigma^2)$).

The choice of a specific learning model is carried out on the basis of the application context (Muth 1986, Schneiderman 1988).

Zangwill and Kantor (1998) introduced a *unifying scheme* of the various models. They presented five postulates that underlie certain types of industrial learning and give rise to a differential equation, which describes that learning. With this interpretation, all learning models become parametric solutions of the Volterra–Lotka differential equation.

With the aim of providing a 'preliminary estimate' of the asymptotic defectiveness of a manufacturing plant, we build a process learning model by means of information gathered during phase 1 (p-chart setting up) and phase 2 (verification of control limits). If $(\bar{t}_1, \bar{p}_1)$ and $(\bar{t}_2, \bar{p}_2)$ are respectively the coordinates of the averages of the fraction non-conforming over time related to the two phases, and $\bar{p} = (\alpha/t) + \gamma + \varepsilon$ a simplified version of the power law model (a), with $\beta = 1$, we may obtain a preliminary estimation of the learning process parameters by the following relationships:

$$a = \frac{\bar{p}_1 - \bar{p}_2}{1/\bar{t}_1 - 1/\bar{t}_2} \tag{4}$$

$$c = \bar{p}_1 - a/\bar{t}_1, \tag{5}$$

where $\bar{p}_1 = \sum_{i=1}^{k} p_i/n$ and $\bar{p}_2 = \sum_{i=1}^{m} p_i/m$ are the averages of fraction non-conforming; $\bar{t}_1$ and $\bar{t}_2$ are the average verification times; and k and m are the number of analysed points for each phase.

It can be shown that a and c are two unbiased estimators of α and γ: $E(a) = \alpha$; $E(c) = \gamma$.

As regards the variance, assuming statistically independent fractions of non-conforming units p_i, we can show that:

$$\sigma_a^2 = \left(\frac{\bar{t}_2 \cdot \bar{t}_1}{\bar{t}_2 - \bar{t}_1}\right)^2 (\sigma_{\bar{p}_1}^2 + \sigma_{\bar{p}_2}^2); \; \sigma_c^2 = \left(\frac{\bar{t}_2}{\bar{t}_2 - \bar{t}_1}\right)^2 \sigma_{\bar{p}_2}^2 + \left(\frac{\bar{t}_1}{\bar{t}_2 - \bar{t}_1}\right)^2 \sigma_{\bar{p}_1}^2. \tag{6}$$

Defined a percentage distance h from the asymptotic target, we can determine the time t^* to its achievement. Substituting this value in the learning model, we find:

$$\gamma + \frac{h\gamma}{100} = \frac{\alpha}{t^*} + \gamma \text{ and therefore } t^* = \frac{\alpha}{h\gamma} \cdot 100. \tag{7}$$

As regards the variance of t^*, we have:

$$\sigma_{t^*}^2 = \left(\frac{1}{h\gamma}\right)^2 \sigma_a^2 + \left(\frac{\alpha}{h\gamma^2}\right)^2 \sigma_c^2. \tag{8}$$

It must be observed that an error in the learning curve estimation may have a far bigger impact in production planning than error in predicting a demand. For that reason, in order to identify model behavior for small variation of input parameters, it could be helpful to perform a quick *sensitivity analysis*.

Differentiating the simplified version of the power law model (a), with $\beta = 1$, we obtain:

$$d\bar{p} = \frac{\partial \bar{p}}{\partial \alpha} \cdot d\alpha + \frac{\partial \bar{p}}{\partial \gamma} \cdot d\gamma = \frac{d\alpha}{t} + d\gamma. \tag{9}$$

This expression shows that the variation of defectiveness is influenced by a proportional term (related to parameter α) and a constant term (related to parameter γ).

Assuming a statistical independence between the two parameters, the range estimation (with $1 - \alpha$ two-sided confidence limits) of the predicted defectiveness is:

$$\bar{p}_{(\text{est})} \pm z_{\alpha/2} \cdot \sqrt{\left(\frac{\partial \bar{p}}{\partial a}\right)^2 \cdot \sigma_a^2 + \left(\frac{\partial \bar{p}}{\partial c}\right)^2 \cdot \sigma_c^2} = \bar{p}_{(\text{est})} \pm z_{\alpha/2} \cdot \sqrt{\frac{\sigma_a^2}{t^2} + \sigma_c^2}, \tag{10}$$

where

$$\bar{p}_{(\text{est})} = \frac{a}{t} + c. \tag{11}$$

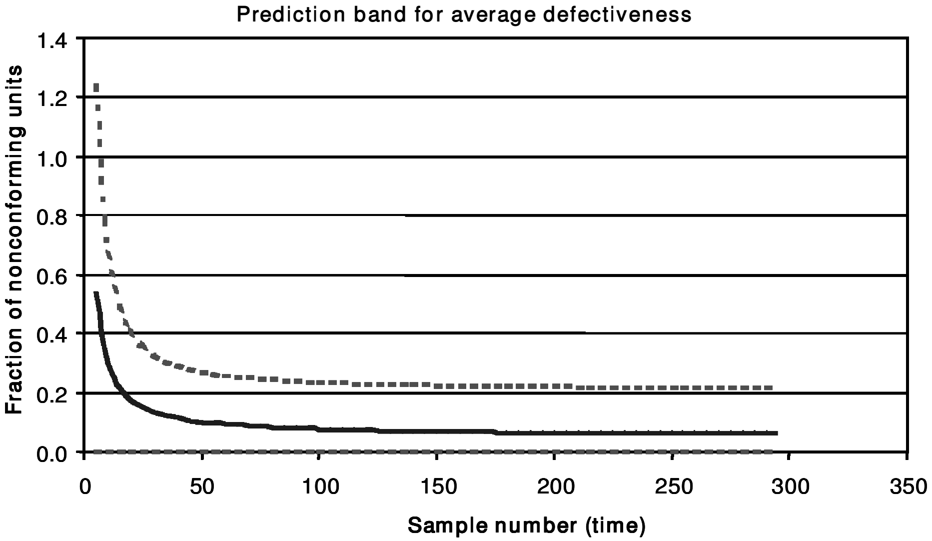

Figure 2. Prediction band (dotted lines) for the fraction of non-conforming units of the process considered in figure 1.

This result leads to the definition of a prediction band for the average defectiveness learning curve. The band amplitude increases with a and c variances and decreases with time (figure 2).

3. Example of application

Consider a process of frozen orange juice concentrate packing in 6-oz cardboard cans. The cans are formed on a machine by spinning them from a cardboard stock and attaching a metal bottom panel (Montgomery 1996).

By inspection of a can, we can determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. We wish to set up a control chart to monitor the process and to improve the fraction of non-conforming cans.

To establish the control chart (phase 1), 30 samples of $n = 50$ cans each were analysed at an hour intervals over a three-shift period in which the machine was in continuous operation. Table 1 shows the gathered data.

Being $\bar{p}_1 = \sum_{i=1}^{30} p_i/n = 0.2313$, a preliminary estimate of the upper and lower control limits of the fraction of non-conforming units control chart (p-chart) is the following:

$$\mathrm{UCL} = \bar{p}_1 + 3\sqrt{\bar{p}_1(1-\bar{p}_1)/n} = 0.2313 + 0.1789 = 0.4102$$

$$\mathrm{LCL} = \bar{p}_1 - 3\sqrt{\bar{p}_1(1-\bar{p}_1)/n} = 0.2313 - 0.1789 = 0.0524,$$

where UCL is the upper control limit and LCL the lower control limit.

As Montgomery states two points plot above the upper control limit (samples 15 and 23). The related assignable causes are detected and removed. Eliminating these points, the new revised control limits become:

$$\bar{p}_1 = \sum_{i=1}^{28} p_i/n = 2150$$

$$\text{UCL} = 0.3893 \text{ and LCL} = 0.0407.$$

Sample 21 exceeds the new upper control limit. However, a further analysis of the data does not produce any reasonable assignable cause. We may conclude that the process is in control. The revised control limits may be adopted for monitoring current production.

We observe that the process fraction of non-conforming units is too high. A detailed analysis of the process indicates that several adjustments can be made on the machine. After these interventions an additional 24 samples are collected with the aim to verify the process improvement (phase 2). Table 2 shows the new gathered data.

As it appears from figure 1, we obtain a considerable reduction of the fraction of non-conforming units. Speaking with Montgomery's words (pp. 258–259), 'It is not unusual to find that the process performance improves following the introduction of formal statistical process-control procedures, often because the operators are more aware of process quality and because the control chart provides a continuing visual display of process performance.'

With these new data the process fraction of non-conforming units becomes:

$$\bar{p}_2 = \sum_{i=31}^{54} p_i/n = 0.1108.$$

The difference between the two average fraction of non-conforming units can be tested by means of the following hypothesis testing:

$$\text{H}_0: p_1 = p_2$$

$$\text{H}_1: p_1 > p_2.$$

Sample no.	Number of non-conforming cans	p_i	Sample no.	Number of non-conforming cans	p_i
1	12	0.24	16	8	0.16
2	15	0.30	17	10	0.20
3	8	0.16	18	5	0.10
4	10	0.20	19	13	0.26
5	4	0.08	20	11	0.22
6	7	0.14	21	20	0.40
7	16	0.32	22	18	0.36
8	9	0.18	23	24	0.48
9	14	0.28	24	15	0.30
10	10	0.20	25	9	0.18
11	5	0.10	26	12	0.24
12	6	0.12	27	7	0.14
13	17	0.34	28	13	0.26
14	12	0.24	29	9	0.18
15	22	0.44	30	6	0.12

Table 1. Fraction of non-conforming units collected in the process for 30 samples of $n = 50$ cans (Montgomery 1996).

Sample no.	Number of non-conforming cans	p_i	Sample no.	Number of non-conforming cans	p_i
31	9	0.18	43	3	0.06
32	6	0.12	44	6	0.12
33	12	0.24	45	5	0.10
34	5	0.10	46	4	0.08
35	6	0.12	47	8	0.16
36	4	0.08	48	5	0.10
37	6	0.12	49	6	0.12
38	3	0.06	50	7	0.14
39	7	0.14	51	5	0.10
40	6	0.12	52	6	0.12
41	2	0.04	53	3	0.06
42	4	0.08	54	5	0.10

Table 2. Fraction of non-conforming data collected for additional 24 samples of $n = 50$ cans (phase 2 control limits verification) (Montgomery 1996).

An approximate test statistic based on the normal approximation to the binomial is:

$$Z_0 = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})(1/n_1 + 1/n_2)}},$$

where

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2}.$$

Substituting the obtained values, we find $Z_0 = 7.10 > Z_{0.05} = 1.645$ Consequently, the null hypothesis is rejected in favour of the alternative hypothesis (Montgomery 1996).

By re-estimating the control limits for the new fraction of non-conforming units, one obtains UCL = 0.2440 and LCL = 0 (figure 1).

Hypothesizing an average learning model $\bar{p} = (\alpha/t) + \gamma + \varepsilon$, we can determine the asymptotic fraction of non-conforming units of the process, and the time to achieve it.

Applying equations (4) and (5), we have respectively:

$$a = \frac{\bar{p}_1 - \bar{p}_2}{1/\bar{t}_1 - 1/\bar{t}_2} = \frac{0.2150 - 0.1108}{1/15 - 1/42} = 2.43$$

$$c = \bar{p}_1 - a/\bar{t}_1 = 0.2150 - 2.43/15 = 0.053.$$

where $\bar{t}_1$ and $\bar{t}_2$ are the average times related to the two phases of p-chart setting up and control limits verification.

We assume for c and a a normal distribution. The c statistic is the estimation of the asymptotic fraction of non-conforming units of the process. It represents the fraction value that can be asymptotically achieved by the process because of the learning mechanism.

By c, we can determine the 'asymptotic control limits' of the p-chart (figure 1):

$$\text{UCL} = c + 3\sqrt{c(1-c)/n} = 0.053 + 0.095 = 0.148$$

$$\text{LCL} = c - 3\sqrt{c(1-c)/n} = 0.$$

The uncertainties associated to the estimation of a and c are (equation 6):

$$s_a = \sqrt{\left(\frac{\bar{t}_2 \cdot \bar{t}_1}{\bar{t}_2 - \bar{t}_1}\right)^2 (s^2_{\bar{p}_1} + s^2_{\bar{p}_2})} = 1.71;$$

$$s_c = \sqrt{\left(\frac{\bar{t}_2}{\bar{t}_2 - \bar{t}_1}\right)^2 s^2_{\bar{p}_2} + \left(\frac{\bar{t}_1}{\bar{t}_2 - \bar{t}_1}\right)^2 s^2_{\bar{p}_1}} = 0.076,$$

where s_a and s_c are respectively the estimation of the standard deviation of a and c statistics, and $s_{\bar{p}_1} = \sqrt{\bar{p}_1(1-\bar{p}_1)/n} = 0.058$, $s_{\bar{p}_2} = \sqrt{\bar{p}_2(1-\bar{p}_2)/n} = 0.044$ the estimation of the standard deviation of $\bar{p}_1$ and$\bar{p}_2$.

The 95% two-sided confidence intervals for the two parameters are:

$$\gamma = c \pm z_{0.025} \cdot s_c = 0.053 \pm 2 \cdot 0.076$$

$$\alpha = a \pm z_{0.025} \cdot s_a = 2.43 \pm 2 \cdot 1.71.$$

From equation (9), we may evaluate, for example, the impact (percentage variation) on $\bar{p}_{(\text{est})}$ due to a 10% variation of the two parameters, at time $t = 200$ h:

$$\frac{\Delta\bar{p}_{(\text{est})}}{\bar{p}_{(\text{est})}} \cdot 100 = \frac{\frac{a \cdot 0.1}{200}}{\frac{a}{200} + c} = 1.87\% \text{ for a 10\% variation of parameter } a;$$

$$\frac{\Delta\bar{p}_{(\text{est})}}{\bar{p}_{\text{est}}} \cdot 100 = \frac{c \cdot 0.1}{\frac{a}{200} + c} = 8.13\% \text{ for a 10\% variation of parameter } c.$$

Furthermore, from equation (10), we can evaluate the prediction band limits for the average fraction of non-conforming units (with 95% two-sided confidence limits):

$$\bar{p}_{(\text{est})} \pm 2 \cdot \sqrt{\frac{1.71^2}{t^2} + 0.076^2}.$$

Figure 2 shows the value of $\bar{p}_{(\text{est})}$ and its prediction band over time (equations (10) and (11)). We set the lower prediction limit to zero, being it constantly lower than zero.

As regards the time t^* to achieve a prefixed percentage distance from the asymptotic value, e.g. $h = 10\%$, we have:

$$t^* = \frac{\alpha}{h\gamma} \cdot 100 = \frac{2.43}{10 \cdot 0.053} \cdot 100 = 460\,\text{h}.$$

This is about 19 days of continuous process operation. The standard deviation of t^* is $s_{t^*} = 736$ h.

4. Conclusions

The paper presents a method for the estimation of asymptotic defectiveness of a manufacturing process and the time to achieve it. The method is based on the prediction of the process learning curve and the information collected during the setting up phases of a p-chart.

The main novelties of the method are given below.

- The possibility to estimate the asymptotic defectiveness of a process, having only a limited set of preliminary information.
- The evaluation of the coherence between the asymptotic process defectiveness and the related design specifications.
- The possibility of choosing among more alternative processes that more 'capable', from the asymptotic defectiveness point of view.
- Providing a simple approach for evaluating the period of revision of process control limits and their asymptotic values.

Further developments of the method are finalized to the definition of a procedure able to automatically adapt a new estimation to continuous information collected by the process over time.

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