Evaluating the Impedance Field through several transport models: a comparison

Evaluating the impedance field through several transport models: A comparison

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The article presents a critical discussion on the evaluation of the impedance field for noise analysis by means of several transport models. The models considered are the drift-diffusion model, the full hydrodynamic model, and the scattered packet method for the direct solution of the Boltzmann transport equation. The comparison is carried out on a canonical Si p^+pp^+ diode at ambient temperature. The agreement found between the approaches is good, thus suggesting that the evaluation of the impedance field does not require any additional care with respect to other, more conventional operating conditions. © *1999 American Institute of Physics.* [S0021-8979(99)01904-0]

I. INTRODUCTION

All classical noise analysis techniques for solid-state electron devices, like the impedance field method (IFM)[1] and its generalizations,[2] are based on a two-step approach. As a first step, the fundamental microscopic fluctuations of carrier population [generation-recombination (GR) noise] and carrier velocity (diffusion noise) are characterized, on the basis of suitable models ultimately derived by the master equation formalism,[3] by their correlation spectra.

The second step amounts to evaluating the effect of the microscopic fluctuations on the induced (open-circuit) voltage or (short-circuit) current at the device terminals. Owing to the small amplitude of the microscopic fluctuations, this effect can be assessed by the linear perturbation theory, i.e., by a Green's function approach; the relevant Green's functions relating charge or current density microscopic fluctuations to terminal open-circuit voltage fluctuations are referred to as the scalar or vector impedance fields, respectively.[1]

Generally speaking, the two aforementioned steps can be carried out by means of different physics-based models. For instance, a full hydrodynamic model can be exploited to estimate the carrier density and average energy, thus deriving a suitable microscopic noise source correlation matrix; then, the simpler, more computationally efficient drift-diffusion (DD) model can be used to evaluate the impedance fields. Therefore, each step of the process can be optimized from the standpoint of computational efficiency versus accuracy or physical soundness.

The accuracy of both the microscopic noise source and the impedance field has to be checked against a reference model, which in principle could be a Monte Carlo solution of the Boltzmann transport equation (BTE). However, noise analysis by means of Monte Carlo techniques[4] does not readily allow for the separation between microscopic local fluctuations and Green's functions, since noise characteristics are directly extracted from the statistical properties of the time-domain simulated response. Recently, however, this problem has been overcome due to the availability of a new technique, the scattered packet method (SPM),[5–8] which performs a time-domain direct solution of the Boltzmann equation in one-dimension (1D). In particular, it allows for the evaluation of the response to a time-impulsive internal charge fluctuation, i.e., the scalar impedance field, thereby providing a suitable comparison solution for the impedance field.

In recent years, a few implementations of the IFM noise analysis within the framework of partial differential equation (PDE) based 1D or multidimensional numerical device simulators have been proposed, thus making the noise analysis of general, realistically complex structures ultimately feasible. In particular, two-dimensional (2D) implementations of the IFM within the framework of a majority-carried drift-diffusion model were proposed in 1989 by Layman[9] and by Ghione *et al.*[10] with application to metal-oxide-semiconductor field effect transistor (MOSFET) and metal-semiconductor field effect transistor (MESFET) noise analysis, respectively. The efficient technique proposed in Ref. 10 for the numerical evaluation of the scalar impedance field (the so-called *adjoint method*, derived from the electrical network noise analysis in Ref. 11) was later extended to general 2D or three-dimensional (3D) two-carrier drift-diffusion models through the technique described in Refs. 2 and 12 and implemented in the general-purpose PADRE[13] simula-

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tor. On the other hand, a fully hydrodynamic IFM approach to the 2D noise simulation of MESFET devices was proposed in Ref. 14, and, later, a 1D model was presented by the Vilnius group[15] and applied to the simulation of canonical structures. A one-dimensional energy-balance model based on the so-called active line approach[16] was also exploited within the framework of the quasi-2D high electron mobility transistor (HEMT) simulator HELENA of the Lille University.[17]

In the present article, a direct comparison between some of the aforementioned approaches to the evaluation of the impedance field is attempted. The purpose is not only to validate, in simple structures, PDE-based models versus reference solutions derived from the BTE, but also to assess whether computationally efficient lower-order models, such as the DD model, are accurate enough for the evaluation of the impedance field, at least in those cases in which overly critical nonstationary transport features do not occur.

II. THE PHYSICAL MODELS

This section is devoted to a brief review of the IFM for noise analysis and an introduction to both hydrodynamic (HD) and SPM approaches. In the DD impedance-field-based noise calculation, noise sources appear as stochastic forcing terms in the current continuity equations.[3] Such sources are related to electron and hole number fluctuations (GR noise, γ) or to electron and hole velocity fluctuations (diffusion noise, often interpreted in terms of current density fluctuations, $\boldsymbol{\xi}$), thus yielding the system

$$\nabla^2\psi = -\frac{q}{\epsilon}(p-n+N^+), \tag{1a}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla\cdot\mathbf{J}_n - U_n + \gamma_n + \frac{1}{q}\nabla\cdot\boldsymbol{\xi}_n, \tag{1b}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla\cdot\mathbf{J}_p - U_p + \gamma_p + \frac{1}{q}\nabla\cdot\boldsymbol{\xi}_p. \tag{1c}$$

Owing to the small amplitude of such fluctuations, the response can be conveniently obtained by linearization and frequency-domain transformation of the time-domain drift-diffusion system

$$\nabla^2\delta\tilde{\psi} = -\Lambda_\psi(\delta\tilde{\psi},\delta\tilde{n},\delta\tilde{p}), \tag{2a}$$

$$\mathrm{j}\omega\delta\tilde{n} = -\Lambda_n(\delta\tilde{\psi},\delta\tilde{n},\delta\tilde{p}) + \tilde{\gamma}_n + \frac{1}{q}\nabla\cdot\tilde{\boldsymbol{\xi}}_n, \tag{2b}$$

$$\mathrm{j}\omega\delta\tilde{p} = -\Lambda_p(\delta\tilde{\psi},\delta\tilde{n},\delta\tilde{p}) + \tilde{\gamma}_p + \frac{1}{q}\nabla\cdot\tilde{\boldsymbol{\xi}}_p, \tag{2c}$$

where Λ_α is a linear operator. The solution of the linearized system enables evaluation of the response (in terms of induced terminal open-circuit voltages) to a spatially impulsive density source for carrier α ($\alpha=n,p$), i.e., the scalar impedance field Z_α, and to a spatially impulsive current density source, i.e., the vector impedance field $\mathbf{Z}_\alpha$. As a matter of fact, only the scalar fields for electrons and holes are needed, since the vector impedance fields can be recovered from the gradient of the scalar ones.[1] Therefore, from a physical standpoint, the evaluation of the impedance fields amounts to computing the response to a spatially impulsive *scalar current source* injected into the right-hand side of the relevant continuity equation

$$\nabla_{\mathbf{r}}{}^2 Z_\alpha(\mathbf{r},\mathbf{r}_1;\omega) = -\Lambda_\psi(Z_\alpha,\delta\tilde{n},\delta\tilde{p}), \tag{3a}$$

$$\mathrm{j}\omega\delta\tilde{n}(\mathbf{r},\mathbf{r}_1;\omega) = -\Lambda_n(Z_\alpha,\delta\tilde{n},\delta\tilde{p}) + \frac{1}{q}\delta_{\alpha,n}\delta(\mathbf{r}-\mathbf{r}_1), \tag{3b}$$

$$\mathrm{j}\omega\delta\tilde{p}(\mathbf{r},\mathbf{r}_1;\omega) = -\Lambda_p(Z_\alpha,\delta\tilde{n},\delta\tilde{p}) + \frac{1}{q}\delta_{\alpha,p}\delta(\mathbf{r}-\mathbf{r}_1), \tag{3c}$$

where $\alpha=n,p$ and $\delta_{\alpha,\beta}$ is Kronecker's δ. From the impedance fields, the spectra of the fluctuations of the voltages at the device terminals $S_{\delta v_i \delta v_j}$ are expressed according to

$$S_{\delta v_i \delta v_j} = \sum_{\alpha=n,p}\int_\Omega \mathbf{Z}_\alpha^{\mathrm{T}}(\mathbf{r}_i,\mathbf{r},\omega)\cdot\mathbf{K}_{\delta\mathbf{J}_\alpha\delta\mathbf{J}_\alpha}(\mathbf{r},\omega)\cdot\mathbf{Z}_\alpha^*(\mathbf{r}_j,\mathbf{r},\omega)\mathrm{d}\mathbf{r} + \sum_{\alpha,\beta=n,p}\int_\Omega Z_\alpha(\mathbf{r}_i,\mathbf{r},\omega)K_{\gamma_\alpha\gamma_\beta}(\mathbf{r},\omega)Z_\beta^*(\mathbf{r}_j,\mathbf{r},\omega)\mathrm{d}\mathbf{r}, \tag{4}$$

where $\mathbf{K}_{\delta\mathbf{J}_\alpha\delta\mathbf{J}_\alpha}$ is the local noise source for diffusion noise, $K_{\gamma_\alpha\gamma_\beta}$, $\alpha,\beta=n,p$, is the local (scalar) noise source for GR noise.

The same physical interpretation of the scalar impedance fields can be extended to other, more complex transport models, meaning that Z_n and Z_p can be evaluated, according to the same definition as in the drift-diffusion case, within the framework of an energy transport, full hydrodynamic or even Boltzmann-based model. Clearly, an improvement in accuracy is obtained in such a way with respect to the simple DD approach. In this work, we shall compare the impedance field evaluated with the DD transport model to the same physical quantity extracted through the SPM. This is basically a time-domain direct solution of the BTE, exploited for the impedance field (IF) evaluation according to the technique detailed in Ref. 8. The frequency-domain IF is then recovered through a fast Fourier transform (FFT) performed on the time series representing the terminal open circuit potential. The FFT analysis is used for the HD-based IF as well, according to the method proposed in Ref. 18. Further details on the numerical techniques exploited by the three simulators, which are beyond the scope of this article, can be found in Refs. 2, 8, and 18.

III. RESULTS AND DISCUSSION

As a case study, we consider a well-known canonical structure, a p^+pp^+ silicon diode at ambient temperature, simulated at different frequencies and bias conditions. In all models under consideration (DD, HD, SPM), care was taken to ensure the physical consistency of the transport models, which were all derived from SPM data. Thus, energy and momentum relaxation models, as well as the velocity-field curve needed in the DD model, were derived from the solution of the homogeneous Boltzmann equation.

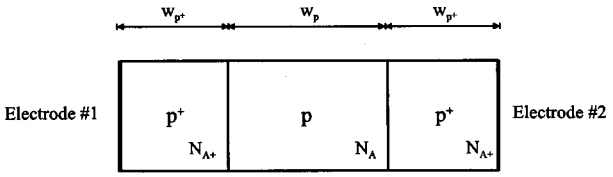

FIG. 1. Structure of the p^+pp^+ Si diode considered in the simulations.

The structure of the p^+pp^+ diode is shown in Fig. 1. The total device width has been held constant throughout all the simulations ($w_p+2w_{p^+}=1$ μm), as well as the ratio between the doping values in the p^+ and p regions ($N_{A+}/N_A=10$).

Figure 2 shows the square magnitude of the hole vector impedance field for a p^+pp^+ diode with $w_p=0.4$ μm, simulated at $f=1$ GHz (a) and $f=235$ GHz (b) for a bias $V_d=0.6$ V and a doping level $N_A=4\times10^{16}$ cm^{-3}. The results for the same device structure and bias, with doping levels $N_A=10^{16}$ cm^{-3} and $N_A=10^{17}$ cm^{-3} are reported in Figs. 3(a) and 3(b), respectively, for $f=1$ GHz. The agreement among the three methods (DD, HD and SPM) is good for all the doping levels. From a physical standpoint, one can observe that a higher doping implies a higher conductivity, and, thus, a comparatively lower electric field since the applied bias is the same for the three devices. This explains the enhanced symmetry observed in the IF behavior as the doping level is increased. Furthermore, since the impedance field in an isolated uniformly doped region is proportional to the region's small-signal impedance, the higher conductivity also explains the lower values of the IF.

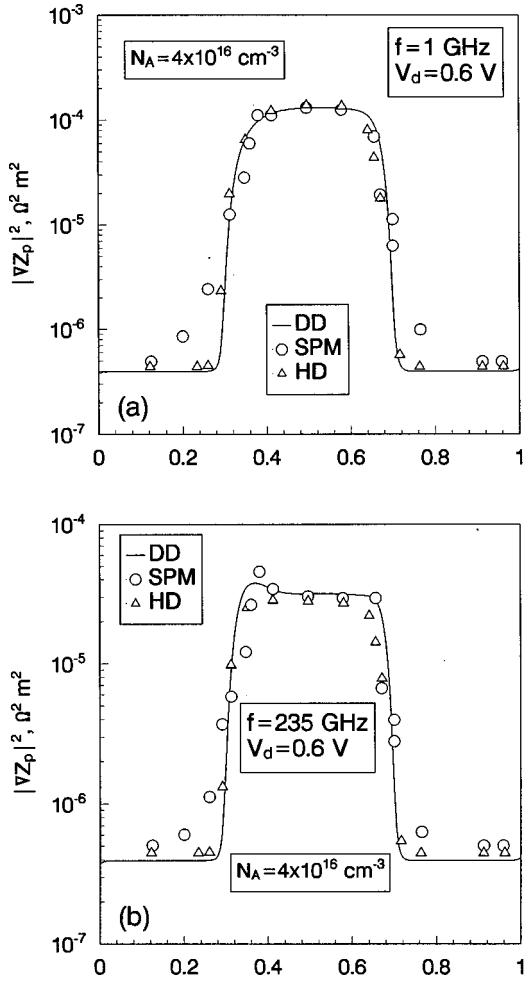

FIG. 2. Spatial dependence of the square magnitude of the vector impedance field in a p^+pp^+ Si diode. The doping value for the p region is $N_A=4\times10^{16}$ cm^{-3} and $N_{A+}=10N_A$. The device structure parameters are: $w_{p^+}=0.3$ μm, $w_p=0.4$ μm; the bias is 0.6 V and the frequency is 1 GHz (a) and 235 GHz (b).

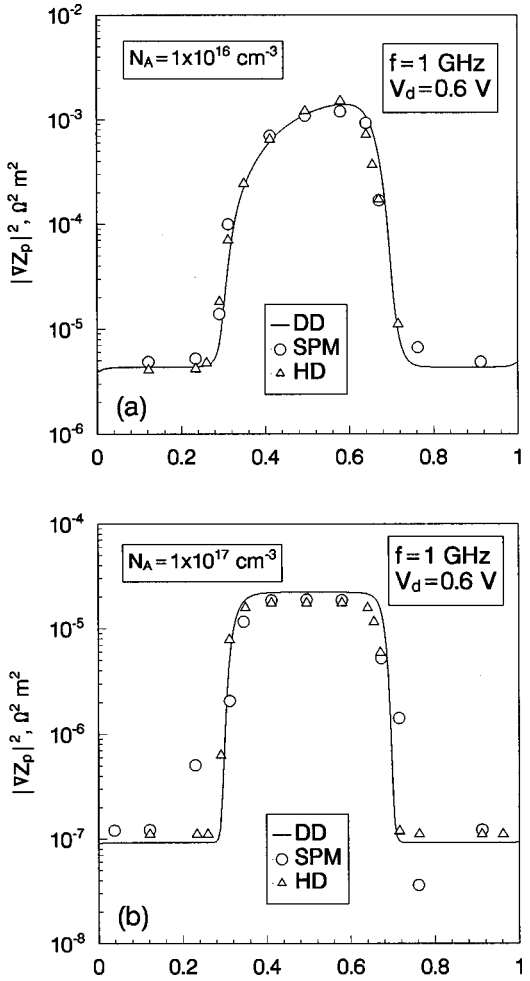

FIG. 3. Spatial dependence of the square magnitude of the vector impedance field in a p^+pp^+ Si diode. The doping value for the p region is $N_A=10^{16}$ cm^{-3} (a) and $N_A=10^{17}$ cm^{-3} (b), and $N_{A+}=10N_A$. The device structure parameters are: $w_{p^+}=0.3$ μm, $w_p=0.4$ μm; the bias is 0.6 V and the frequency is 1 GHz.

The influence of the width w_p of the central region has also been investigated. The results are reported in Fig. 4, where the $|\nabla Z_p|^2$ spatial behavior ($f=1$ GHz) for three values of w_p is shown ($w_p+2w_{p^+}=1$ μm): $w_p=0.2$ μm (a), $w_p=0.4$ μm (b) and $w_p=0.72$ μm (c). Again, the agreement among the three transport models is good. In this case, the electric field influences only the spatial behavior of the IF rather than the magnitude. The narrower the central region, the higher the electric field will be (the bias is kept constant to $V_d=0.6$ V) and the less symmetric the IF distribution will be. This behavior is also confirmed by the results shown in

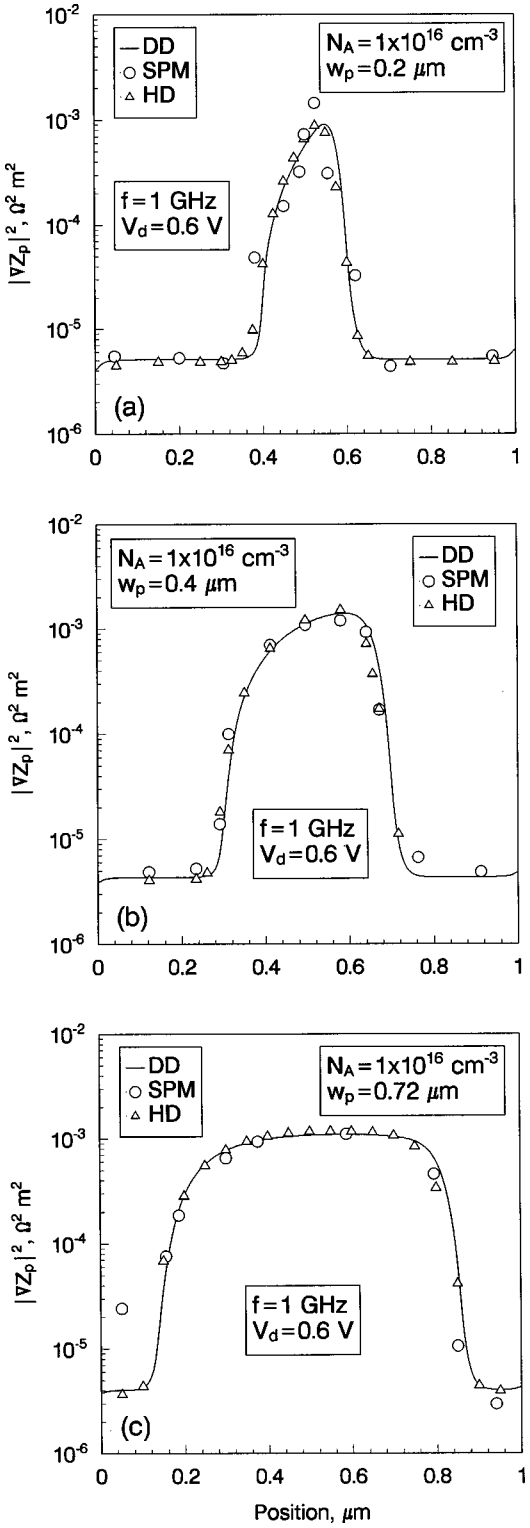

FIG. 4. Spatial dependence of the square magnitude of the vector impedance field in a p^+pp^+ Si diode. Three values of w_p are considered (keeping $w_p+2w_{p^+}=1$ μm): $w_p=0.2$ μm (a), $w_p=0.4$ μm (b) and $w_p=0.72$ μm (c). The doping values are $N_{A+}=10^{17}$ cm^{-3} and $N_A=10^{16}$ cm^{-3}. The bias is 0.6 V and the frequency is 1 GHz.

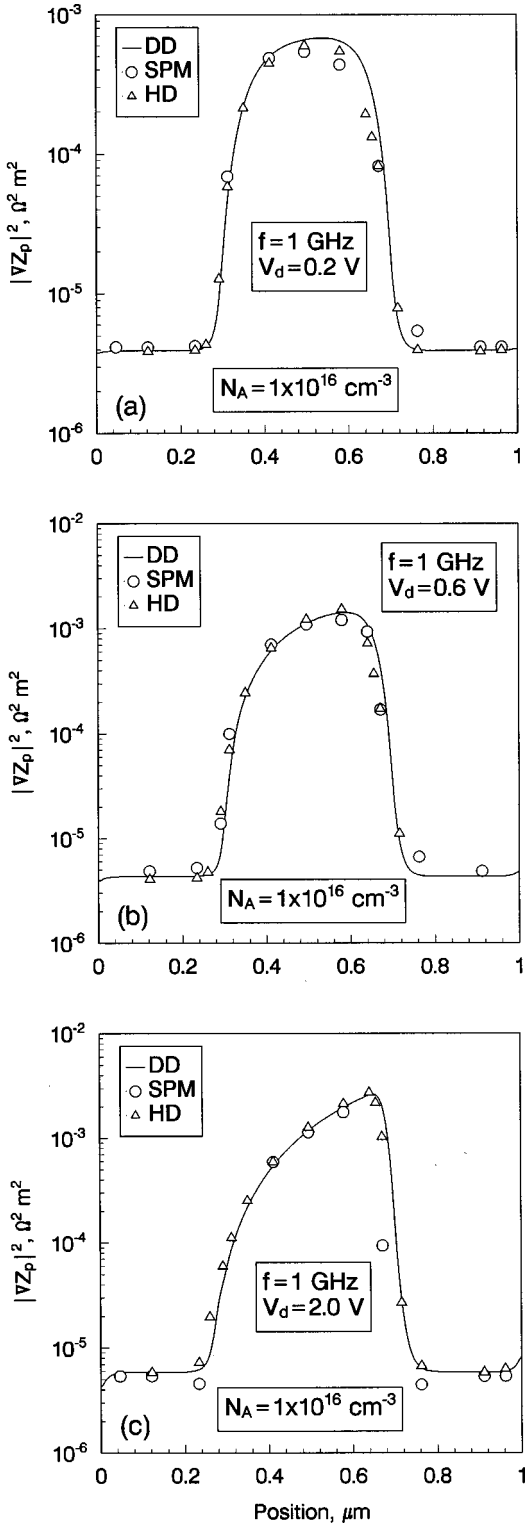

FIG. 5. Bias dependence of the square magnitude of the vector impedance field in a p^+pp^+ Si diode. Three bias values are considered: $V_d=0.2$ V (a), $V_d=0.6$ V (b) and $V_d=2.0$ V (c). The device structure parameters are: $w_p=0.4$ μm, $w_{p^+}=0.3$ μm, $N_{A+}=10^{17}$ cm^{-3} and $N_A=10^{16}$ cm^{-3}. The frequency is 1 GHz.

Fig. 5, where the device with $w_p=0.4$ μm and $N_A=10^{16}$ cm^{-3} has been simulated ($f=1$ GHz) for three values of dc bias: $V_d=0.2$ V (a), $V_d=0.6$ V (b) and $V_d=2$ V (c). It should be noted that not only does the IF spatial behavior become less symmetric as the electric field is increased, but also that its peak value increases as the bias is increased. This suggests that at very high electric field the peak value of the IF also becomes field dependent.

IV. CONCLUSION

This work has presented a comparison among a complete hierarchy of transport models applied to the evaluation of the IF for physics-based noise analysis of solid-state electron devices. A canonical test structure, a Si p^+pp^+ diode was considered, and results were reported concerning simulations performed through the DD, HD and SPM models. They are, at least for this canonical device, in good agreement for several bias, doping level and geometric conditions. Therefore, if heavy nonstationary transport effects do not appear in the dc solution, it may be expected that the impedance field evaluated through the DD model is also accurate enough, at least for frequencies up to the order of the inverse of the energy relaxation time (that is, in the usual operating range for most practical device applications).

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[1] W. Shockley, J. A. Copeland, and R. P. James, *Quantum Theory of Atoms, Molecules and Solid State*, edited by P. O. Lowdin (Academic, New York, 1966), p. 537.
[2] F. Bonani, G. Ghione, M. R. Pinto, and R. K. Smith, IEEE Trans. Electron Devices **45**, 261 (1998).
[3] C. M. van Vliet, IEEE Trans. Electron Devices **41**, 1902 (1994).
[4] L. Varani, L. Reggiani, T. Kuhn, T. González, and D. Pardo, IEEE Trans. Electron Devices **41**, 1916 (1994).
[5] L. Hlou, Ph.D. thesis (in French), Université Montpellier II, 1993.
[6] J. C. Vaissiere, L. Hlou, J. P. Nougier, and A. Achachi, *Proceedings of 12th International Conference on Noise in Physics Systems and 1/f Fluctuations*, edited by P. H. Handel and A. L. Chung (AIP, New York, 1993), p. 73.
[7] J. P. Nougier, L. Hlou, P. Houlet, J. C. Vaissiere, and L. Varani, *Proceedings of the Third International Workshop on Computational Electronics*, edited by S. M. Goodnick (Corvallis, Oregon State University, 1994), p. 15.
[8] P. Houlet, Ph.D. thesis (in French), Université Montpellier II, 1995.
[9] P. A. Layman, Ph.D. thesis, University of Waterloo, 1989.
[10] G. Ghione and F. Filicori, IEEE Trans. Comput.-Aided Des. **12**, 425 (1993).
[11] R. A. Roher, L. Nagel, R. Meyer, and L. Weber, IEEE J. Solid-State Circuits **6**, 204 (1971).
[12] F. Bonani, G. Ghione, M. R. Pinto, and R. K. Smith, Proceedings of the International Electron Devices Meeting, Washington, DC, December, 1995 (unpublished), p 777.
[13] M. R. Pinto, C. S. Rafferty, R. K. Smith, and J. Bude, Proceedings of the International Electron Devices Meeting, Washington, DC, December, 1993 (unpublished), p 701.
[14] A. Abou-Elnour and K. Schuenemann, Microwave Eng. **19**, 43 (1992).
[15] P. Shiktorov, E. Starikov, V. Gružinskis, L. Reggiani, and L. Varani, Appl. Phys. Lett. **69**, 1516 (1996).
[16] A. Cappy and W. Heinrich, IEEE Trans. Electron Devices **36**, 403 (1989).
[17] H. Happy and A. Cappy, *HELENA: HEMT Electrical Properties and Noise Analysis Software and User's Manual* (Artech, Boston, 1993).
[18] V. Gružinskis, E. Starikov, and P. Shiktorov, *Proceedings of 13th International Conference on Noise in Physics Systems and 1/f Fluctuations*, edited by V. Bareikis and R. Katilius (World Scientific, Singapore, 1995), p. 185.