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This is the author version of an article published in
S. Zucca, D. Botto, M.M. Gola, Faster on-line calculation of thermal stresses by time integration, International Journal of Pressure Vessels and Piping, Volume 81, Issue 5, May 2004, Pages 393-399, ISSN 0308-0161, 10.1016/j.ijpvp.2004.03.012.
The article is available on-line at:
http://www.sciencedirect.com/science/article/pii/S0308016104000808

Faster on-line calculation of thermal stresses by time integration

Zucca* S. , Botto D. , Gola M.M.
Politecnico di Torino – Department of Mechanical Engineering
C.so Duca degli Abruzzi 24, 10129 Torino - Italy

Abstract
The Green's function technique (GFT) is largely used for on-line calculation of thermal stresses in machines and plants, because it allows turning parameters such as fluid temperatures, pressures and flow rates directly in thermal stresses.
Recently the use of the GFT has been extended by the authors to thermo-mechanical models having variable convective coefficients. The novel methodology is made of two steps. First of all, boundary temperatures are evaluated by time integration of a reduced thermal model and then thermal stresses are calculated by means of the GFT using as inputs the boundary temperatures previously evaluated. The new approach implies a large number of convolution integrals to be solved for thermal stress calculation.
In order to reduce computation time it is here proposed to replace the convolution integrals which characterise the GFT with a reduced model of uncoupled first order differential equations, whose coefficients are estimated fitting the Green's functions of the thermo-mechanical model with a sum of exponential terms. Thermal stresses are obtained by time integration of the model.

Keywords: *Green's functions, non-linear fitting, on-line calculation, reduced model, thermal stresses*

1 Introduction
On-line calculation of thermal stresses is currently performed in machines and structures, characterised by high safety requirements and/or expensive maintenance costs (e.g. aircraft engines, nuclear power plants) in order to directly relate the fatigue damage accumulation to the actual load histories.
Ad-hoc low order models are to be developed because detailed FE models are too large to be employed for on-line processing.
A largely used methodology is the Green's Function Technique (GFT). The time histories of the outputs are evaluated from the time histories of the inputs solving a set of convolution integrals. The only data necessary to perform the calculation are the Green's functions of the thermo-mechanical model, i.e. time histories of thermal stresses due to unit step inputs.
In linear thermo-mechanical models, the GFT is used to evaluate on-line thermal stresses from the time history of fluid temperatures (Figure 1a) around the component, by-passing the metal temperature evaluation (Refs. [1]-[3]).
On the contrary, in applications characterised by variable convective coefficients, the direct calculation of thermal stresses from fluid temperatures is not possible, because the system is not linear and the superposition principle does not apply.
For this class of applications a novel methodology has been recently proposed by the authors [4]. It is based on two steps:
1. temperature calculation at nodes located at the boundaries of the body by time integration of a reduced thermal model obtained by means of Component Mode Synthesis (Ref. 5);
2. thermal stress calculation by means of the GFT using as inputs the time history of the boundary temperatures (Figure 1b) evaluated at step 1.

This approach allows splitting the thermal problem and the thermo-mechanical problem, confining the non-linearity within the former.

* Corresponding author: tel +390115646953; fax: +390115646999
email address: stefano.zucca@polito.it

The comparison of Figure 1a and Figure 1b shows that the main implication of this approach is that the number of inputs for thermal stress calculation increases with respect to the linear case.
As a consequence, a large number of convolution integrals has to be performed at step 2, making the calculation very time consuming for applications characterised by many variable convective coefficients.
In order to reduce the numerical burden due to calculation of the convolution integrals at step 2, in this paper a method is proposed to replace the GFT with a reduced model made of uncoupled first-order differential equations. It is a different version of a method developed for feedback control in thermal applications ([6]).
The off-line procedure necessary to build the reduced model consists of three parts:

1. calculate, by means of a detailed thermo-mechanical model, the Green's functions (responses to unit step inputs) for thermal stresses that are to be calculated on-line;
2. fit each Green's function with a sum of exponential terms by means of a non-linear least squares fitting algorithm based on the method of conjugate gradients;
3. evaluate coefficients of the reduced model from amplitudes and exponents of series estimated at point 2.

Time integration of the reduced model allows calculating thermal stresses on-line. The number of operations per time unit is one to two orders of magnitude lower than that necessary to perform convolution integrals with the GFT.

2 Development of the reduced model

Let's take a thermo-elastic model with a number *I* of temperature inputs. If the GFT is used for on-line calculation of thermal stresses $\sigma(t)$ at a point P(x,y,z) of the model, the equation

$$\sigma(t) = \bar{\sigma} + \sum_{i=1}^{I} \int_0^t G_i(P, t-\tau) \cdot \frac{dT_i(\tau)}{d\tau} d\tau = \sum_{i=1}^{I} \left(\bar{G}_i(P) \cdot \bar{T}_i + \int_0^t G_i(P, t-\tau) \cdot \frac{dT_i(\tau)}{d\tau} d\tau \right) \tag{1}$$

has to be numerically solved (Ref. 1), where

$\bar{\sigma}$: initial steady-state value of thermal stress,
$T_i(t)$: i^{th} temperature input of the system,
$\bar{T}_i$: i^{th} temperature input of the system at time t=0,
$G_i(P, t)$: Green's function at point P of i^{th} input,
$\bar{G}_i(P)$: steady-state value of the Green's function at point P of i^{th} input.

The time derivative $d\sigma(t)/dt$ can be calculated by an equation having the same structure, where the Green's functions $G_i(P, t)$ are replaced with their time derivatives $H_i(P, t)$:

$$\frac{d\sigma(t)}{dt} = \frac{d\sigma(0)}{dt} + \sum_{i=1}^{I} \int_0^t H_i(P, t-\tau) \cdot \frac{dT_i(\tau)}{d\tau} d\tau, \tag{2}$$

with $d\sigma(0)/dt = 0$, because the calculation is started at steady-state conditions.
Green's functions $G_i(P, t)$ of thermal stress are always characterised by a transient which leads to a steady-state asymptotic value. Their analytical expression is

$$G_i(P, t) = \sum_{j=1}^{N} g_{ij}(t) = \sum_{j=1}^{N} \eta_{ij} \cdot (1 - e^{\lambda_{ij} \cdot t}), \tag{3}$$

with

λ_{ij}: j^{th} thermal eigenvalues of the model (real and negative numbers),
η_{ij}: contribution of the j^{th} thermal eigenvector to thermal stress at point P,
N: number of degrees of freedom of the thermo-mechanical model.

The key idea of the method described in this paper is to replace the analytical expression of the Green's functions written at Eq. (3) with the approximate expression

$$G_i(P, t) \cong \sum_{j=1}^{J_i} \tilde{g}_{ij}(t) = \sum_{j=1}^{J_i} \tilde{\eta}_{ij} \cdot (1 - e^{\tilde{\lambda}_{ij} \cdot t}), \tag{4}$$

where J_i is smaller than N.
If $G_i(P, t)$ in Eqs. (1) and (2) is replaced by the right hand side of Eq. (4), thermal stress and its time derivative can be written as

$$\sigma(t) = \sum_{i=1}^{I} \sum_{j=1}^{J_i} \tilde{\sigma}_{ij}(t), \tag{5}$$

$$\frac{d\sigma(t)}{dt} = \sum_{i=1}^{I} \sum_{j=1}^{J_i} \frac{d\tilde{\sigma}_{ij}(t)}{dt} \tag{6}$$

with

$$\tilde{\sigma}_{ij}(t) = \tilde{\eta}_{ij} \cdot \overline{T}_i + \int_0^t \tilde{\eta}_{ij} \cdot (1 - e^{\tilde{\lambda}_{ij} \cdot (t-\tau)}) \cdot \frac{dT_i(\tau)}{d\tau} d\tau \tag{7}$$

$$\frac{d\tilde{\sigma}_{ij}(t)}{dt} = -\int_0^t \tilde{\eta}_{ij} \cdot \tilde{\lambda}_{ij} \cdot e^{\tilde{\lambda}_{ij} \cdot (t-\tau)} \cdot \frac{dT_i(\tau)}{d\tau} d\tau \, . \tag{8}$$

If the integral in equation (7) is transformed as follows:

$$\int_0^t \tilde{\eta}_{ij} \cdot (1 - e^{\tilde{\lambda}_{ij} \cdot (t-\tau)}) \cdot \frac{dT_i(\tau)}{d\tau} d\tau = \int_0^t \tilde{\eta}_{ij} \cdot \frac{dT_i(\tau)}{d\tau} d\tau - \tilde{\eta}_{ij} \cdot \int_0^t e^{\tilde{\lambda}_{ij} \cdot (t-\tau)} \cdot \frac{dT_i(\tau)}{d\tau} d\tau =$$

$$= \tilde{\eta}_{ij} \left(T_i(t) - \overline{T}_i\right) - \tilde{\eta}_{ij} \cdot \int_0^t e^{\tilde{\lambda}_{ij} \cdot (t-\tau)} \cdot \frac{dT_i(\tau)}{d\tau} d\tau \, ,$$

$\tilde{\sigma}_{ij}(t)$, defined in Eq. (7), becomes

$$\tilde{\sigma}_{ij}(t) = \tilde{\eta}_{ij} \cdot F_i(t) - \int_0^t \tilde{\eta}_{ij} \cdot e^{\tilde{\lambda}_{ij} \cdot (t-\tau)} \cdot \frac{dT_i(\tau)}{d\tau} d\tau \, . \tag{9}$$

Comparison of Eq. (8) and Eq. (9) leads to the following relation between $\tilde{\sigma}_{ij}(t)$ and its time derivative $d\tilde{\sigma}_{ij}(t)/dt$:

$$\frac{d\tilde{\sigma}_{ij}(t)}{dt} = \tilde{\lambda}_{ij} \cdot \tilde{\sigma}_{ij}(t) - \tilde{\lambda}_{ij} \cdot \tilde{\eta}_{ij} \cdot T_i(t) \, . \tag{10}$$

Equation (10) is valid for any term $\tilde{\sigma}_{ij}(t)$. As a consequence, for any temperature input T_i a reduced model of size J_i

$$\dot{\hat{\boldsymbol{\sigma}}}_{\mathbf{i}} = \boldsymbol{\alpha}_{\mathbf{i}} \cdot \hat{\boldsymbol{\sigma}}_{\mathbf{i}} + \boldsymbol{\beta}_{\mathbf{i}} \cdot T_i \tag{11}$$

can be obtained, where

$\hat{\boldsymbol{\sigma}}_{\mathbf{i}}$: ($J_i$ x 1) vector having $\tilde{\sigma}_{ij}(t)$ at the j^th^ position,

$\dot{\hat{\boldsymbol{\sigma}}}_{\mathbf{i}}$: ($J_i$ x 1) vector having $d\tilde{\sigma}_{ij}(t)/dt$ at the j^th^ position,

$\boldsymbol{\alpha}_{\mathbf{i}}$: ($J_i$ x J_i) diagonal matrix containing the $\tilde{\lambda}_{ij}$,

$\boldsymbol{\beta}_{\mathbf{i}}$: ($J_i$ x 1) vector having the j^th^ term equal to $-\tilde{\lambda}_{ij}\,\tilde{\eta}_{ij}$.

In the case of *I* temperature inputs, the *I* reduced models of Eq. (11) can be assembled together to form

$$\begin{cases} \dot{\hat{\boldsymbol{\sigma}}} = \mathbf{A} \cdot \hat{\boldsymbol{\sigma}} + \mathbf{B} \cdot \mathbf{T} \\ \sigma = \mathbf{1}^T \cdot \hat{\boldsymbol{\sigma}} \end{cases} \tag{12}$$

with

$$\hat{\boldsymbol{\sigma}} = \begin{Bmatrix} \hat{\boldsymbol{\sigma}}_{\mathbf{1}} \\ \hat{\boldsymbol{\sigma}}_{\mathbf{2}} \\ \vdots \\ \hat{\boldsymbol{\sigma}}_{\mathbf{I}} \end{Bmatrix}; \quad \mathbf{A} = \begin{bmatrix} \boldsymbol{\alpha}_{\mathbf{1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\alpha}_{\mathbf{2}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\alpha}_{\mathbf{I}} \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \boldsymbol{\beta}_{\mathbf{1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\beta}_{\mathbf{2}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\beta}_{\mathbf{I}} \end{bmatrix}; \quad \mathbf{T} = \begin{Bmatrix} T_1 \\ T_2 \\ \vdots \\ T_I \end{Bmatrix}; \quad \mathbf{1} = \begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{Bmatrix}.$$

If a new state variable

$$\boldsymbol{\sigma}' = \hat{\boldsymbol{\sigma}} + \mathbf{A}^{-1} \cdot \mathbf{B} \cdot \mathbf{T} \tag{13}$$

is introduced, the reduced model of Eq. (12) becomes

$$\begin{cases} \dot{\boldsymbol{\sigma}}' = \mathbf{A} \cdot \boldsymbol{\sigma}' + \mathbf{C} \cdot \dot{\mathbf{T}} \\ \sigma = \mathbf{1}^T \cdot \boldsymbol{\sigma}' + \mathbf{G}^T \cdot \mathbf{T} \end{cases} \tag{14}$$

with

$$\mathbf{C} = \mathbf{A}^{-1} \cdot \mathbf{B} \text{ and } \mathbf{G}^T = -\mathbf{1}^T \cdot \mathbf{A}^{-1} \cdot \mathbf{B} \, . \tag{15}$$

The new state variable has been introduced in order to separate and put into evidence the two main constituents of thermal stress: the transient part $\mathbf{1}^T \cdot \boldsymbol{\sigma}'$ and steady-state part $\mathbf{G}^T \cdot \mathbf{T}$.

Verifying that $\mathbf{G}^T \cdot \mathbf{T}$ is the steady-state part of thermal stress is simple. Let us suppose to neglect $\mathbf{1}^T \cdot \boldsymbol{\sigma}'$ in the thermal stress calculation, thermal stress σ(t) would reach instantaneously the steady-state value corresponding to vector **T**(t). On the contrary if $\mathbf{1}^T \cdot \boldsymbol{\sigma}'$ is included the delay due to thermal capacity of the material is taken into account.

These considerations imply that in order to evaluate exactly thermal stress at steady-state, the i^th^ term of the vector **G** has to be equal to $\overline{G}_i(P)$, i.e. the steady-state thermal-stress due to a unit amplitude of i^th^ input.

3 Calculation of the coefficients of the model

In the previous section the reduced model has been derived and it has been shown how to calculate the terms of the vector **G**.

In order to define completely the model, matrices **A** and **C** are also to be obtained. As shown in Eqs. (10-15) they both depend on coefficients $\tilde{\eta}_{ij}$ and $\tilde{\lambda}_{ij}$. This means that once the terms of the series introduced at Eq. (4) are calculated, then the reduced model is completely defined.

In this paper it is proposed to calculate these coefficients by means of a non-linear least-square fitting.

In detail, for any Green's function $G_i(t)$, the problem is to find a set of coefficients $\tilde{\eta}_{ij}$ and $\tilde{\lambda}_{ij}$ which minimises the function

$$f\left(\tilde{\lambda}_{ij}, \tilde{\eta}_{ij}, t_k\right) = \sum_{k=1}^{K} R_i^2\left(\tilde{\lambda}_{ij}, \tilde{\eta}_{ij}, t_k\right), \tag{16}$$

with residual R_i defined as

$$R_i\left(\tilde{\lambda}_{ij}, \tilde{\eta}_{ij}, t_n\right) = G_i\left(t_n\right) - \sum_{j=1}^{J_i} \tilde{\eta}_{ij} \cdot \left(1 - e^{\tilde{\lambda}_{ij} \cdot t_k}\right) \tag{17}$$

and K equal to the number of samples of the Green's function.

The problem is solved using the MATLAB-lsqnonlin non-linear least square fitting algorithm based on the methods of conjugate gradients.

The number J_i of exponential terms necessary to properly fit the curve $G_i(t)$ is not known a priori; therefore a simple iterative procedure is employed.

For each Green's function $G_i(t)$:

1. The first attempt is made with a series made of just one term ($J_i = 1$).
2. Amplitudes and exponents are found finding the minimum of Eq. (16).
3. If the resulting series does not fit the Green's function with the required accuracy, another term is added ($J_i = J_i + 1$) and step 2 is performed again. The calculation stops when the series fits properly the Green's function.

During the fitting procedure some boundaries can be imposed to the unknown parameters $\tilde{\eta}_{ij}$ and $\tilde{\lambda}_{ij}$ to allow for some physical features of the thermo-mechanical model:

- The exponent range is set by the minimum and maximum eigenvalues of the thermal model;
- For any Green's function $G_i(t)$ the sum of the J_i amplitudes $\tilde{\eta}_{ij}$ is imposed equal to the corresponding steady-state value $\overline{G}_i(P)$.

4 Application to a thick pipe

A simple application will now be presented. Although the method has been developed for applications characterised by a large number of thermal inputs, this simple study case explains better how the procedure works.

In detail, the case study is a thick pipe subjected to convective boundary conditions at both inner and outer radii. The critical point of the pipe is placed at its inner radius and the monitored stress component at that point is the hoop stress.

Geometry, material properties and time history of fluid temperatures are plotted in Figure 2. Both models of the pipe, the thermal and the thermo-elastic, are supposed to be perfectly axi-symmetric. The variation in time of the inner and the outer convective coefficients has been separately determined and is shown in Table 1.

As stated in the introduction, employment of the GFT for thermal stress calculation in thermo-mechanical models characterised by variable convective coefficients is possible only if boundary temperatures are previously evaluated and then used as inputs.

In this case boundary temperatures correspond to temperatures at the inner and at the outer radius of the pipe, henceforth called $T_i(t)$ and $T_o(t)$ respectively.

The detailed description of the calculation of $T_i(t)$ and $T_o(t)$ is beyond the scope of this paper. For more details see Ref. [4]. Here the time histories of $T_i(t)$ and $T_o(t)$ (Figure 3) are given as data and used as inputs for thermal stress calculation.

The Green's functions $G_i(t)$ and $G_o(t)$, corresponding to unit step variation of $T_i(t)$ and $T_o(t)$ respectively, are evaluated by means of the detailed thermal and thermo-elastic models (Figure 4). Then they are fitted by means of a sum of exponential terms, following the procedure described in Section 4. For the proposed case study 3 terms are necessary for $G_i(t)$ and 2 terms for $G_o(t)$.

The resulting reduced model is

$$\dot{\boldsymbol{\sigma}}' = \begin{bmatrix} -0.0475 & 0 & 0 & 0 & 0 \\ 0 & -0.669 & 0 & 0 & 0 \\ 0 & 0 & -5.920 & 0 & 0 \\ 0 & 0 & 0 & -0.0491 & 0 \\ 0 & 0 & 0 & 0 & -1.114 \end{bmatrix} \cdot \boldsymbol{\sigma}' + \begin{bmatrix} -1.424 & 0 \\ 0.411 & 0 \\ 3.003 & 0 \\ 0 & -1.646 \\ 0 & -0.344 \end{bmatrix} \cdot \begin{Bmatrix} \dot{T}_i \\ \dot{T}_o \end{Bmatrix}. \quad (18)$$

$$\sigma = \mathbf{1}^T \cdot \boldsymbol{\sigma}' + \lfloor 1.990 \quad -1.990 \rfloor \cdot \begin{Bmatrix} T_i \\ T_o \end{Bmatrix}$$

Time integration of the reduced model allows calculation of the thermal stress at the critical point.
Figure 5 shows the thermal stress obtained by means of the detailed FE thermo-mechanical model and the difference between the results from the reduced model and those from FEM.
The number of operations is reduced with respect to the GFT. Time integration of the reduced model performed by an Euler implicit method with a constant time step $\Delta t = 0.5$ s requires 34 operations per second, whilst the solution of the convolution integrals would need 1600 (see Appendix).

5 Application to a nozzle

The second application is of a more practical relevance. It is a junction between a nozzle and the spherical head of a pressure vessel, whose geometry and material properties are shown in Figure 6a. The critical location of the component is the point A and the stress to monitor is the hoop stress σ_{HA}.
The component is subjected to convective boundary conditions along the boundaries AF and AE. A transient starts from an nitial uniform temperature T_0. The time histories of the variation of fluid temperatures facing the component are plotted in Figure 6b.
Convective coefficients are:

- constant and equal to $h_2 = 567.4$ W/m^2/K along the side AF;
- variable according to time history plotted in Figure 6c along the side AE.

Fluid temperature T_{fl2} facing the AF side can be used for thermal stress calculation because the corresponding convective coefficient h_2 is constant.
On the contrary, in order to take into account the effect of T_{fl1} it is necessary to evaluate the temperature profile along the side AE and then use such a distribution as input for thermal stress.
A detailed description of the procedure adopted to evaluate the time history of the temperature distribution along the AE side is given in [4].
Here it is sufficient to say that the temperature distribution along AE is supposed to be piecewise-linear. A Lagrange basis made of 5 first-order polynomials, called L_A, L_B, L_C, L_D and L_E, is used to describe the temperature distribution. T_A, T_B, T_C, T_D and T_E (Figure 6a) are the amplitudes of the corresponding Lagrange functions.
Time histories of T_A, T_B, T_C, T_D and T_E (Figure 7), which represent the inputs for thermal stress calculation together with T_{fl2}, are evaluated by time integration of a reduced thermal model ([4]) and henceforth are given as data.
In order to use time histories of T_A, T_B, T_C, T_D, T_E and T_{fl2} as inputs for thermal stress calculation a set of Green's functions are to be calculated, called respectively G_A, G_B, G_C, G_D, G_E and G_{fl2}.
G_A, G_B, G_C, G_D and G_E are the time histories of σ_{HA} due to a unit step input of the Lagrange polynomials L_A, L_B, L_C, L_D and L_E respectively, imposing null T_{fl2}. G_{fl2} is the time history of σ_{HA} due to unit step input of T_{fl2} and null temperature along AE.
Once the Green's functions are evaluated, they are fitted with a series of exponential terms. As a result of the non linear fitting procedure the exponential series listed in Tab. 2 are obtained and a reduced model made of 10 uncoupled differential equations is built. FEM results and error due to time integration of the reduced model are plotted in Figure 8. Time integration of the reduced model, performed with the Euler implicit method (time step 0.5 s) requires 72 operations per time unit; the same calculation with the GFT would require 480 operations (see Appendix).

6 Conclusions

A reduced model made of uncoupled first order differential equations has been proposed as a tool for on-line calculation of thermal stresses.
The approach here proposed for the definition of the reduced model consists of:

- fitting the Green's functions of the thermo-elastic model with a series of exponential terms;
- defining the reduced model by means of the amplitudes and the exponents of the series.

In this paper a non-linear least square fitting procedure based on the method of conjugate gradients has been successfully used to estimate amplitudes and exponents of the series.

The methodology has been applied to two study cases: a thick pipe and a nozzle.
Results presented in this paper show that the accuracy of stress calculated via the reduced model vs. reference FE model are very satisfactory.
It has also been shown that the reduced model compared to the Green's function technique reduces the number of operations by between one to two orders of magnitude (34 vs. 1600 and 72 vs. 480 respectively).

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Appendix

The method proposed in this paper reduction, with respect to the GFT, of the number of operations necessary to perform on-line calculation of thermal stresses.
For each thermal stress which has to be monitored, the amount of this reduction depends on several elements:

1. the number I of temperature inputs,
2. the time length t_G of the Green's functions,
3. the time step Δt chosen for on-line calculations,
4. the size M of the reduced model.

If the GFT is used, the number of operations per second necessary to perform on-line calculation is

$$N_{op}^{GFT} = I \cdot \frac{t_G}{\Delta t} \cdot \frac{1}{\Delta t} = \frac{I \cdot t_G}{\Delta t^2} \, ,$$

where $t_G/\Delta t$ is the length of vectors involved in the convolution integrals and $1/\Delta t$ is the frequency at which on-line calculation occurs.
With reference to Eq. (14), the reduced model requires a number of operations per second equal to

$$N_{op}^{RM} = \left(2 \cdot M + M + N\right) \cdot \frac{1}{\Delta t} \, ,$$

where $1/\Delta t$ is the frequency at which on-line calculation occurs and each term in the parentheses represents the number of operations necessary to:

– perform time integration of the differential equations with Euler implicit method,
– calculate the transient part of thermal stress $\mathbf{1}^T \cdot \boldsymbol{\sigma}'$,
– calculate the steady-state part $\mathbf{G}^T \cdot \mathbf{F}$

respectively.

Figures

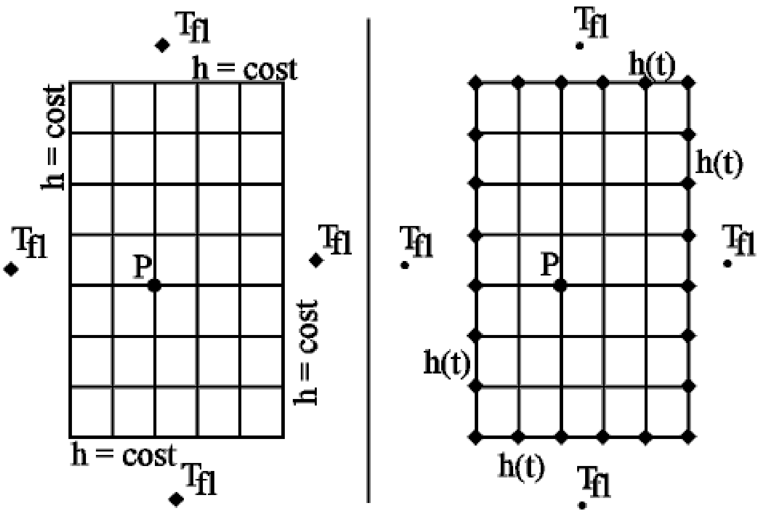

Figure 1: Temperature inputs (♦) for σ(t) calculation with the GFT; linear models (a) and models with variable convective coefficients (b).

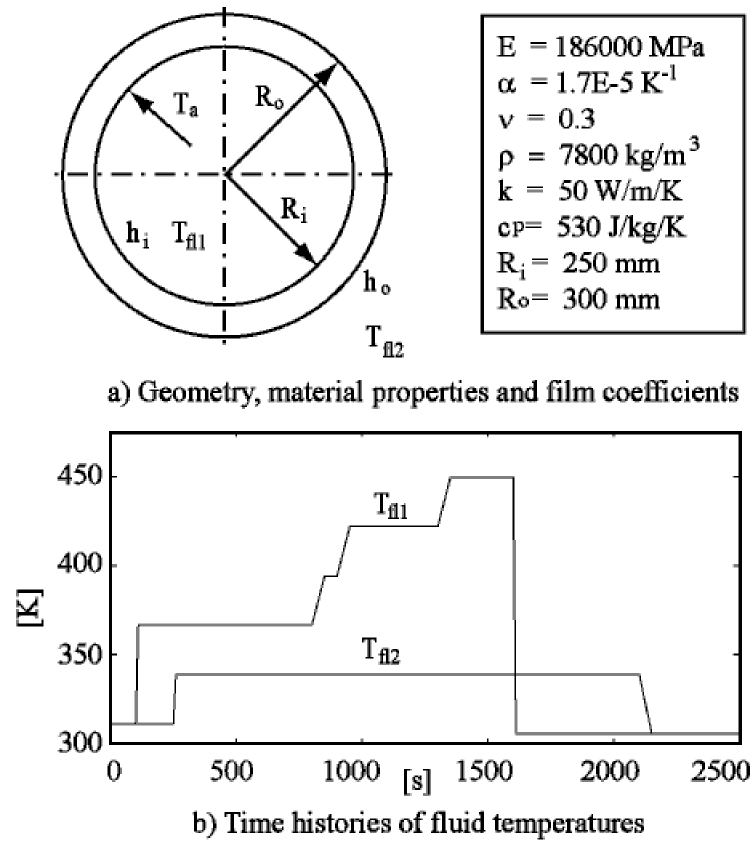

Figure 2: Thick pipe geometry, material properties (a) and boundary conditions (b)

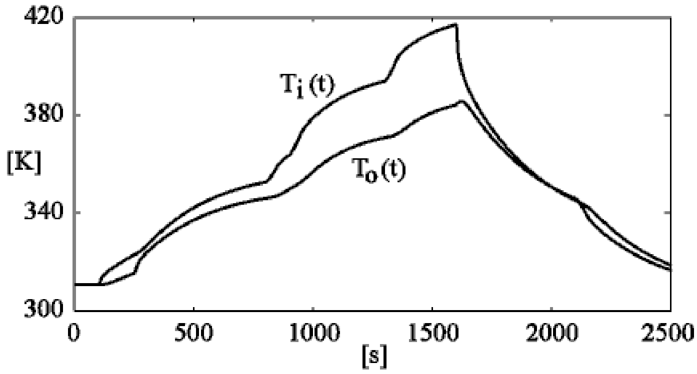

Figure 3: Temperature at the inner and outer radius of the pipe.

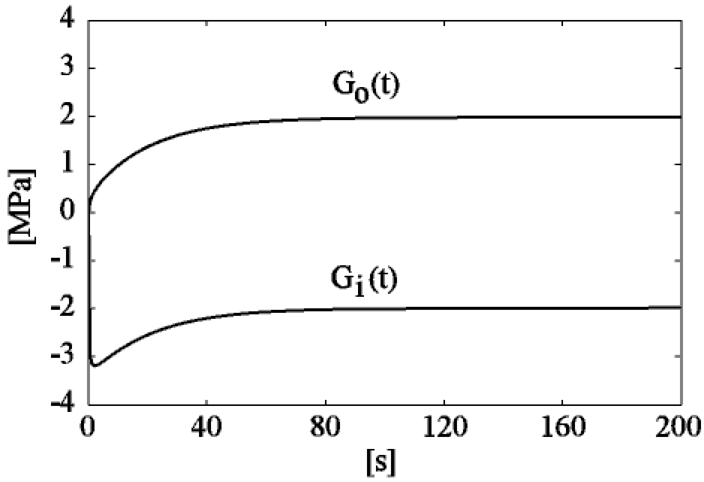

Figure 4: Green's function $G_i(t)$ and $G_o(t)$ of the thick pipe.

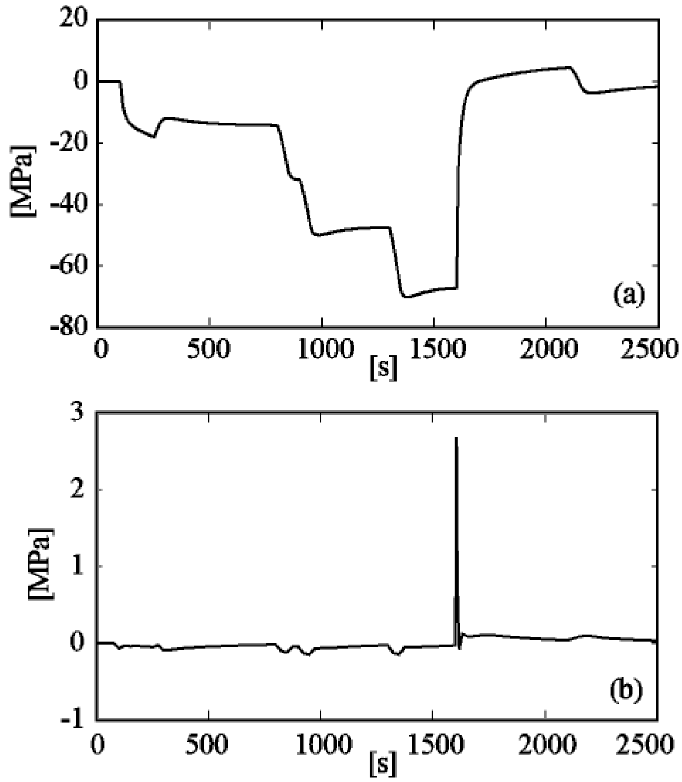

Figure 5: Hoop stress at the inner radius of the pipe; FEM results (a) and difference between results from the reduced model and those from FEM (b)

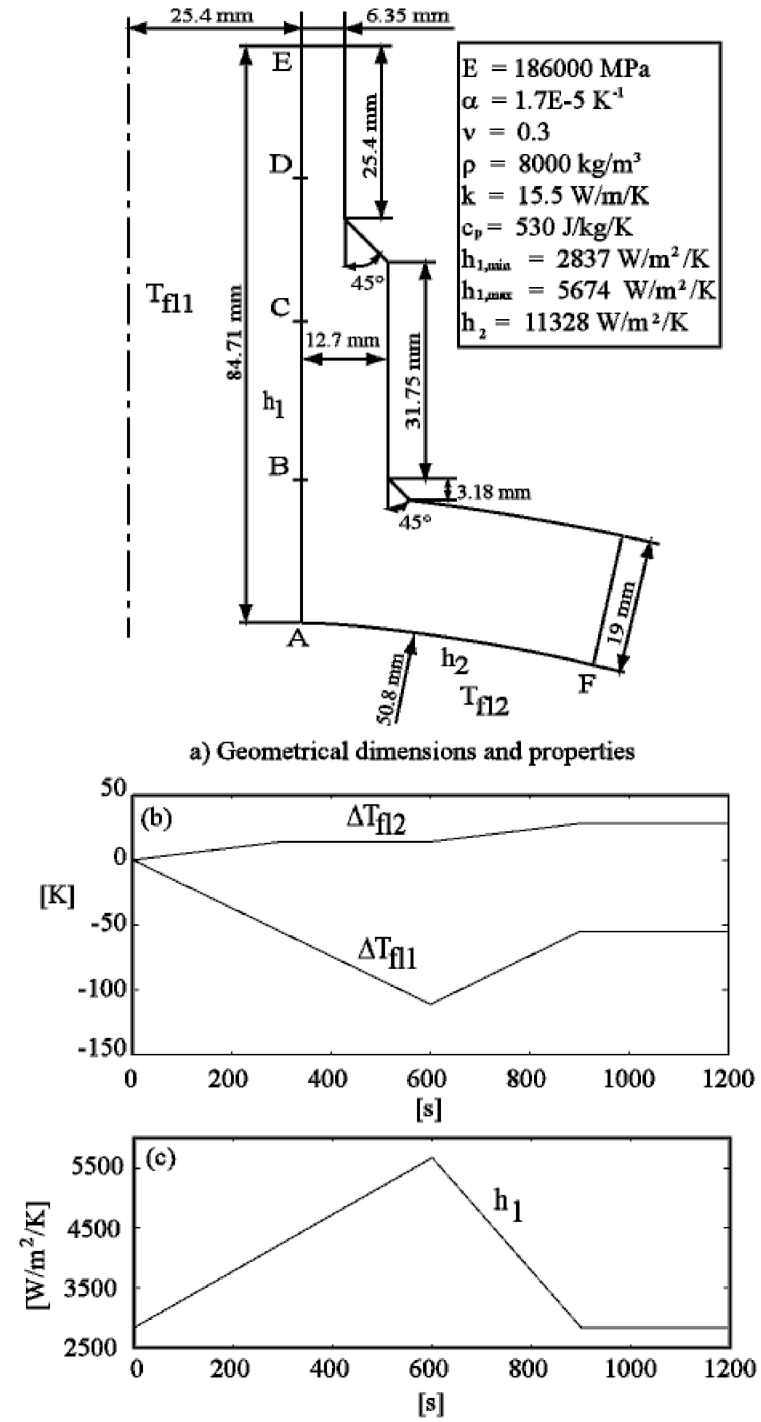

Figure 6: Nozzle geometry (a), material properties (a), fluid temperatures (b) and heat transfer coefficient over AE side (c)

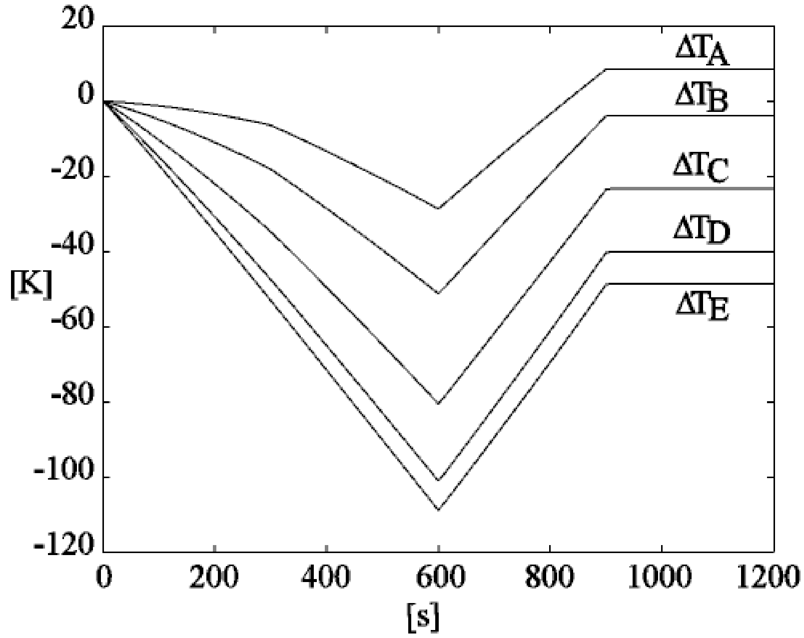

Figure 7: Time history of metal temperatures over AE side of the nozzle.

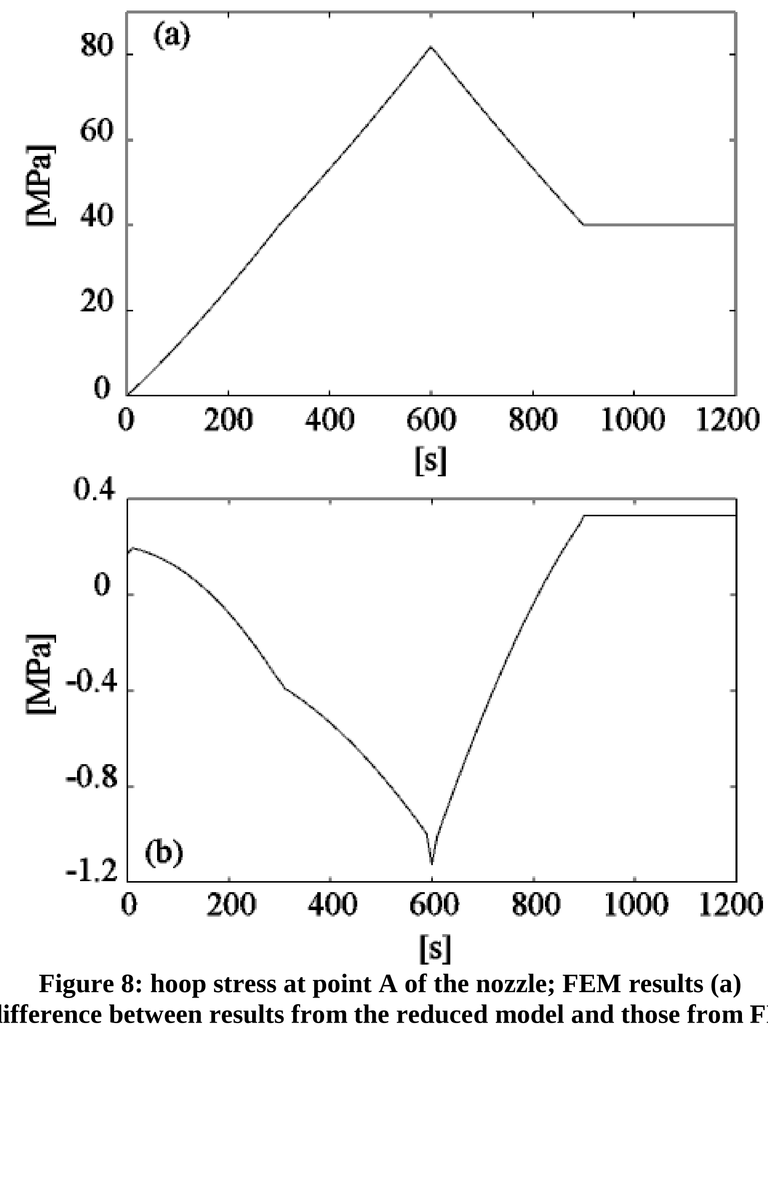

Figure 8: hoop stress at point A of the nozzle; FEM results (a)
and difference between results from the reduced model and those from FEM (b)

Tables

Table 1: variation of film coefficients of the thick pipe.

time [s]	0	1600	1610	2500
h_i [W/m^2/K]	150	1200	150	150
h_o [W/m^2/K]	500	600	500	500

Table 2: Green's function exponents and amplitudes.

Green's function	η_1	λ_1	η_2	λ_2
$\mathbf{G_A}$	0.107	-1.277	-2.513	-307.5
$\mathbf{G_B}$	0.260	-1.277	0.496	-124.1
$\mathbf{G_C}$	-0.271	-8.544	0.202	-1949.0
$\mathbf{G_D}$	-0.1896	-5.284	-0.0125	-895.8
$\mathbf{G_E}$	-1.7e-4	-1.277	-	-
$\mathbf{G_{fl2}}$	1.921	-1.277	-	-