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Design of a Broadband Multiprobe Reflectometer

Gianluigi Madonna, *Student Member, IEEE*, Andrea Ferrero, *Member, IEEE*, and Marco Pirola, *Member, IEEE*

***Abstract*—A new design approach for low-cost multiprobe reflectometers is presented. While traditional circuits adopt equally spaced probes, the solution presented here provides a method to greatly enhance the bandwidth of the measuring system by a proper choice of each probe position. As an example, a five-probe (1–16) GHz reflectometer was designed and the measurement results compared to those obtained with commercial vector network analyzers.**

***Index Terms*—Microwave measurements, microwave network analyzer, microwave reflectometer.**

I. Introduction

MODERN automatic network analyzers use complex and expensive heterodyne and mixing circuitry to obtain the amplitude and phase of unknown reflection coefficients at microwave frequencies. The development of six-port reflectometers provided an alternative method, which reconducts the problem to the measurements of power ratios. Six-port systems [1] still require many directional couplers and hybrids and they are quite complicated. The simplest method is probably the multiprobe reflectometer, which samples, with fixed probes, the standing wave pattern on a transmission line that feeds the device under test [2], [3]. This solution can be built easily in microstrip and monolithic versions [4], providing a really low-cost way to have vector information inside larger subsystems. Equally spaced probes were designed to operate at a single frequency or for small bandwidth applications [5], [6]. In order to extend the operating range, Chang *et al.* [7], [8] proposed to raise the number of probes and to modify the spacing between them, but a general design method has not been presented yet.

Here a novel approach is investigated, which obtains the probe positions as a function of the required bandwidth. To validate the method, comparisons were made with the results already published in the case of equally spaced probes.

II. Multiprobe Juncton Design

A. Theoretical Analysis

Fig. 1 shows a simplified scheme of the measurement setup. N probes are distributed along the transmission line that connects the source to the device under test. Each probe is loosely coupled to the main line to avoid perturbing the standing wave pattern. A detector is connected at the end of each probe, while an additional one is mounted on an external directional coupler.

Manuscript received July 2, 1998.

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Publisher Item Identifier S 0018-9456(99)02940-X.

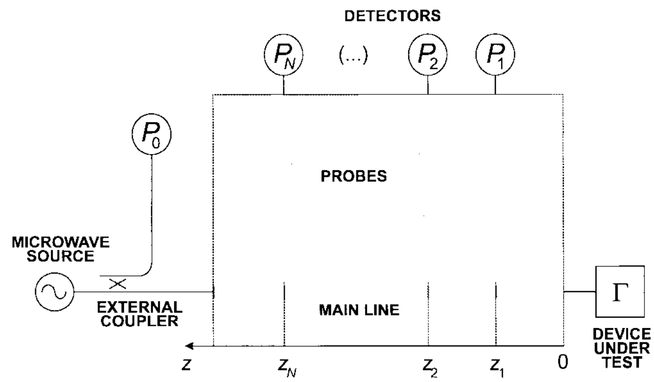

Fig. 1. Simplified measurement scheme.

Probes, transmission line, and coupler can be seen as a generic multiport junction. The theory of operation is carried out regardless of the actual circuit implementation. From six-port theory [1], it is well-known that ratios between generic detected powers P_i are related to the unknown reflection coefficient Γ by a simple quadratic equation. In particular

$$p_i = \frac{P_i}{P_0} = k_i \frac{|Q_i\Gamma + 1|^2}{|Q_0\Gamma + 1|^2} \quad i = 1, \ldots, N \tag{1}$$

where k_i are real numbers, while Q_0 and Q_i are complex quantities. These coefficients depend only on the parameters of the junction and the power detectors, and can be computed by a proper calibration procedure.

Once coefficients k_i, Q_0, and Q_i are known, the equations of (1) form a linear system that can be solved to compute the reflection coefficient Γ from the N detected powers. In particular, if

$$\mathbf{x} = [|\Gamma|^2 \, \mathrm{Re}(\Gamma) \, \mathrm{Im}(\Gamma)] \tag{2}$$

is chosen as the vector of unknowns, system (1) is written in the following matrix form (likewise, [9]):

$$\mathbf{Ax} = \mathbf{b}. \tag{3}$$

If N is greater than three, the system of equations defined by (3) is overdetermined and it can be solved in a least-square sense as

$$\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}. \tag{4}$$

Measurement errors can be seen as small perturbations in matrix $\mathbf{A}$ and vector $\mathbf{b}$; thus, the accuracy can be studied by means of the condition number $\kappa_2(\mathbf{A})$ of matrix $\mathbf{A}$ [10], defined as

$$\kappa_2(\mathbf{A}) \equiv ||\mathbf{A}||_2 \, ||(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T||_2 \tag{5}$$

where $||\mathbf{A}||_2$ is the matrix two-norm of $\mathbf{A}$. The smaller $\kappa_2(\mathbf{A})$ is, the more $\mathbf{x}$ is insensitive from perturbations in the system coefficients.

Both $\mathbf{A}$ and $\mathbf{b}$ depend on the junction parameters and on the power readings, i.e., on the reflection coefficient to be measured. The analysis of the system defined by (3) is made independent from the device under test by introducing a new complex variable γ, defined by the following relationship:

$$\frac{Q_1\Gamma + 1}{Q_0\Gamma + 1} \equiv \gamma + 1. \tag{6}$$

Equation (6) is similar to the bilinear transformation introduced in [11] for six-to-four port reduction. Power ratios of (1) become

$$p_i = k_i|q_i\gamma + 1|^2 \tag{7}$$

where

$$q_i = \frac{Q_i - Q_0}{Q_1 - Q_0}. \tag{8}$$

Measurement system (3) assumes the form

$$\mathbf{C}\xi = \mathbf{b} \tag{9}$$

where the coefficient matrix $\mathbf{C}$ depends only on the junction parameters. The vector of unknowns becomes

$$\xi = [|\gamma|^2 \,\mathrm{Re}(\gamma)\, \mathrm{Im}(\gamma)]. \tag{10}$$

Then, the analysis is completed by determining the error propagation from γ to Γ. It is straightforward from (6) that a small error $\delta\gamma$ affecting γ propagates to Γ as

$$\delta\Gamma = \frac{\partial\Gamma}{\partial\gamma}\delta\gamma = \frac{(Q_0\Gamma + 1)^2}{Q_0 - Q_1}\delta\gamma. \tag{11}$$

The key idea underlying this work was to study the behavior of $\kappa_2(\mathbf{C})$, rather than $\kappa_2(\mathbf{A})$, over the specified frequency range in order to properly design the multiprobe junction. $\kappa_2(\mathbf{C})$ versus frequency was computed for an *ideal multiprobe reflectometer* where

- the junction is lossless;
- all probes are identical and symmetrical with respect to their longitudinal axis;
- the main line is perfectly matched.

Under these assumptions, coefficients q_i depend only on the electrical length between probes [12], i.e.,

$$q_i = \exp\left(-j4\pi\frac{d_i}{\lambda}\right) \tag{12}$$

where $d_i = z_i - z_1$ is the distance between the first and the ith probe, while λ is the wavelength in the main transmission line. Thus, coefficients q_i are points on the unit circle in the complex plane.

For a three-probe junction, operating at single frequency, it is well-known from six-port theory that the best accuracy is achieved by 120° phase differences and equal magnitudes for all q_i. This corresponds to a spacing of a sixth of the wavelength between probes. In general, single frequency perfomance of an N-port reflectometer is optimized by q-points dividing the unit circle into equal sectors [9].

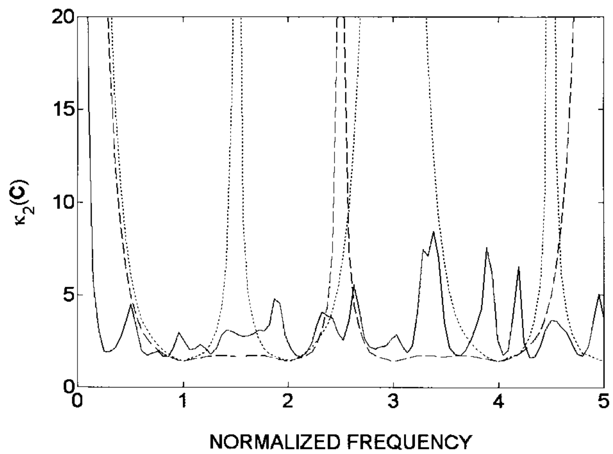

Fig. 2. Condition number of system (5), for three equally spaced probes (dotted line), five equally spaced probes (dashed line), and five optimally spaced probes (continuous line).

As frequency changes, electrical spacings between probes vary, too, and the q-points move along the unit circle. As an example, for the N equally spaced probe junction optimized at center frequency f_0, matrix $\mathbf{C}$ is singular at frequency $(N/2)f_0$ and all q-points coincide.

We investigated a new solution where nonequally spaced probes are used and their positions are computed to avoid singularities over a specified frequency range. Spacing values $d_2, \ldots, d_N$ were chosen to minimize functional F defined as the mean square condition number over the required band

$$F(d_2, \ldots, d_N) = \frac{1}{m}\sum_j [\kappa_2(\mathbf{C}_j)]^2. \tag{13}$$

$\mathbf{C}_j$ is the matrix $\mathbf{C}$ evaluated at the jth frequency, and the sum is extended to m specified frequencies.

To validate this approach, a five unequally spaced junction was designed. Theoretical results of $\kappa_2(\mathbf{C})$ versus normalized frequency f/f_0 are shown in Fig. 2, together with the performances of equally spaced reflectometers (respectively, with three and five probes). The latter solutions present singularities at frequency $f = 1.5f_0$ (three port case) and $f = 2.5f_0$ (five port case). On the contrary, the use of properly positioned probes allows extending the bandwidth up to $f = 3.2f_0$.

To complete the design, error propagation from γ to Γ was taken into account. Assuming ideal both junction and directional coupler, coefficient Q_0 is easily proved to be null. This greatly simplifies the analysis, since the error propagation expressed by (11) is no more dependent on the reflection coefficient Γ and it is limited in magnitude.

B. Circuit Design and Experimental Results

After the probe spacing optimization, a microstrip junction circuit was designed to verify the theoretical hypotheses. Theoretical assumptions prescribe that all probes shall be identical,

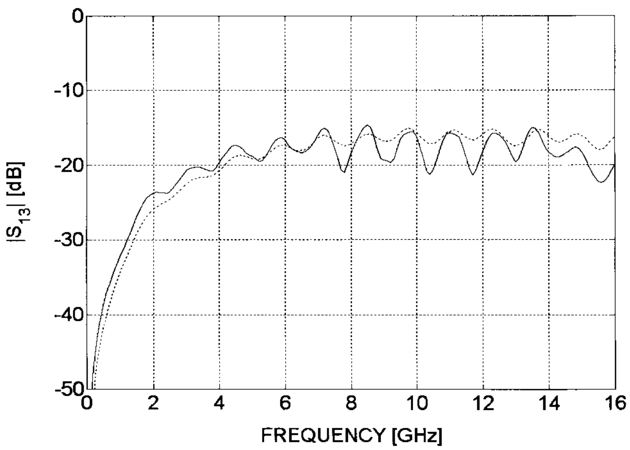

Fig. 3. Probe coupling factor: simulation data (dotted line) and measurement results (continuous line).

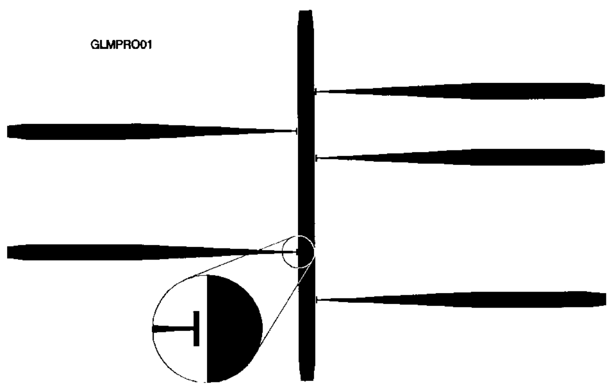

Fig. 4. Layout of the five optimally spaced probe reflectometer. A T-shaped probe tip is shown in detail.

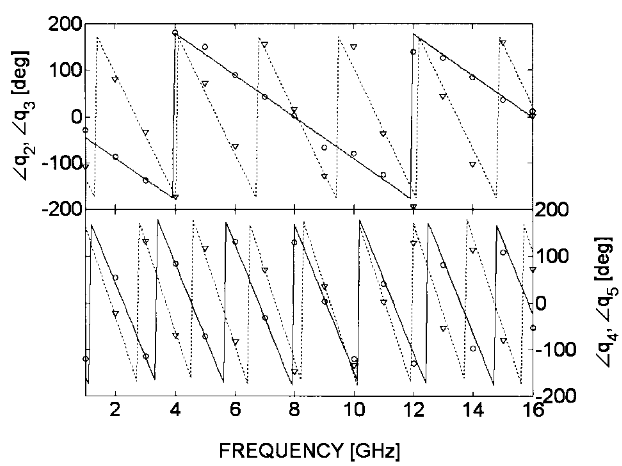

Fig. 5. Phase of the q-points versus frequency. Calibration results (q_2, q_4: $\circ$, q_3, q_5: $\bigtriangledown$) are shown together with values specified by theoretical analysis (q_2, q_4: continuous line, q_3, q_5: dotted line).

symmetrical, and not significantly affect main line fields. Furthermore, power sensor dynamic range imposes a lower limit to probe coupling. Chang *et al.* [8] used tapered probes coupled about -25 dB in the (8–12) GHz frequency range, but this solution presents a too-low coupling below 5 GHz. For broader bandwidth, we introduced T-shaped probe tips with

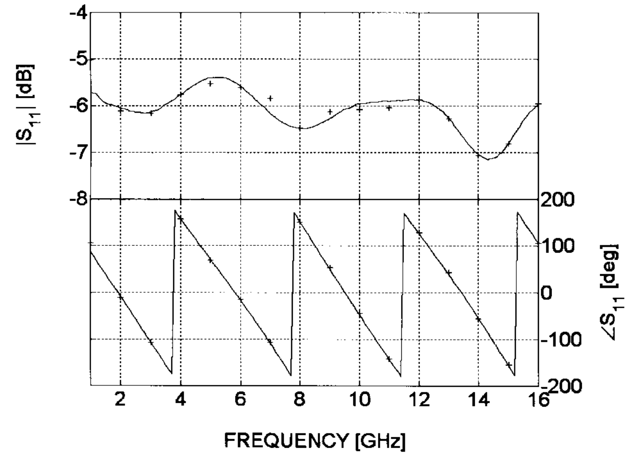

Fig. 6. Reflection coefficient of a 3 dB pad terminated by a standard short circuit: results from the five probes reflectometer (crosses) and an HP8510 VNA (continuous line).

a short section parallel to the main path. Fig. 3 shows the HP-MDS simulation and corresponding measurement results of a single probe coupling factor. The five probe microstrip junction layout is shown in Fig. 4. The circuit was mounted with SMA connectors and external diode detectors as power sensors.

After a proper calibration [6], actual junction q-points were obtained and compared with the theoretical values. Fig. 5 shows the angular positions of the q-points versus frequency. Finally, Fig. 6 shows the measured reflection coefficient of a 3 dB pad terminated by a standard short circuit. Results from the designed five probes system are shown against the reflection coefficient obtained with an HP8510 network analyzer.

III. Conclusion

A new design approach for broadband multiprobe reflectometers is given. To the authors' knowledge, the bandwidth of the realized system is three times broader than previously published multiprobe solutions. Comparisons with commercial vector network analyzers prove the effectiveness of this technique for low-cost reflection measurements inside larger subsystems.

References

[1] G. Engen, "A (historical) review of the six-port measurement technique," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 2414–2417, Dec. 1997.
[2] A. L. Samuel, "An oscillographic method of presenting impedances on the reflection-coefficient plane," *Proc. IRE*, vol. 35, pp. 1279–1283, Nov. 1947.
[3] R. Caldecott, "The generalized multiprobe reflectometer and its application to automated transmission line measurements," *IEEE Trans. Antennas Propagat.*, vol. AP-21, pp. 550–554, July 1973.
[4] R. E. Neidert, "Monolithic circuit for reflection coefficient measurement," *IEEE Microwave Guided Waves Lett.*, vol. 1, pp. 195–197, Aug. 1991.
[5] C. J. Hu, "A novel approach to the design of multiple-probe high-power microwave automatic impedance measuring system," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 1422–1428, Dec. 1980.
[6] S. Li and R. Bosisio, "Calibration of multiport reflectometers by means of four open/short circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1085–1089, July 1982.

[7] K. Chang, M. Li, and T. Sauter, "A three port microstrip impedance measurement system," *Microwave Opt. Technol. Lett.*, vol. 1, pp. 90–93, May 1988.
[8] K. Chang, M. Li, and T. Sauter, "Low cost microwave/millimeter-wave impedance scheme using a three-probe microstrip circuit," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1455–1460, Oct. 1990.
[9] P. Probert and J. Carroll, "Design features of multi-port reflectometers," *Proc. Inst. Elect. Eng.*, vol. 129, pp. 245–252, Oct. 1982.
[10] G. Golub and C. Van Loan, *Matrix Computations*, 2nd ed. Baltimore, MD: Johns Hopkins Univ. Press, 1989.
[11] G. Engen, "Calibrating the six-port reflectometer by means of sliding terminations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 951–957, Dec. 1978.
[12] J. Hunter and P. Somlo, "Simple derivation of six-port reflectometer equations," *Electron. Lett.*, vol. 21, pp. 370–371, Apr. 1985.

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