

On the dissipated energy density in compression

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On the dissipated energy density in compression / Ferro, GIUSEPPE ANDREA. - In: ENGINEERING FRACTURE MECHANICS. - ISSN 0013-7944. - 73:(2006), pp. 1510-1530.

Availability:

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On dissipated energy density in compression for concrete

Giuseppe Ferro *

Politecnico di Torino, Department of Structural Engineering and Geotechnics, Corso Duca degli Abruzzi 24, I-10129 Torino, Italy

Received 29 August 2005; received in revised form 17 January 2006; accepted 24 January 2006

Abstract

An experimental investigation on drilled cylindrical concrete specimens in compression over a large scale range (1:19) has been carried out to evaluate the variation of some mechanical parameters by varying specimen size. The peculiarity of the present investigation consists in exploring very small specimen dimensions. The experimental results show scale effects on dissipated energy density rather than on uniaxial compressive strength. A theoretical explanation for such a phenomenon, based on fractal hypothesis, is presented and a comparison between experimental and theoretical values is discussed. © 2006 Elsevier Ltd. All rights reserved.

Keywords: Fracture mechanisms; Powder compaction; Concrete; Structures; Mechanical testing

1. Introduction

The advent of computers has considerably changed the capabilities in design and analysis of concrete structures. The extensive use of powerful computers and finite element codes in structural analysis is meaningful only if suitable and reliable constitutive laws for the material are available. In design, however, concrete is generally classified on the basis of its compressive strength. A correct evaluation is therefore fundamental.

In general, the constitutive relations and the mechanical parameters for concrete are obtained from standard specimens. The sizes and shapes of compressive strength test specimens of concrete vary from one country to another. Commonly used standard sizes are 150 mm for cubes and 150 × 300 mm for cylinders. The introduction of high-strength concrete, with compression strength up to five times the standard strength, suggests the use of smaller specimens, with the advantages of maintaining the standard test machines available in the laboratories, easy handling, and using less concrete. Another important application of reducing specimen sizes is constituted by the determination of the concrete strength for existing structures by drilling small specimens. This technique is very useful, the deterioration of the mechanical properties for concrete structures being one of the main problems in civil engineering.

The choice of the standard size is affected by the variation of the compressive strength with size and height/diameter (or slenderness) ratio. This variation is high when the rigid test machine platens are in direct contact

* Tel.: +39 11 5644885; fax: +39 11 5644899.

E-mail address: ferro@polito.it

Nomenclature

δ	displacements
ϵ	strain
ϵ_{peak}	strain at the peak stress
d	specimen diameter
h	specimen height chosen as the characteristic specimen size
ϵ_m	mean deformation
f'_c	compression strength
C1	smallest specimen set with $h = 10$ mm
C2	specimen set with $h = 23$ mm
C3	specimen set with $h = 46$ mm
C4	specimen set with $h = 100$ mm
C5	largest specimen set with $h = 190$ mm
C33	third specimen of the set with $h = 46$ mm
N	number of fragments in fragmentation process
r	characteristic linear dimension of fragments
B	constant of proportionality
D	fractal dimension of the fragmentation process
V_f	total volume (mass) of fragments
r_{max}	characteristic linear dimension of the largest fragment
r_{min}	characteristic linear dimension of the smallest fragment
k	constant of proportionality
V	volume of the un-fragmented specimen
A_f	total surface area of the fragments
C	geometrical factor depending upon the average shape of the fragments
W	energy dissipated to produce a new free surface in the fragmentation process
β_F	specific energy absorbing capacity;
\mathcal{G}	elastic energy release rate or specific energy necessary to generate the unit area of fracture
d_ω	fractal dimension of the fragmented set = 3-D
h^*	measure of the fractal set representing the fragmented configuration
S	dissipated energy density
\mathcal{G}_F^*	fractal dissipated energy density parameter
ϵ^*	renormalized fractal strain
E^*	renormalized fractal elastic modulus

32 with the concrete specimen, the lateral deformation of concrete being restrained at the specimen ends. A wide
33 investigation has been carried out by Carpinteri et al. [1].

34 Very interesting results have been obtained in a round robin test organized by the RILEM Committee
35 148 SSC “Strain-Softening of Concrete” [2], whose aim was to investigate the softening behavior of concrete
36 by varying specimen dimensions, boundary conditions, feed-back signals and testing machine characteris-
37 tics. They observed the independence of the slenderness (or size) on the compressive strength, when the
38 boundary conditions of the concrete specimens were characterized by no friction (or reduced friction) at
39 the ends.

40 The effect of size on the mechanical properties of concrete is also important when small scale models are
41 used to predict the behavior of real structures. Early work on the size effect in compression dates back to
42 the 1920s. Gonnermann [3] emphasized the size effects through an extensive investigation on the compressive
43 strength of cylinders with a height/diameter ratio equal to two.

44 Many other authors fronted the problem of size effects on nominal strength for concrete in compression.
45 Blanks and McNamara [4] performed tests on cylindrical specimens with slenderness of $h/d = 2$ in a large scale

46 range (1:12), while Jishan and Xixi [5] performed experiments on cubes (scale range 1:4) and on prisms with $h/d = 3$ (scale range 1:3). Other analyses on size effects in compression were conducted by considering particular
47 geometries, as compact compression tests [6], reinforced concrete columns [7] and high-strength concrete rein-
48 forced with randomly distributed fibers [8]. Several assessments were made on the size effects through the
49 extensive experimental work reported in the literature [9].

51 The nominal compressive strength is obtained by dividing the peak force by the initial specimen cross-sec-
52 tion area. This operation has the meaning of estimating a global material property, ignoring at the same time
53 the material structure as well as the material failure behavior during the test. Momber [10] analyzed the frag-
54 mentation of standard concrete cylinders under compression. He observed that the standard codes (ASTM
55 C39-86) consider types of failure which involve only large primary fracture debris, while after compression
56 testing of any concrete specimen, fragments of fine-grained material are generated. Slate and Hover [11]
57 showed pervasive internal crack growth up to the peak load by studying the interior of concrete specimens
58 that were loaded up to a certain level and that were subsequently unloaded. From their experimental obser-
59 vations, it is believed that energy dissipation in the pre-peak regime is a global continuum-dominated process
60 that may be attributed to microcracking throughout the entire specimen. On the other hand, energy dissipa-
61 tion in the post-peak regime is a localized surface-dominated fracture process after the coalescence of micro-
62 cracks in the peak regime. Vardoulakis et al. [12] proposed a continuum fracture mechanics of uniaxial
63 compression on brittle materials to arrive at a continuum description of the observed post-peak phenomena.
64 Their conclusions, based on experimental evidence and dimensional analysis, state that the post-peak axial
65 stress is a function of the axial displacement normalized by the radius of the specimen, and not by its height.
66 Van Vliet and van Mier [13], observed that post-peak data from uniaxial compression experiments on plain
67 concrete suggest a stress–displacement rather than a stress–strain relation.

68 An extensive experimental investigation on geometrically similar cylindrical concrete specimens in compres-
69 sion obtained by a unique concrete block is herein presented to evaluate the variation of some mechanical
70 parameters by varying specimen size on a very large scale range (1:19) and by avoiding frictional restraint
71 between the loading platens and the specimen. The peculiarity of the present investigation consists in exploring
72 very small specimen dimensions. One of the main goals of the present paper is the measure of the energy dis-
73 sipated in destroying or fragmenting a volume of concrete.

74 From a phenomenological point of view, the softening branch of the load–displacement curve is governed
75 by macrocracking, after the coalescence of the initial microcracks. As shown by the experimental results, the
76 ultimate compressive strength of concrete depends on the type of testing machine, the specimen size and the
77 nature of the contact between the machine platens and the specimen. In this paper, it is shown how,
78 avoiding friction, the strength is almost independent of specimen dimension, whereas strong variations
79 are observed for dissipated energy density in compression. A theoretical explanation, recently proposed
80 by Carpinteri and Pugno [14,15], for the scale effects on the dissipated energy density in compression is dis-
81 cussed and applied to the experimental results. This is based on the concept of fractal geometry [16,17], and
82 on the fragmentation approach [18]. From the theory, it can be shown how, in the scale range of the tested
83 specimens, the energy dissipation is a surface-dominated phenomenon and damage localization occurs in
84 small concentrated zones. This statement is valid only for small specimen sizes, while for larger structural
85 dimensions the energy dissipation should be a volume-dominated process and damage is more spatially
86 distributed.

87 2. Experimental set-up

88 The ambition of testing concrete specimens in compression in a very wide size range strongly impacts with
89 the laboratory set-ups, which opposes physical limits. The fundamental idea was to use a very simple standard
90 testing apparatus composed only of a closed-loop servo-hydraulic system and strain gauges glued on the spec-
91 imen to record the longitudinal as well as the transverse deformation in the pre-peak part of the force versus
92 displacement curve. As the specimens were very different in size, two different set-ups have been adopted, even
93 if this could cause some inconveniences due to different stiffnesses of the frames.

94 2.1. Test specimens

95 The first problem was that of defining the size and the slenderness of the test specimens. The size is limited by
96 the dimension of the aggregates (lower limit) and by the potentialities of the available equipment (upper limit).

97 All the cylinders were obtained by drilling from a unique concrete block with sizes $800 \times 500 \times 200$ mm. The
98 microconcrete used for the specimens is characterized by a maximum aggregate size of 4 mm. The porosity is
99 equal to 17.7 vol% and the distribution of pores is 15.8% > 10000 nm, 52.9% 30–10000 nm and
100 31.3% < 30 nm. The nominal strength is 51.8 N/mm^2 while the compression strength f'_c , obtained from cubes
101 ($150 \times 150 \times 150$ mm) after 28 days, is equal to 33 N/mm^2 . The water–cement ratio is equal to 0.65.

102 Five different diameters were considered in relation to the disposable drilling core-bits in a scale range of
103 1:19. The specimens were cylinders with a height/diameter ratio $h/d = 1$ and h chosen as the characteristic
104 dimension equal to 10, 23, 45, 100, 190 mm, respectively. Four specimens have been tested at each size.
105 Two extra specimens for $h = 10, 23$ and 45 mm were used to check the electrical parameters (impedance, gain).
106 The geometries of the tested specimens are presented in Fig. 1a, while an overview of all the specimen sizes is
107 reported in Fig. 1b. The geometrical characteristics are reported in Table 1.

108 2.2. Testing equipment

109 For the three smallest sizes, the tests were carried out on a uniaxial compression machine with a capacity of
110 100 kN. The machine is controlled by a closed-loop servo-hydraulic system. Of the two loading platens, the
111 lower is fixed, while the upper is connected to the machine hinge. In this way, the upper platen can adjust to
112 the geometrical imperfections of the specimen. All compression tests with this machine have been performed
113 under displacement control, by imposing a constant rate of the displacement of the upper loading platen.
114 The displacement rate has been set in order to impose the same stress rate for all the specimen sizes. A stress
115 rate equal to $0.5 \text{ N/mm}^2/\text{s}$ was adopted, as prescribed by UNI Standard 6130 for cubic strength evaluation.
116 To obtain this stress rate, the displacement rates have been set equal to $2 \times 10^{-3} \text{ mm/s}$ for the smallest specimens
117 ($h = 10$ mm, C1), $4.6 \times 10^{-3} \text{ mm/s}$ for the specimens with $h = 23$ mm (C2) and $10 \times 10^{-2} \text{ mm/s}$ for $h = 46$ mm
118 (C3). On each specimen two bidirectional strain gauges were glued, the length of which was taken proportional
119 to the specimen height. More specifically, the strain gauge length was 1.5 mm for $h = 10$ mm (strain gauge HBM
120 1.5/120 xy 11), 3 mm for $h = 23$ mm (strain gauge HBM 3/120 xy 11) and 6 mm for $h = 45$ mm (strain gauge
121 HBM 6/120 xy 11). The axial deformations as well as the lateral deformations in the middle part of the specimen
122 were measured with these strain gauges, and the volumetric variation in the pre-peak part was determined. A
123 detail of the three smallest specimens with the glued strain gauges is reported in Fig. 1c.

124 For the two remaining specimen sizes, $h = 100$ (C4) and 190 mm (C5), a manual load-controlled uniaxial
125 compression machine with a capacity of 3000 kN was used. The choice of this kind of machine was chosen for
126 the following reasons. First of all, the height of the specimens do not permit control of the post-peak σ – ϵ
127 diagram under displacement control, due to the more brittle structural behavior. This aspect could be
128 overpassed if the tests were controlled over a central part of the specimens, as performed by van Vliet and
129 van Mier [19], or through lateral deformations [20]. The latter would have comported a very sophisticated test,
130 which was not the author's intention. Secondly, the control of the explosive behavior of the specimens and the
131 determination of the fragment sizes pushed the author toward the aforementioned solution. For these two
132 larger sizes, loading cycles around the peak-load (characterized by a decrease of the slope in the load versus
133 displacement diagram) were performed in order to capture the post-peak branch and to plot the entire curve.
134 Unfortunately, as should have been easy to predict, only for one specimen (C44) it was possible to capture the
135 softening part. On the other hand, two specimens (C41 and C42) were tested in displacement control with a
136 different closed-loop servo-hydraulic machine with a capacity of 1000 kN. The stiffness of this machine was
137 not enough and an explosive failure occurred.

138 2.3. Boundary conditions

139 In uniaxial compression tests it is well-known how the boundary conditions play an important role. When
140 a concrete specimen is loaded between rigid loading platens (steel), the lateral deformation of concrete is

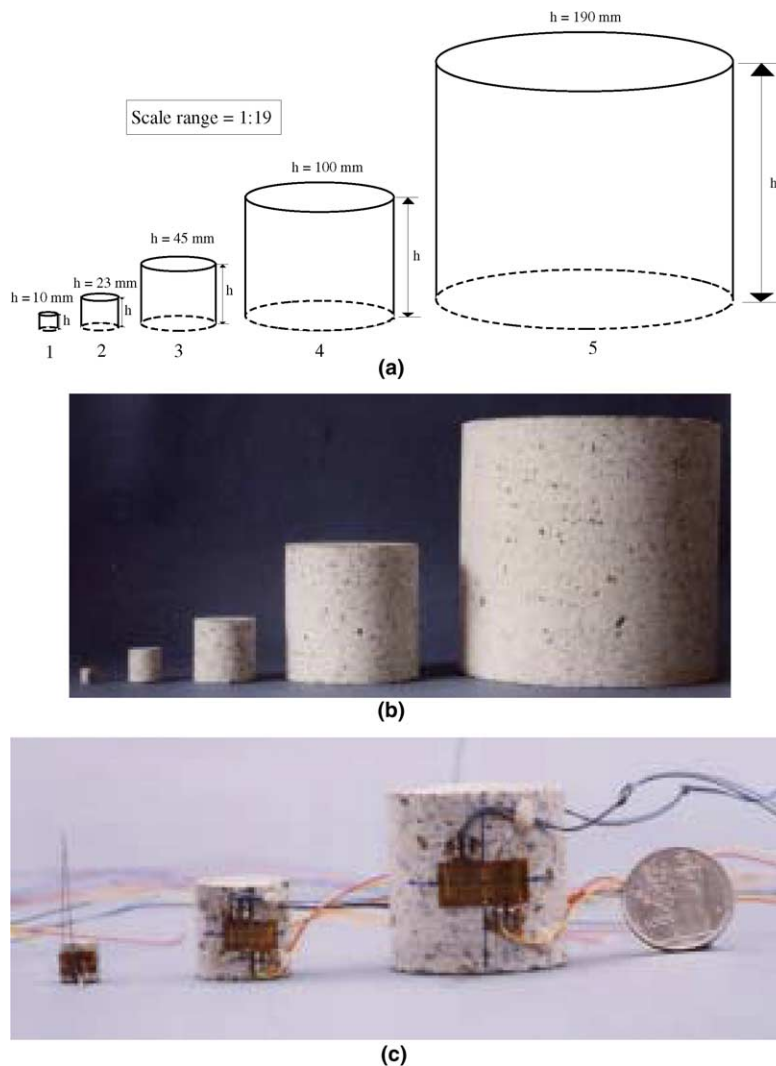


Fig. 1. (a) Geometry of the five different concrete specimens; (b) overall view of the five specimen sizes; (c) particular of the three smallest specimens with the glued strain gauges.

141 confined to the specimen ends (Fig. 2a), which are forced to have the same lateral deformation as the rigid
 142 platens. In this case, shear-stresses develop between specimen and loading platen, causing a three-dimensional
 143 state of stress at the specimen ends.

144 An opposite trend comes out when loading platens with a low stiffness and a high Poisson's ratio are used
 145 (rubber or brushes) (Fig. 2b). The platens are subjected to large lateral deformations and outward-directed
 146 shear forces develop at the interfaces, producing local splitting cracks.

147 Kotsovos [21] performed experiments on cylinders with an aspect ratio of 2.5 and with different frictional
 148 systems. He observed the same pre-peak behavior (in dimensionless form) for the different choices of loading
 149 system, and post-peak dimensionless stress–displacement curves characterized by increasing slope with
 150 decreasing the coefficient of friction. In order to minimize the interface friction, van Mier [22] and Vonk
 151 [23] developed brush platens. Wittmann et al. [24] tested normal-strength concrete cylinders by attaching at
 152 the ends two high-strength concrete disks of the same diameter. This system was also adopted by Lee and
 153 Willam [25].

Table 1
Summary of geometrical characteristics of specimens

Specimen no.	Diameter (mm)	High (mm)	A (mm ²)	Weight (g)	Volume (mm ³)
C11	9.9	9.5	76.98	1.9	731.31
C12	9.9	9.9	76.98	2.0	762.10
C13	9.8	10.0	75.43	2.1	754.30
C14	9.8	9.8	75.43	1.9	739.21
C15	9.8	10.0	75.43	2.0	754.30
C16	9.8	9.9	75.43	1.7	746.76
C21	23.7	24.2	441.15	24.0	10676
C22	23.7	24.2	441.15	24.0	10676
C23	23.7	24.2	441.15	23.5	10676
C24	23.7	24.6	441.15	24.3	10852
C25	23.7	24.4	441.15	23.5	10764
C26	23.7	23.9	441.15	23.9	10543
C31	44.9	45.3	1583.4	160.0	71728
C32	45.1	44.6	1597.5	160.0	71248
C33	45.1	45.1	1597.5	161.0	72047
C34	44.8	45.4	1576.3	159.0	71564
C35	45.0	45.3	1590.4	161.0	72045
C36	45.1	45.6	1597.5	161.5	72846
C41	99.5	100.3	7775.6	1795	779893
C42	99.6	100.2	7791.3	1770	780688
C43	99.5	100.6	7775.6	1790	782225
C44	99.5	100.5	7775.6	1805	781448
C51	192.2	192.9	29013	12585	5596608
C52	192.1	191.7	28983	12580	5556041
C53	192.2	193.6	29013	12645	5616917
C54	192.2	191.4	29013	12585	5553088

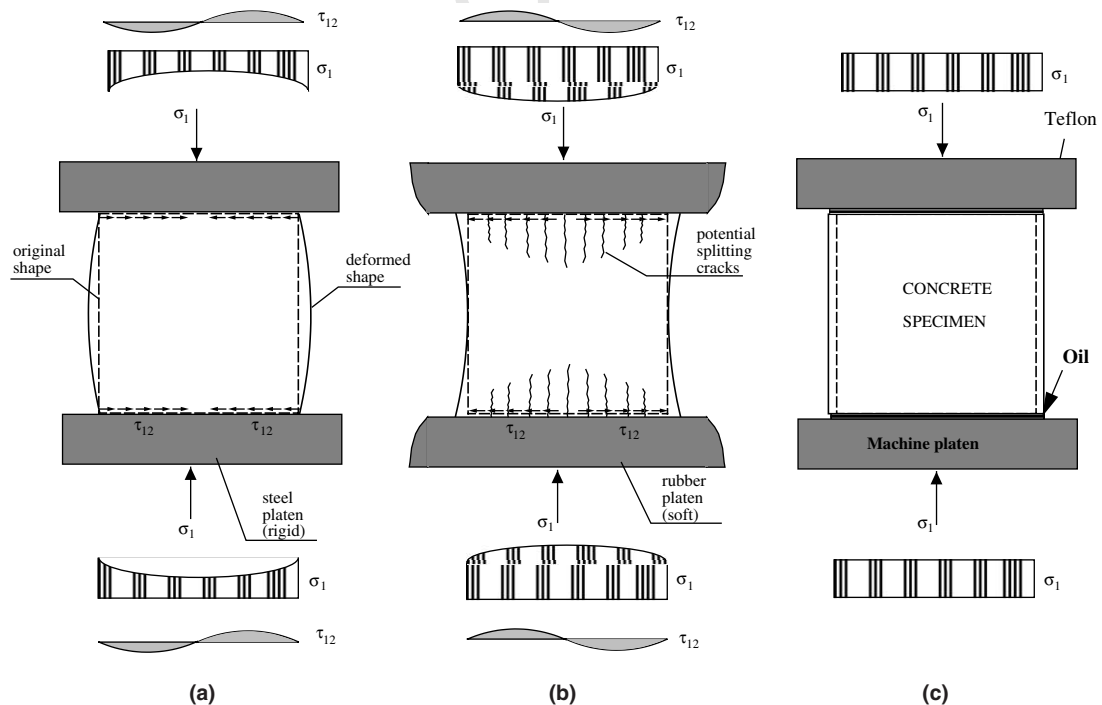


Fig. 2. Effect of loading platen response: (a) steel rigid platens; (b) soft platens and (c) adopted teflon layer proposed by van Vliet and van Mier [19].

154 The system adopted in the present compression tests comes from the analysis of the RILEM Technical
 155 Committee 148 SSC results [2]. The loading platens, they found, not only affect the post-peak behavior,
 156 but also the peak stress. Van Vliet and van Mier [13] observed an increase in peak stress by decreasing the
 157 slenderness (up to 200%), when the specimens were loaded between rigid steel platens, whereas an almost con-
 158 stant peak stress with the application of teflon interlayers. The softening branch, with both the systems,
 159 becomes steeper with increasing specimen height. These results suggested the use of two teflon layers of
 160 150 μm thickness with oil in between and a specimen slenderness equal to one.

161 3. Experimental observations

162 Only one representative curve for each of the five sizes has been selected, for graphical reasons. The sum-
 163 mary of the experimental results for peak-load, stress at peak load and specific compressive energy is reported
 164 in Table 2. The experimental load versus displacement diagrams for smaller specimens ($h = 10$ (C1), 23 (C2)
 165 and 46 mm (C3)) are reported in Fig. 3a, while those related to the larger specimens ($h = 100$ (C4) and 190 mm
 166 (C5)) are reported in Fig. 3b. Moreover, a typical failure in the post-peak softening regime is shown in Fig. 4
 167 (specimen C33).

168 3.1. Stress–deformation response

169 The nominal stress versus nominal deformation curves are plotted in Fig. 5. These curves have been
 170 obtained from the load versus displacement curves by dividing the load by the initial specimen cross-section
 171 and the displacement by the initial specimen height. The curves show an initial steadily increasing slope, due to
 172 the lower stiffness at the beginning of the test. This fact is due to the adjustment of the loading platens to the
 173 specimen surfaces and to the compressibility of the teflon interlayers. This transition can be appreciated from
 174 Fig. 6, in which the axial and lateral strains obtained from strain gauges placed in the middle third of the spec-
 175 imen are plotted together with the axial strains obtained by dividing the piston-stroke by the specimen height.
 176 In the latter, the measured axial strains (called mean deformation, ϵ_m , Fig. 6) also contain the deformation of
 177 the steel loading platens. This part is often substituted by a straight line with slope equal to the maximum pre-
 178 peak slope of the force–displacement curve. In this paper, the experimental curves have been presented as they
 179 have been recorded without any correction. Another possible correction that could be done on the experimen-

Table 2

Summary of experimental results for peak-load, stress at peak load and critical compressive energy density

Specimen no.	Peak load (da N)	Stress at peak load (N/mm ²)	Stress–strain area (N/mm ²)
C11	295	38.32	6.12
C12	328	42.61	7.32
C13	327	43.35	7.12
C14	335	44.41	8.34
C21	1289	29.22	2.51
C22	1457	33.03	3.45
C23	1229	27.86	2.75
C24	1404	31.83	2.78
C31	5298	33.46	1.33
C32	6136	38.41	2.90
C33	5813	36.39	2.62
C34	6311	40.04	3.24
C41	27247	35.04	0.32
C42	39194	50.30	0.33
C43	39231	50.45	0.20
C44	29923	38.48	0.42
C51	142400	49.08	–
C53	132210	45.57	0.35

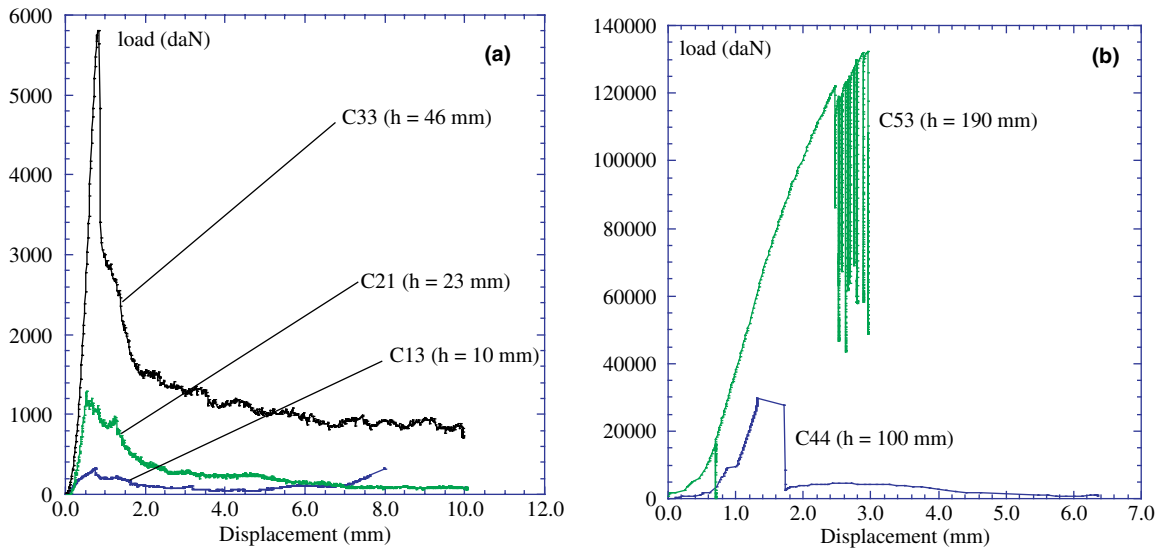


Fig. 3. Experimental load versus displacement curves: (a) small specimens, $h = 10$ (C1), 23 (C2) and 46 mm (C3); (b) and large specimens, $h = 100$ (C4) and 190 mm (C5).

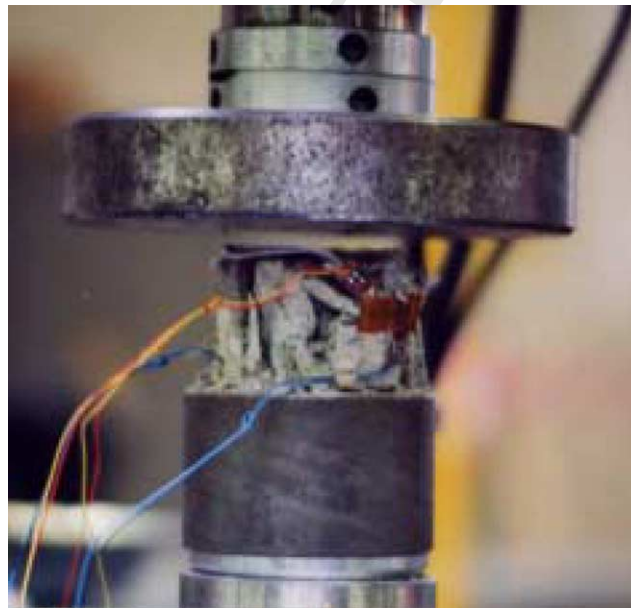


Fig. 4. Typical failure of a specimen in the post-peak softening regime.

180 tal curves is related to the different testing machine used for different specimen sizes. In this case, the two dif-
 181 ferent experimentally evaluated stiffnesses for the set-ups, obtained with the teflon layers without any speci-
 182 men, could be subtracted from the experimental diagrams.

183 After this initial part, the stress–strain path is nearly linear and this linear part is as more pronounced as the
 184 specimen is larger (Fig. 5). The smaller the specimen, the more pronounced pre-peak nonlinearities are. After
 185 the peak stress, a gradual descending branch has been detected. To appreciate the shape of the stress–strain
 186 curve better, its normalized version obtained by dividing the stresses by the peak stress (Fig. 7a) and the strains

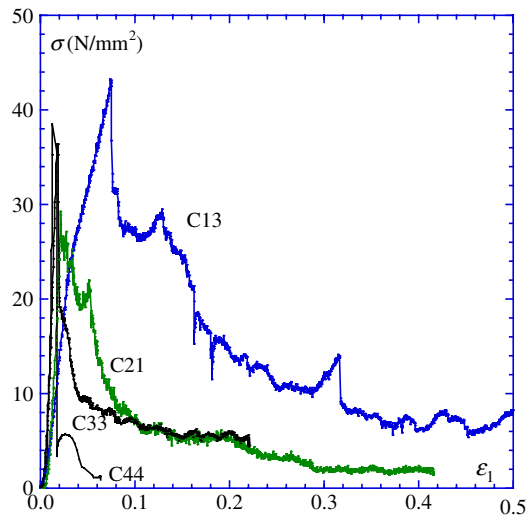


Fig. 5. Nominal stress versus nominal strain diagrams for four different cylindrical specimen sizes.

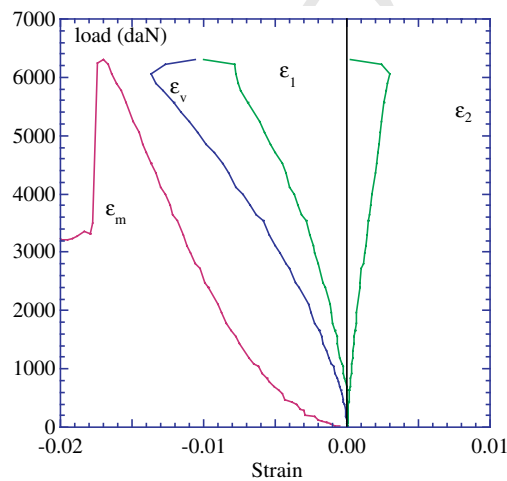


Fig. 6. Axial (ϵ_1), lateral (ϵ_2), volumetric (ϵ_v) and mean (ϵ_m) deformations for specimen C34.

187 by the strain at the peak stress (Fig. 7b) are reported. As can be deduced from Fig. 7, the stress–strain curve
 188 for different specimens are almost the same in the pre-peak regime, but, beyond the peak, the slope of the post-
 189 peak part decreases with decreasing specimen height. Van Mier [22] plotted the normalized stress versus post-
 190 peak displacement curves, in which the displacements are calculated as

$$192 \quad \delta = (\epsilon - \epsilon_{\text{peak}})h \quad (1)$$

193 and obtained nearly overlapped curves. He concluded that, as the same displacement is needed to fracture the
 194 specimens, the post-peak deformation must be localized in a small zone, and cannot be interpreted as an aver-
 195 age strain. This fracture localization of concrete uniaxial compression implies that strain cannot be used as
 196 state variable in constitutive laws. The dimensionless stress versus post-peak deformation diagrams for four
 197 cylindrical specimen sizes are plotted in Fig. 8. It can be effectively observed that these curves are close to each
 198 other, even if different initial slopes, indicating an increase in brittleness with size, is present.

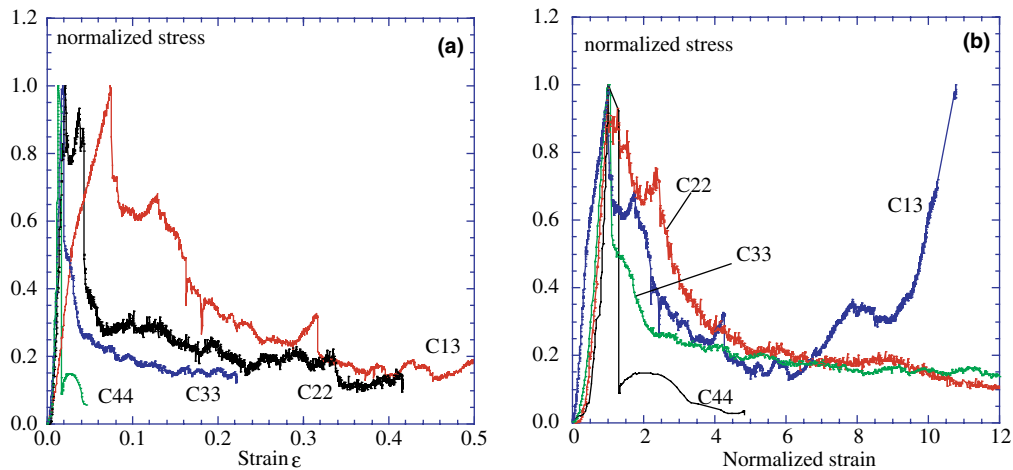


Fig. 7. (a) Normalized stress–strain diagrams for four different cylindrical specimen sizes; (b) normalized stress versus normalized strain diagrams for four different cylindrical specimen sizes.

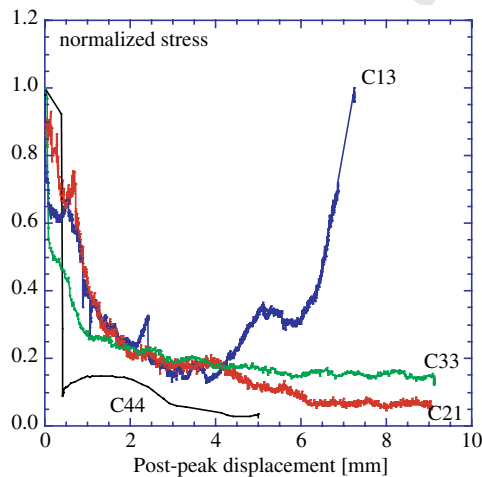


Fig. 8. Dimensionless stress versus post-peak displacement for four different cylindrical specimen sizes.

199 3.2. Scatter in peak stress

200 The values of the peak stresses, which are commonly called *compressive strength*, are reported in Fig. 9 by
 201 varying specimen sizes. It can be noticed how in compression a marked size effect does not come out and no
 202 relation with the specimen size is evident, as instead can be observed in tension [26–28] or in compression when
 203 localization is present [1]. The same results were obtained experimentally by the RILEM Committee 148 and
 204 numerically by Carpinteri et al. [29,30] by simulations with a boundary element approach. The scatter in the
 205 results is pronounced. What is interesting to observe is that the values even for the smallest size are compa-
 206 rable to the compressive strength of standard cubes. This permits us to affirm that, if friction is avoided or
 207 drastically reduced, the compressive strength of an existing concrete structure can be evaluated using very
 208 small drilling core specimens (nondestructive test method).

209 3.3. Scatter in dissipated energy density

210 The dissipated energy density can be evaluated by considering the area under the P – δ curve divided by the
 211 volume of the specimen. This is equivalent to considering the area under the stress–strain curve. For the small-

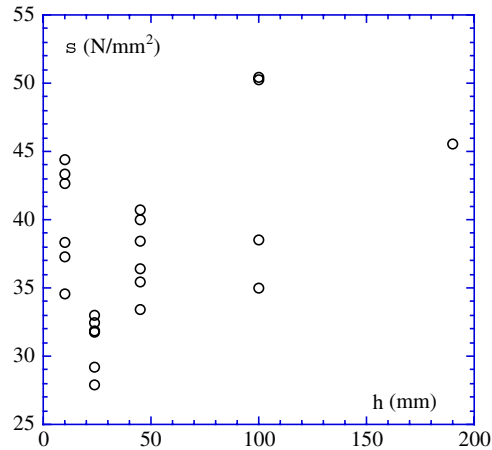


Fig. 9. Peak stresses by varying specimen size.

est specimen size, the dissipated energy density has been evaluated by considering the area under the stress–strain curve up to the minimum value of the stress. The numerical values of these areas for each specimen are reported in Table 2. The values are also plotted versus the characteristic specimen size in Fig. 15. This dissipated energy density undergoes severe scale effects, and the trend is a decrease by increasing the specimen dimension. This interesting result is discussed in the next section and a theoretical explanation is presented based on a fractal hypothesis for the fragment size distribution generated during the compression test.

4. Full stress–strain curves

In this section the full stress–strain curves for the smallest cylindrical specimens ($h = 10$ mm) are presented and some considerations are proposed. An interesting discussion of full stress–strain curve is proposed in the paper by Armer and Grimer [31]. They considered the re-ascending stress–strain branch which is not usually considered. This phenomenon was also described by Nikitin [32] by using dynamic analysis.

The complete curve for specimen C13 is plotted in Fig. 10. After the specimen has been crushed down more or less to a heap of aggregates, the resistance to further deformation reaches a minimum for $\epsilon \simeq 0.5$, and then begins to increase once more to higher stresses. Armer and Grimer affirm the existence of a new higher peak, at which the aggregate itself begins to break down. From the present experimental tests (Figs. 10 and 11) the author's opinion is that the re-ascending curve tends to an oblique asymptote rather than to a new peak. The increase of load will continue and when all the aggregates will be pulverized, the slope of the asymptote has to coincide with the test machine stiffness.

Physically, the valley zone BCD of the nominal stress versus average strain for the specimen C12 (Fig. 11) reflects the macroscale breakdown of the specimen and the following restructuring of the material into a new stable form.

The knowledge of the full load versus displacement (or nominal stress versus average strain) diagram can be very useful in a load-controlled experimental test. In this case, after the peak load (point B) a snap-through instability is evidenced, and a horizontal jump up to point D occurs. The energy under the curve BCD is released suddenly in a blasting way. Of course, point D corresponds to a new equilibrium configuration, which is usually not achievable, as the instantaneous release of energy is transformed into kinetic energy with expulsion of fragments. The curve, therefore, can be used to determine the specific energy released in the case of load-controlled test.

In concrete diamond drilling it can be observed how the snap-trough phenomenon just described can be very useful to simulate numerically the necessary energy (elastic and kinetic) to break concrete and to determine the chip size. For crushing phenomena (in concrete recycling, for example) it is very interesting to observe that the energy required to fragment a specimen (or in general a concrete element) is much larger than

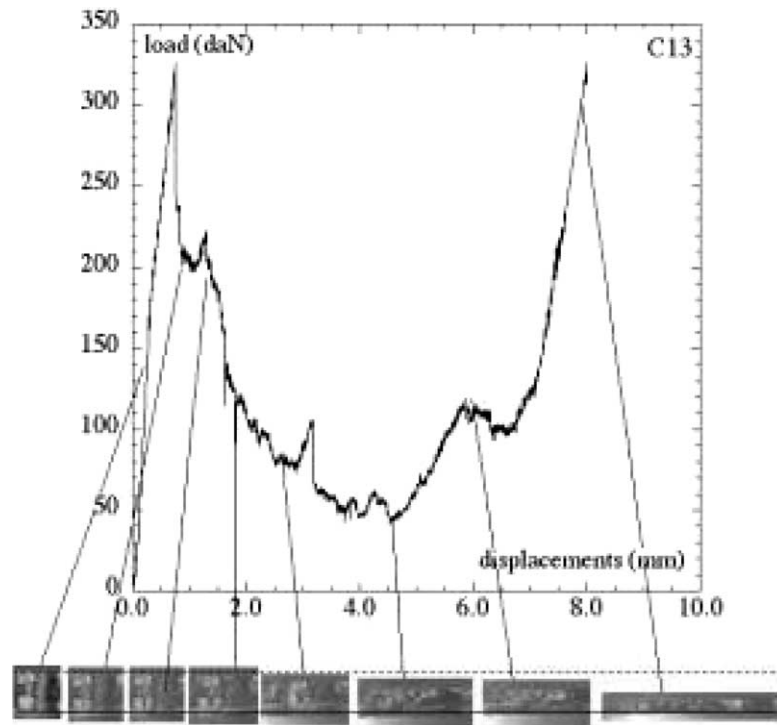


Fig. 10. Complete load versus displacement diagram up to compaction for the smallest specimen.

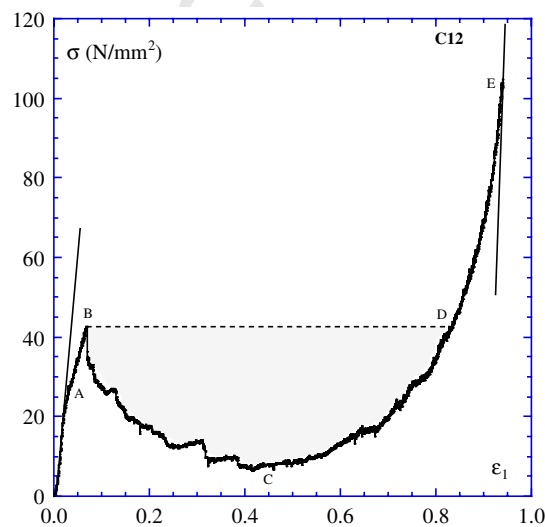


Fig. 11. Complete nominal stress versus nominal strain curve up to compaction for the smallest specimen size.

244 the elastic energy or the energy under the softening curve. This is due to the fact that the fragmentation and
 245 the formation of smaller chips is related to the surface area of the chips and this strongly increases by decreas-
 246 ing the size of the chips.

247 Finally, the last part of the curve can be influenced by the confinement due to the interaction of the aggre-
 248 gates with the machine platens. The interaction can be compared with the confinement effect due to the steel

249 reinforcement in building columns which undergo earthquakes. In this case, if the confinement is well
 250 designed, the evaluation of re-ascending curve for concrete can be very useful.

251 5. Fractal explanation of size effects on dissipated energy density in compression

252 The performed compression tests have shown an evident decrease of dissipated energy density with increas-
 253 ing specimen dimension (Fig. 15). This interesting phenomenon can be interpreted by considering the fragmen-
 254 tation and the comminution theories. In this field, fractal geometry represents a very helpful tool to explain
 255 such a phenomenon [16,17]. Turcotte [18] in the formulation of his fragmentation theory explains the difficul-
 256 ties in developing comprehensive theories. A primary reason is that fragmentation involves initiation and
 257 propagation of fractures. Fracture propagation is a highly nonlinear process requiring complex models even
 258 for the simplest configuration. Fragmentation involves the interaction between fractures over a wide range of
 259 scales. If fragments are produced over a wide range of sizes and if natural scales are not associated with either
 260 the fragmented material, fractal distribution of fragment number versus size would seem to be expected [33].

261 Let us consider a concrete specimen which undergoes a compression test. As is shown in Fig. 4, in the post-
 262 peak softening regime the specimen is characterized by the generation of a large number of fragments. After
 263 fragmentation, the number of fragments N with a characteristic linear dimension greater than r should satisfy
 264 the relation:
 265

$$267 \quad N = \frac{B}{r^D}, \quad (2)$$

268 where B is a constant of proportionality, and D is the fractal dimension.

269 In order to describe the mechanical meaning of the fractal exponent D , in Fig. 12 some examples of discrete
 270 fragmentation model are presented, where fragmentation is a scale-invariant process that leads to a fractal dis-
 271 tribution of chip sizes. Let consider a fractal cube and use it as the basis for a fragmentation model. The frag-
 272 mentation is such that some blocks are retained at each scale but others are fragmented. In Fig. 12a two
 273 diagonally opposed blocks are retained at each scale. For this configuration we have $N_1 = 2$ for $r_1 = \frac{h}{3}$,
 274 $N_2 = 50$ for $r_2 = \frac{h}{9}$, and $N_3 = 1250$ for $r_1 = \frac{h}{27}$. In order to determine D , Eq. (2) can be written as

$$276 \quad D = \frac{\log(N_{n+1}/N_n)}{\log(r_n/r_{n+1})} \quad (3)$$

277 and then we can find for this case that $D = \log 25 / \log 3 = 2.93$. This is the fractal distribution of a discrete set.
 278 The cumulative number of blocks larger than a specified size for the three highest orders are $N_{1c} = 2$ for $r_1 = \frac{h}{3}$,
 279 $N_{2c} = 52$ for $r_2 = \frac{h}{9}$ and $N_{3c} = 1302$ for $r_2 = \frac{h}{27}$, obtaining a value $D = 2.95$.

280 In Fig. 12b eight angular diagonally opposed blocks are retained at each scale. For this configuration we
 281 have $N_1 = 8$ for $r_1 = \frac{h}{3}$, $N_2 = 152$ for $r_2 = \frac{h}{9}$, and $N_3 = 2888$ for $r_1 = \frac{h}{27}$, so that $D = \log 19 / \log 3 = 2.68$. The
 282 cumulative number of blocks larger than a specified size for the three highest orders are $N_{1c} = 8$ for $r_1 = \frac{h}{3}$,
 283 $N_{2c} = 160$ for $r_2 = \frac{h}{9}$ and $N_{3c} = 3048$ for $r_2 = \frac{h}{27}$, obtaining a value $D = 2.70$.

284 In Fig. 12c the limit case of localization is presented in which 18 angular blocks are retained at each scale,
 285 while only nine central blocks are fragmented. For this configuration we have $N_1 = 18$ for $r_1 = \frac{h}{3}$, $N_2 = 162$ for
 286 $r_2 = \frac{h}{9}$, and $N_3 = 1458$ for $r_1 = \frac{h}{27}$, so that $D = \log 9 / \log 3 = 2.00$. The cumulative number of blocks larger than
 287 a specified size for the three highest orders are $N_{1c} = 18$ for $r_1 = \frac{h}{3}$, $N_{2c} = 180$ for $r_2 = \frac{h}{9}$ and $N_{3c} = 1638$ for
 288 $r_2 = \frac{h}{27}$, obtaining a value $D = 2.05$.

289 The same value for D can be obtained by retaining at each scale 18 blocks and fragmenting nine blocks
 290 placed this time in the configuration displayed in Fig. 12d. In this case, $D = 2$, but differently from Fig. 12c
 291 when we obtained a surface in correspondence of the central part of the largest block, we observe that the
 292 fragmentation phenomenon is localized in different small zones. Localization does not mean in this case dis-
 293 sipation on a surface (fracture or shear band), but rather localization of failure in concentrated zones.

294 The fractal dimensions for the discrete set and for the cumulative statistics are nearly equal. In Fig. 13 the
 295 cumulative statistics are reported for three fragmentation models.

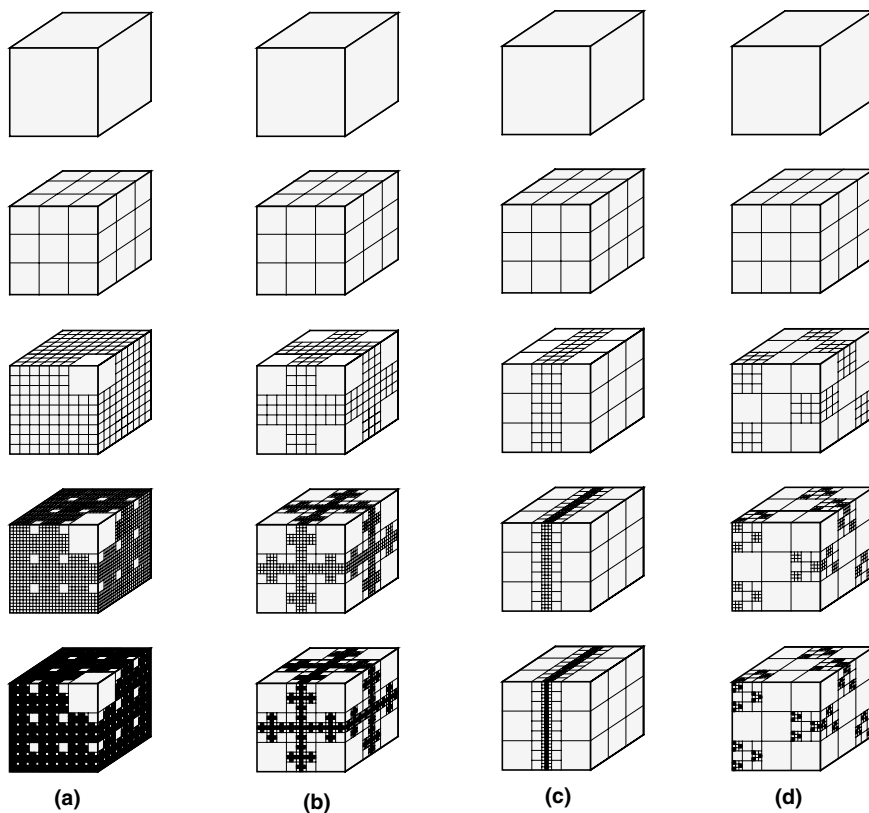


Fig. 12. Physical meaning of exponent D ; (a) at each step only one cube is retained, while all the others are divided into 27 equal-sized cubes with $r_n = \frac{1}{3}r_{n-1}$ ($D = 2.93$), very close to a volumetric fragmentation; (b) at each step the eight angular cubes are retained, while all the others 19 are divided into 27 equal-sized cubes with $r_n = \frac{1}{3}r_{n-1}$ ($D = 2.70$); (c) and (d) at each step the nine central cubes are divided into 27 equal-sized cubes with $r_n = \frac{1}{3}r_{n-1}$, while the others 18 are retained ($D = 2.00$), showing a localization of the dissipation energy.

296 The total volume (mass) of fragments is given by [33]

299
$$V_f = \int_{r_{\min}}^{r_{\max}} r^3 dN, \tag{4}$$

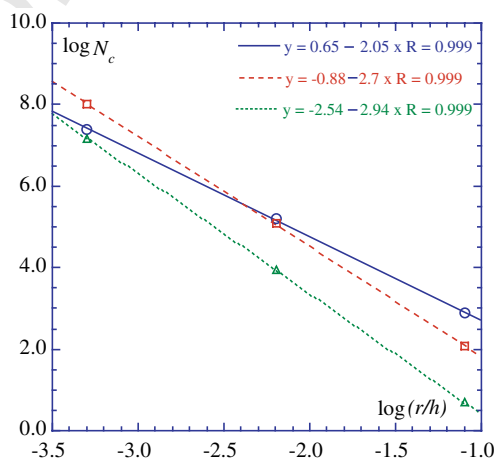


Fig. 13. Cumulative statistics for the fragmentation models proposed in Fig. 12.

300 since r has been defined to be the cube root of the fragment volume. It is expected that there will be upper and
 301 lower limits to the validity of the fractal (power-law) relation for fragmentation. The upper limit r_{\max} is gen-
 302 erally controlled by the size of the fragmented object ($r_{\max} = k\sqrt[3]{V}$ [34]). The lower limit r_{\min} is likely to be con-
 303 trolled by the scale of the heterogeneities responsible for fragmentation (grain size). For a power-law
 304 (monofractal hypothesis) distribution of sizes, substituting $dN = -BDr^{(-D-1)} dr$, with B a constant of integra-
 305 tion, into Eq. (4) and integrating gives:

$$307 \quad V_f = \frac{-DB}{3-D} (r_{\max}^{3-D} - r_{\min}^{3-D}), \quad (5)$$

308 where, for $0 < D < 3$, the specification of r_{\min} is not necessary. In this case the volume (mass) of fragments is
 309 predominantly provided by the largest fragments, so that V_f can be rewritten as

$$312 \quad V_f = \frac{-DB}{3-D} r_{\max}^{3-D} \quad \text{for } 0 < D < 3, \quad (6)$$

313 and then

$$316 \quad V_f = \frac{-DB}{3-D} k^{3-D} V^{1-\frac{D}{3}}. \quad (7)$$

317 On the other hand, the total surface area A_f of the fragments is given by

$$320 \quad A_f = C \int_{r_{\min}}^{r_{\max}} r^2 dN, \quad (8)$$

321 where C is a geometrical factor depending upon the average shape of the fragments. For a power-law distri-
 322 bution, substitution of $dN = -BDr^{-D-1} dr$ into Eq. (8) and integrating gives:

$$324 \quad A_f = \frac{-DBC}{D-2} \left(\frac{1}{r_{\min}^{D-2}} - \frac{1}{r_{\max}^{D-2}} \right). \quad (9)$$

325 If $0 < D < 2$, it is necessary to specify r_{\max} in order to obtain a finite total surface area for fragments. If $D > 2$,
 326 it is necessary to specify r_{\min} in order to constrain the total surface area to a finite value. Usually the surface
 327 area of the smallest fragments dominates:

$$329 \quad A_f = \frac{-BDC}{D-2} \frac{1}{r_{\min}^{D-2}}. \quad (10)$$

330 It can be assumed that the energy dissipated to produce a new free surface in the fragmentation process is
 331 provided by the product of specific energy absorbing capacity β_F and the total surface area A_f , for
 332 $2 < D < 3$ [17]:

$$334 \quad W = \beta_F A_f = \beta_F A_f \frac{V}{V} \quad (11)$$

335 in which β_F should have dimension of $[F][L]^{(D-1)}$. From Eq. (7), V can be expressed as

$$337 \quad V = V^{D/3} V_f \frac{3-D}{-DB} k^{D-3} = \frac{r_{\max}^3}{k^3}. \quad (12)$$

338 In this case it is possible to have, from Eqs. (6) and (7):

$$341 \quad W = \beta_F A_f \frac{V}{V} = \beta_F A_f \frac{V_f V^{D/3} \frac{3-D}{-DB} k^{D-3}}{\frac{r_{\max}^3}{k^3}} = \left(\beta_f \frac{-BCD}{D-2} r_{\min}^{2-D} r_{\max}^D k^D \right) V^{\frac{D}{3}} = \mathcal{G}_F^* V^{\frac{D}{3}}. \quad (13)$$

342 The two extreme cases contemplated by Eq. (13) are $D = 2$, surface theory [35,36], when the dissipation really
 343 occurs on a surface ($W \propto V^{\frac{2}{3}}$) and by $D = 3$, volume theory [37], when the dissipation occurs in a volume
 344 ($W \propto V$). In this case \mathcal{G}_F^* presents the following physical dimension:

$$346 \quad [\mathcal{G}_F^*] = \left(\beta_f \frac{-BCD}{D-2} r_{\min}^{2-D} r_{\max}^D k^D \right) = [F][L]^{D-1} [L]^{2-D} [L]^{-D} = [F][L]^{1-D}. \quad (14)$$

347 For $D = 2 \rightarrow [\mathcal{G}_F^*] = [\mathbf{F}][\mathbf{L}]^{-1}$, which is the canonical dimension for fracture energy, while for
 348 $D = 3 \rightarrow [\mathcal{G}_F^*] = [\mathbf{F}][\mathbf{L}]^{-2}$, which is the physical dimension of stress. The experimental cases of fragmentation
 349 are usually intermediate ($D \cong 2.5$) [18], as well as the size distribution for concrete aggregates due to Fuller
 350 [38].

351 If we consider $V = h^3$, we can write the expression of the dissipated energy density, from Eq. (13):

$$353 \quad S = \frac{W}{V} = \mathcal{G}_F^* h^{D-3}. \quad (15)$$

354 The relationship of dissipated energy density related to different sizes can be posed in logarithmic form:

$$357 \quad \log S = \log \mathcal{G}_F^* + (D - 3) \log h. \quad (16)$$

358 Eq. (16) represents a straight line with slope $(D - 3)$ in the $\log S$ versus $\log h$ plane (Fig. 14). If $D = 2$, the slope
 359 is -1 , as well as $D = 3$ implies a vanishing slope.

360 The same results can be obtained in a different way, by considering a sequence of scales of observation [39].
 361 Considering W as the global dissipated energy measured by the experimental set-up, \mathcal{G} as the elastic energy
 362 release rate or the specific energy necessary to generate the unit area of fracture, which is by hypothesis invari-
 363 ant with respect to the scale of observation, we have:

$$365 \quad W = \mathcal{G}A, \quad (17)$$

366 and then:

$$368 \quad \mathcal{G} = \frac{W}{A} = \frac{SV}{A} = \frac{Sh^3}{h^2} = Sh. \quad (18)$$

369 If we consider a sequence of scale of observation, we have:

$$371 \quad \mathcal{G} = S_1 h_1 = \dots = S_{n-1} h_{n-1} = S_n h_n = S_{n+1} h_{n+1} = \dots = S_\infty h_\infty, \quad (19)$$

372 where the first scale of observation could be the macroscopic one, with $S_1 h_1 = Sh$, h being the characteristic
 373 linear dimension of the specimen, and the asymptotic scale of observation could be the microscopic one, with
 374 $S_\infty h_\infty = \mathcal{G}_F^* h^*$, h^* being the measure of the fractal set representing the fragmented configuration. It is impor-
 375 tant to underline that the measure h^* assumes finite value only for one particular value of d_ω equal to the frac-
 376 tal dimension of the set (Hausdorff dimension). For any other values of d , $h^* = 0$ for $d < d_\omega$ and $h^* = +\infty$ for
 377 $d > d_\omega$. From the equality between the extreme members we can write:

$$379 \quad S = \mathcal{G}_F^* \left(\frac{h^*}{h} \right), \quad (20)$$

380 or

$$381 \quad S = \mathcal{G}_F^* \left(\frac{h^{1-d_\omega}}{h} \right), \quad (21)$$

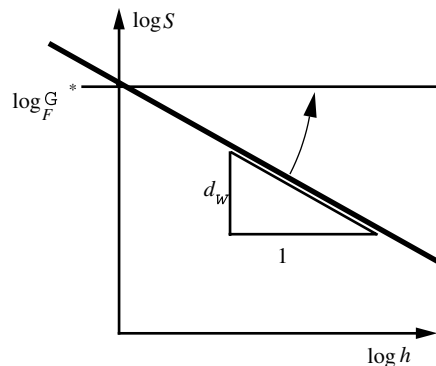


Fig. 14. Size effect on dissipated energy density in compression.

384 where $0 < d_\omega < 1$ is the decrement of the topological dimension due to fragmentation. Taking the logarithms
 385 of both members of Eq. (21) we obtain:

$$388 \quad \log S = \log \mathcal{G}_F^* - d_\omega \log h, \quad (22)$$

389 where $d_\omega = 3 - D$ can be considered as the decrement of the topological dimension of the set in which energy
 390 dissipation occurs. When $d_\omega = 1$ we obtain $D = 2$ (localization); when $d_\omega = 0$ we have $D = 3$ (volumetric dis-
 391 sipation). Localization assumes here the meaning of dissipation localized in concentrated zones. Eq. (22) is
 392 identical to Eq. (16).

393 The values of dissipated energy density for the three smallest sizes are plotted in Fig. 15a against the speci-
 394 men size in a bilogarithmic plane. The values for the four available sizes are instead reported in Fig. 15b. The
 395 size effect is represented by the slope of the linear regression of the points of the diagram. It is evident how the
 396 dissipated energy density decreases with increasing specimen size.

397 As may be seen from Fig. 15, the slope of the dissipated energy density decrease proves to be equal to 0.67
 398 when only three specimen sizes are considered, and to 0.97 when considering the fourth size. We have consid-
 399 ered the two different cases as the fourth size has been tested with a different procedure which can cause vari-
 400 ations in energy estimation. In the former case, the physical meaning reveals an energy dissipation on a fractal
 401 space of dimension 2.33, while in the latter case the dissipation occurs on a fractal space of dimension 2.03, i.e.,
 402 very close to a two-dimensional surface. In the second case, as the fractal space is close to a two-dimensional
 403 surface, different interpretations, by using the classical euclidean geometry, could be also proposed. In this
 404 approach, however, the fractal approach has been chosen. It is therefore possible to obtain a constant (uni-
 405 versal) dissipated energy density equal to $31 \text{ N mm}^{-1.33}$ and to $74 \text{ N mm}^{-1.03}$, respectively (Fig. 16). The gra-
 406 phic interpretation of the renormalization procedure is given in Fig. 14. The assumption of a fractal (or
 407 anomalous) physical dimension allows the determination of the dissipated energy density parameter \mathcal{G}_F^* , which
 408 results in independence of the scale. As it is easy to observe, in the latter case the renormalized dissipated
 409 energy density tends to be a fracture energy, the dissipation occurring on a fractal set very close to a two-
 410 dimensional surface. Such a result confirms the localization of the dissipation on a surface [25]. The fractal
 411 nature of the fragments generated by the compressive test emerges very clearly at the size scale of the speci-
 412 mens. Momber [10] applied fragmentation theory to the study of compression and analyzed the fragments,
 413 determining a fractal exponent D close to 2. On the other hand, the property of self-similarity is very likely
 414 to vanish or change at higher or lower scales, owing to the limited character of the particle size curve. The
 415 price to pay for obtaining a constant value is the loss of the classical physical dimensions for dissipated energy
 416 density. It is obviously very difficult to use these results in a structural analysis, a noneuclidean (or fractal)
 417 mechanics being not yet available, even if very important steps have been moved forward by Carpinteri
 418 et al. [40].

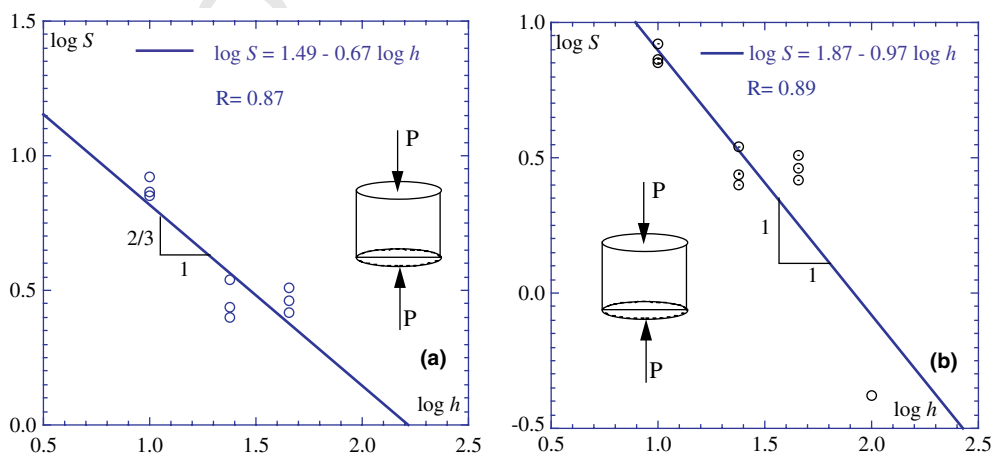


Fig. 15. Size effect on dissipated energy density (experimental tests).

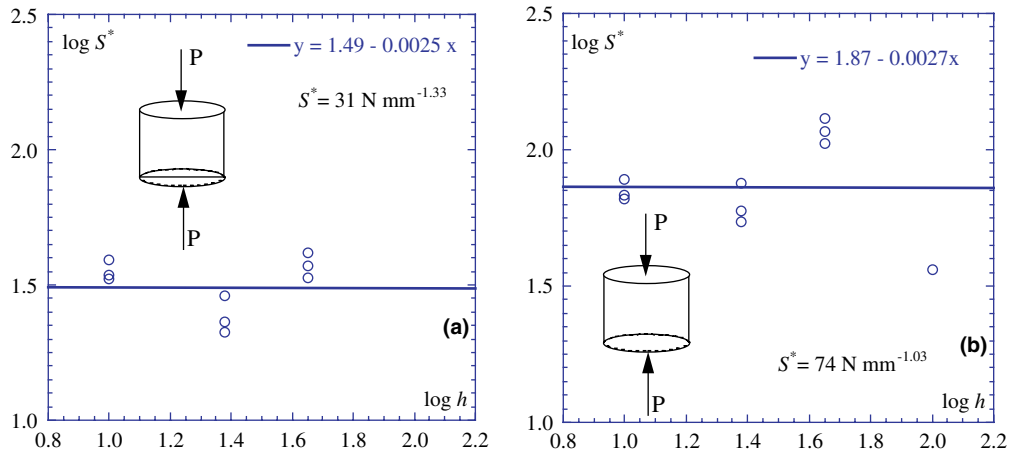


Fig. 16. Renormalized value of dissipated energy density (experimental tests).

419 6. Scale-independent constitutive law for concrete in compression

420 The experimental curves σ versus ϵ , reported in Fig. 5, show a marked scale dependence, in particular for
 421 what concerns the post-peak part. These curves σ versus ϵ , or F versus δ , are in fact characterized by two dif-
 422 ferent regimes. The first regime corresponds to the pre-peak elastic behavior, when microcracks form ran-
 423 domly in the specimen. At this stage the external force linearly increases until it reaches the peak value and
 424 the statistical fluctuations are very small. In the second regime, which could be called “catastrophic”, the inter-
 425 actions between the microcracks begin to rule the process, until macrofractures form and propagate through
 426 the whole specimen. In large specimens this phenomenon could occur with a sudden release of stored elastic
 427 energy.

428 In this section, a renormalization procedure is proposed to obtain a unique constitutive relationship for
 429 softening in compression. By assuming damage occurring in a fractal sub-domain inside the specimen, energy
 430 dissipation becomes scale-dependent. Hence it should be substituted by a fractal quantity, which is the true
 431 material constant. The assumption that the energy dissipation occurs in a sub-domain characterized by a frac-
 432 tal dimension, imposes the definition of fractal strain (or dilatation).

433 Let us consider the external work W , which presents the physical dimension of $[F][L]$. The nominal dissi-
 434 pated energy density, $S = W/V$, is usually the dissipated energy over the specimen volume, so that it presents
 435 the physical dimension of $[F][L]^{-2}$ and can be evaluated by integration:
 436

$$438 \quad S = \frac{W}{V} = \int_0^{\epsilon_{\max}} \sigma(\epsilon) d\epsilon, \quad (23)$$

439 which represents the area under the σ - ϵ curve. Supposing that the energy dissipation does not occur in the
 440 specimen volume ($V \propto l^3$) but in a fractal domain of dimension D ($V \propto l^D$), and considering $[\sigma] = [F][L]^{-2}$
 441 as the nominal stress, in order to obtain a constant specific compression energy, the strain has to assume a
 442 physical dimension of $[L]^{-(D-3)} = [L]^{d_w}$ [40,41]. In fact, in this hypothesis, if W is dissipated over a domain with
 443 physical dimension of $[L]^D$, we obtain:

$$445 \quad [S] = \frac{[W]}{[V]} = \frac{[F][L]}{[L]^D} = [F][L]^{1-D}. \quad (24)$$

446 For $D = 2$ (surface theory, dissipation occurring on a surface) $\rightarrow S = [F][L]^{-1}$, while for $D = 3$ (volume the-
 447 ory, dissipation occurring on a volume) $\rightarrow S = [F][L]^{-2}$. Assuming to maintain the nominal stress σ with phys-
 448 ical dimension of $[F][L]^{-2}$, from Eq. (23) we have:

$$450 \quad [S] = [\sigma][\epsilon^*] = [F][L]^{-2}[L]^x = [F][L]^{1-D}, \quad (25)$$

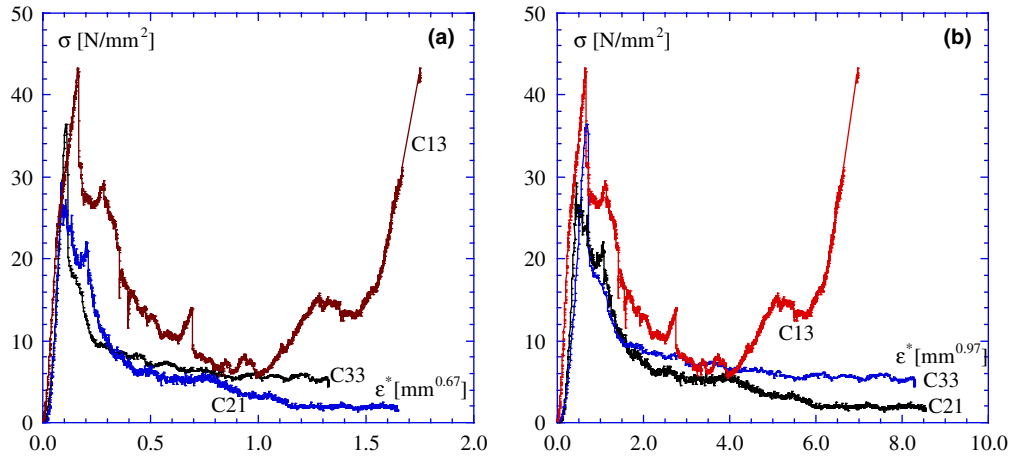


Fig. 17. Stress versus renormalized strain for three different specimen sizes: (a) $D = 2.33$; (b) $D = 2.03$.

451 and than:

$$453 \quad x = 3 - D = d_o. \quad (26)$$

454 In the monofractal hypothesis, the renormalized strain therefore assumes the physical dimension of $[L]^{3-D}$,
 455 defined as the displacements δ divided by $l = [L]^{D-2}$.

456 By considering the fractal strain, a scale-invariant constitutive relationship can be obtained. In other words,
 457 the experimental diagrams related to the different sizes can be rescaled by considering the strain renormaliza-
 458 tion, and a clear superposition of the curves is evidenced. In Fig. 17a the strains are renormalized for $D = 2.67$
 459 as obtained from the fitting that considers only three sizes, while in Fig. 17b the dimension $D = 2.03$ has been
 460 used. It is possible to observe how the curves tend to superpose one on each other and in particular how the
 461 variation in structural behavior disappears.

462 Lastly, from Fig. 17a and b, it can be observed how a renormalization (or a new definition) of the elastic
 463 modulus comes out. In fact, the elastic modulus is defined, from the classical Hooke law's, as the ratio between
 464 the stress and the strain. In the present analysis we obtain:

$$466 \quad [E^*] = \frac{[\sigma]}{[\epsilon^*]} = \frac{[F][L]^{-2}}{[L]^{-(D-3)}} = [F][L]^{D-5}, \quad (27)$$

467 and in the two limit cases for $D = 2$ (surface theory, dissipation occurring on a surface) $\rightarrow E = [F][L]^{-3}$, and
 468 assumes the physical dimension of a density, while for $D = 3$ (volume theory, dissipation occurring on a vol-
 469 ume) $\rightarrow E = [F][L]^{-2}$ and we obtain the classical elastic modulus.

470 The renormalization previously presented is in good agreement with the methodology proposed by van
 471 Mier [42] in order to obtain a unique empirical stress–displacement relationship and applied to the present
 472 experimental curves (Fig. 8). In fact, in our results (Fig. 17) the renormalization strain has a physical dimen-
 473 sion equal to 0.97, very close to 1, and then very close to a displacement. What is important to emphasize at
 474 this stage is that in compression we have dissipation of the energy over an area at small scales, while at large
 475 scales the energy dissipation occurs in a volume. This appears very interesting as it is the opposite trend with
 476 respect to tension, in which localization is evident for large specimens and not at small scales. Eventually, the
 477 renormalization procedure for large specimens ($D = 3$) tends again to a stress–strain diagram, as $\epsilon^* = \epsilon$.

478 7. Conclusions

479 Uniaxial compression tests were performed under displacement control on drilled cylindrical specimens
 480 obtained from a single concrete block over a very large scale range (1:19), the largest range available in the
 481 literature. The friction between specimen ends and testing machine platens was reduced by using two layers

of teflon with oil in between. The experimental results show how, reducing the friction at the ends, the nominal compressive strength is not clearly affected by scale effects as is evident in tension. The tests performed on very small specimen sizes (10 mm diameter) and the independence of the dimension for the compressive strength allows us deep considerations for evaluating the effective strength of real concrete structures, that undergo deterioration of the mechanical properties and represent a dramatic problem in civil engineering. The results can be considered as a starting point in order to reconsider the standard dimension for evaluating the compressive strength of high-strength concretes, using the same testing machines available in the laboratories.

In addition, the full stress–strain curve in compression has been determined, and from those results it can be evidenced a snap-through instability allowing to quantify the energy required for fragmentation. The determination of the full curve is also important in concrete recycling industry to quantify the energy necessary for destroying concrete structures, even in blasting.

Finally, a theoretical explanation for the size effect on the dissipated energy density has been proposed, from which it appears how the dissipation occurs in a fractal sub-space of dimension comprised between a surface and a volume. A renormalization procedure for determining a constant dissipated energy density is also proposed and it comes out that, in the range of the tested specimens, the energy dissipation occurs in a fractal space very close to a two-dimensional surface. This is in a good agreement with the hypothesis proposed by van Mier [22] and by Lee and Willam [25], which stated that energy dissipation in the post-peak regime is a localized surface-dominated fracture process, after the coalescence of microcracks in the peak regime. This hypothesis is however valid only when small specimen sizes are used, whereas for large specimen sizes a volumetric dissipation occurs, as proposed by Carpinteri and Ferro [43] in a forthcoming paper.

Acknowledgements

The present research was carried out with the financial support of the Ministry of University and Scientific Research (MIUR) under the grant PRIN 2003 “Damage mechanics and durability of ordinary and high performance concrete”. The author is thankful to Mr. Vincenzo Angilletta for his technical support in performing the experimental tests.

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