HIGH HOMOGENEITY SOLENOIDAL MAGNET FOR CESIUM FREQUENCY STANDARD

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Abstract- In Cs frequency standards a zone of highly uniform magnetic flux density, lower in value of 0.1 T, is required. The magnetic homogeneity value is tied to the overall accuracy of the standard and, for an accuracy of $10^{-14}$, a design value 1 p.p.m. is required. For this purpose a resistive solenoid 700 mm long with a bore radius of 32.5 mm has been designed and built. This paper reports the design process, the measurements on the manufactured magnet and the shimming strategy used to reach the homogeneity target.

I. INTRODUCTION

The magnetic flux density value required in an high field Cs beam resonator is of 82 mT. For an accuracy of $10^{-14}$, a field homogeneity of the order of 1 p.p.m. is needed over the whole interaction region which is a cylindrical volume 400 mm long and of 8 mm in diameter [1]. This goal is clearly very difficult to reach because of the mechanical constraints it imposes on the manufacturing of the magnet. In a dipole magnet, in fact, this value of homogeneity could be reached with a gap width accurate at $10^{-20}$, considering the gap width of the dipole this would lead to a value of mechanical tolerance on the width of 30 nm.

Therefore this constraint does not allow to use a ferromagnetic dipole magnet for this purpose, even if it would have very attractive characteristics as, for instance, the possibility to build it with permanent magnets, cutting the power consumption and the self shielding of the structure by external fields.

On the contrary, high homogeneity values seem to be more easily obtainable if a solenoidal magnet is used. This can be explained by two main arguments: multilayer windings can have a statistical compensation of different winding layers and homogeneity depends in this case by the area of the cross section of the solenoid. Even if the last argument is in favour of the solenoidal solution, it poses a limit on the mechanical tolerance of the bobbin on which the coil is wound.

Another reason that makes the solenoidal solution more convenient is related to the fact that the longitudinal field matches well with the microwave cavity, in which beam holes can be opened without excessively disturbing the rf field distribution.

On the contrary, the solenoidal solution requires the shielding of the magnet from possible external disturbances. This fact compels to build a ferromagnetic shield around the solenoid that has the twofold purpose of flux return and shield.

In the following the design strategy used will be presented and the results of the measurements performed on the manufactured magnet will be discussed. Finally the shimming procedure used to reach the design homogeneity value will be presented.

II. DESIGN OF THE SOLENOIDAL MAGNET

The proper design of the solenoidal magnet system requires the evaluation and the assessment of several quantities that are related to the magnetic field behaviour. In addition, the presence of the ferromagnetic yoke makes the analysis of the structure impossible to perform without a magnetic analysis tool. Fortunately the structure has a rotational symmetry and then can be simulated in a meridian cross section avoiding a three dimensional analysis of the whole device. A commercial finite element two dimensional code, VF/OPERA2d, was extensively used for the design of the magnet structure.

The overall structure of the solenoidal magnet is shown, out of scale, in its half-meridian cross section in Fig. 1. All the design procedure was based on the homogeneity target of 1 p.p.m. in a cylindrical volume 400 mm long and 8 mm in diameter.

![Fig. 1 Half-meridian cross section of the magnet (out of scale).]
The first quantities to be defined were the internal bore diameter of the solenoid and its length.

It was decided to have the solenoid wound directly on the copper pipe used as vacuum chamber. The external diameter of this pipe was set to 70 mm both to conform to standard vacuum equipment (NW63), and to compromise between power consumption of the magnet, proportional to its internal bore, and useful inner space for the microwave cavity.

The length should not be too short because of the homogeneity value of the field, but its value is limited by the external encumbrance of the magnet. Accordingly to the external constraints on dimensions and time of flight of the Cs atoms, a compromise value of 700 mm was decided.

Around the solenoid a ferromagnetic tube is put to serve both as flux return and as magnetic shield. A high permeability material was required to limit the weight of the structure, so a mumetal material was chosen. The dimensions of the yoke required a magnetic analysis to compute the value of thickness and diameter of the tube and to evaluate the effect of the closing plates.

The analysis had to take into account the nonlinear characteristic of the material, in fact, while the mumetal has very high relative permeabilities values at relatively low magnetic flux density values, its saturation level is of about 0.7 T, the nonlinear characteristic of the material used is shown in Fig. 2.

The aim of the analyses performed was to optimize the efficiency of the material, that is to have a working point near the maximum permeability value. The design variables used in this case were the diameter of the yoke, its thickness and the thickness of the closing plates. The best efficiency was obtained for a tube of 240 mm diameter and 2 mm thick while the end plates had a thickness of 12 mm. In this configuration the working point of the material is about 0.4 T with a value of relative permeability of about $3 \times 10^5$.

Another effect checked by calculations, was the gap in the closing plates at the crossing of the vacuum pipe. This gap, 77 mm thick, causes an increase in the reluctance of the overall magnetic circuit and can locally alter the distribution of the magnetic flux lines. The computations performed allowed to quantify the effect of the gap and these were acceptable in the design. A zooming of the meridian cross section of the magnet in the zone of the end plates is shown in Fig. 3.

![Fig. 3 Grey scale map of the absolute value of magnetic flux density in the end plates.](image)

The magnetic analysis code was also used to quantify the effect of a defect on the copper pipe on which the solenoid is wound. The most probable defect consists of a bulge in the external surface of the pipe that can locally increase the inner diameter of the solenoid leading to a lower value of field on axis. This situation has been simulated by means of a triangular defect extending for 100 mm with a maximum depth of 10 μm. The relative variation got from this configuration is shown in Fig. 4. It is then clear from the magnitude of the relative variation of the field on axis that this is one of the most dangerous causes of error between the theoretical model and the manufactured magnet.

![Fig. 4 Relative variation of the absolute value of magnetic flux density on axis caused by a defect in the copper pipe (defect described in text).](image)

### III. Realization of the Magnet

The magnet has been wound around a cylindrical calibrated copper pipe 700 mm long and with an external diameter of 70 mm. At the extremities of the cylinder two ISO standard stainless steel NW63 conflat flanges are
braided in order to connect them with the vacuum system which must ensure a large free path for the atomic flux travelling along the magnet axis.

The external surface of the pipe was machined at a mechanical tolerance of 10 micrometer; this value was chosen taking into account that the static flexion in the middle is of the same order of magnitude.

The mumetal ferromagnetic pieces were obtained by extrusion and simple mechanical workmanship. After that, a thermal treatment in a vacuum chamber at 900°C allowed to reach a typical magnetic permeability in the order of 10^5.

For the winding a 3x1 mm rectangular cross section copper wire was used. The wire was electrically insulated by a thin enamel layer with a thermal conductivity of about 0.1 W/mK. The wire was wound by a mechanical lathe rotating at 10 rpm and it was stretch out by a vice between two jaws made in bakelite.

The magnet is then constituted by 24 superimposed layer of 220 coils.

During the winding a thermally conductive silicon paste was smeared to fill the air gaps between the layers and then to help the radial heat flux.

The latter was a significant problem because the Joule effect generates 0.8 KW and the surface heat flux density is 0.2 W/cm².

In account of this fact, a water cooled jacket extracts the dissipated heat by means of two double copper pipes, with an inner diameter of 3 mm, which are wound around the external surface of the magnet and fixed with two copper cylindrical shells.

A 3 liter per minute flux of room temperature water is adequate to ensure a temperature of 46 C along the axis with a difference of 0.1°C between the center and the extremities of the winding.

At this moment an open cooling circuit is used, but a servo digital control of the pumping speed to achieve a thermal stability of the bobbin better than 0.1 C during the field measurement is foreseen.

The magnetic field uniformity of 10⁻⁶ requires also an equivalent current stability of 10⁻⁶. Because the relatively high power necessary to create the 82 mT of the magnetic flux density and for the better line ripple control, a switching power supply with a frequency modulation of 100 KHz was designed.

The transfer function of the inductance and the shielding factor of the copper pipe reduce at -200 dB, at least, the 100 KHz frequency inside the vacuum chamber. The 300 Hz line ripple, coming from a six-phase rectifier bridge, is reduced by the switching servo control under the 80 mV level.

This power supply has a 10⁻⁶ stability over an integration time of 1s. To reduce the current drift a series regulator, working like a variable resistor, controls the power supply by a manganin resistor which converts the exciting current in a voltage signal. The latter is then amplified and compared with a LTX1000 voltage reference.

Grounds problems and switching interference with the control loop limit at few 10⁻⁶ the current stability. The work in progress is an E.M. compatibility approach to reduce the noise investigating also in the ripple problems caused by a triangle connection in the three-phase transformer.

In the same time a linear power supply with a classical scheme is also realized.

IV. MEASUREMENTS ON THE MAGNET

Once the magnet has been manufactured a magnetic induction measurement session was started. The measurements were performed by means of a commercial NMR equipment, with a transfer function of 43 MHz/T and a resolution of 1 Hz. In this way a resolution of 3x10⁻⁷ was realized at an operating magnetic flux density value of 82 mT which gives raise to an NMR frequency of 3.5 MHz.

One unresolved problem in this measures was the long term stability of the power supply that did not allowed to reach the 1 p.p.m. required because of current instabilities. The last fact anyway was not a problem because the field homogeneity was by far worse, in the order of 10⁻⁴ rms.

The measured values of homogeneity are thought to be mainly caused by variations in the outer diameter of the copper pipe because the obtained values matches quite well with the ones theoretically predicted.

In Fig. 5 the measured pattern of the homogeneity of field along the axis of the magnet is shown.

![Fig. 5 Measured values of magnetic flux density along the axis in the central area of the solenoid.](image)

V. SHIMMING STRATEGY

Because the measured value of the field homogeneity was two order of magnitude greater than the target value of 1 p.p.m. rms a shimming strategy was developed. This shimming can be performed in two ways: active shimming by means of current turns placed on the outer surface of the main winding, passive shimming by means of ferromagnetic rings placed inside the vacuum chamber.

Both techniques should give good results but have different characteristics. Current shims have a wider action length but can be easily set and moved, on the contrary, iron shims can give finer corrections but, since they are put inside the vacuum chamber, their placing and movement can be
cumbersome. For this reason the active shimming has been chosen in this first phase, even if it is possible in the future that a combined strategy, using both kinds of shimming, will be used.

The active shimming strategy can use two degrees of freedom for each turn, that are the position on the outside surface of the winding and the exciting current values.

A very simple mathematical model for a single circular coil shows that the intensity of the magnetic field on the axis halves at a distance equal to the radius. This large effect of a single coil around its proximities makes hard the correction with high accuracy when high local variations occur. Another problem is the positioning of the coils, in fact, an error of 1 mm in position causes a $10^{-3}$ error on the correcting field.

The positions and the current values for a given set of correcting coils have to be determined by a proper optimization technique. Fortunately, the system showed to be linear with the current values used for the correction. This fact allowed to optimize the field by means of a least square approach. Considering the position of the turns assigned, for instance equispaced along the area of interest or placed in correspondence of the greater variations of the field pattern, their contribute to the magnetic field is linear with the current value. This property can be used to set up a least squares problem that tries to minimize the error between the field obtained by the sum of the main field and of the shims and the uniform desired value.

This approach gives very good results in terms of the resulting homogeneity but, unfortunately, the resulting current values are very high.

The solution can be improved by solving a constrained least squares problem that minimize, besides the error on the field, also the energy associated to the shimming coils.

This approach turned out to be successful and was then used to compute the coil currents.

Several shimmings with different number of turns were computed and realized. With seven independent power supplies a 50 p.p.m. homogeneity, with current values less than 10 A was obtained. To improve this result 18 coils were used, but at the same time, the number of current supplies was reduced, trying to obtain them as a linear combination of some base values. This process leads to a deterioration of the homogeneity since the current values are not the theoretical evaluated ones. Anyway this worsening turned out to be roughly in the order of 10% with a significantly reduction in complexity of the power supply.

The best result achieved, at this moment, is an homogeneity value of 15 p.p.m. rms obtained using 7 currents supplies and 18 shim coils. The main limitation to the improvement of this value seems to be the stability of the exciting current of the magnet because the starting problem about the positioning of the coils has been circumvented getting the position from the field measurements on the axis. Furthermore, theoretical calculations show that an homogeneity value better than 10 p.p.m. can be get with 25 independent coils.

The best results obtained in terms of homogeneity of magnetic flux density are shown in Fig. 6, while in Fig. 7 the values of current as function of the turn position are depicted.

![Fig. 6 Original (o) and corrected (△) measured patterns of field homogeneity along the axis of the magnet.](image)

![Fig. 7 Shimming current as function of turn number for the homogeneity distribution shown in Fig. 6.](image)

**VI. CONCLUSIONS**

The design process and the manufacturing of an high homogeneity solenoid has been presented. The design tools used have given reliable predictions on field values and pattern. The limited budget in manufacturing unfortunately did not allow to reach the design target on homogeneity, anyway the work is going on to perform a correction system that should meet the design requirements.

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