Buckling and Fundamental Frequency Optimization of Tow-Steered Composites Using Layerwise Structural Models

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Variable-angle-tow (VAT) composite laminates can eventually improve the mechanical performance of lightweight structures by taking advantage of a larger design space compared to straight-fiber counterparts. Here, we provide a scalable low- to high-fidelity methodology to retrieve the tow angles that maximize the buckling load and the fundamental frequency of VAT plates. A genetic algorithm is used to solve the optimization problem in which the objective function is mimicked using a surrogate model. Both unconstrained and manufactured-constrained problems are solved. The surrogates are built with outcomes from numerical models generated by means of the Carrera unified formulation, which enables to obtain straightforwardly different degrees of accuracy by selecting the order of the structural theory employed. The results show both the validity and flexibility of the proposed design approach. It is shown that, although the optimal design fiber angle orientations are consistently similar, discrepancies in the prediction of the buckling load or fundamental frequency can be found between high-fidelity layerwise and low-to-refined equivalent-single-layer models, of which classical laminated plate or first-shear deformation theories are degenerate examples.

Nomenclature

\( \mathbf{C} \) = material stiffness matrix, Pa
\( D \) = differential operator
\( F_{\text{ct}} \) = buckling load, N
\( F_{\text{t}}, F_{\text{s}} \) = through-the-thickness expansion functions
\( f_1 \) = fundamental frequency, Hz
\( K_{\text{ct}} \) = buckling load factor
\( K_T \) = tangent stiffness matrix
\( K_g \) = geometric stiffness matrix
\( K_0 \) = stiffness matrix
\( M \) = mass matrix
\( N_i, N_j \) = finite element shape functions
\( q \) = unknown nodal vector
\( T_0, T_1 \) = fiber path angle parameters, °
\( u \) = displacement vector
\( \beta \) = design variables vector
\( \mathbf{\beta} \) = polynomial coefficient vector
\( \delta \) = virtual variation
\( \varepsilon \) = strain tensor
\( \theta \) = fiber angle orientation, °
\( \kappa \) = curvature, m\(^{-1}\)
\( \rho \) = density, kg/m\(^3\)
\( \sigma \) = stress tensor
\( \phi \) = fiber path rotation angle, °
\( \omega \) = natural frequency, rad/s

Subscripts

ext = external
ine = inertia
int = internal
\( L \) = lower bound
max = maximum
\( U \) = upper bound

I. Introduction

The irruption of novel manufacturing techniques, such as automated fiber placement (AFP) and automated type laying (ATL), brought the emergence of new families of laminated structures, namely, variable-angle-tow (VAT) composites or variable-stiffness composites (VSC), in which fiber tows are steered conforming curvilinear paths [1]. Although VAT composites have been recently introduced, the concept has existed for over three decades. Leissa and Martin [2] studied the free vibration and buckling of straight-fiber composites having nonuniformly spaced fibers and found that these two characteristics can be improved by as much as 21 and 38%, respectively. Gürald and Olmedo [3] proposed the VSC concept and studied its in-plane response modifying the parameters that define the varying fiber path. These analyses were conducted by means of closed-form and numerical solutions based on the classical laminated plate theory (CLPT). Similarly, Gürald et al. [4] investigated the buckling of VAT plates for different boundary conditions and rotations of the fiber path, resulting in a parametric analysis of the influence of the fiber path parameters on the nondimensional buckling load factor. Improvements up to 19 and 80%, with regard to classic laminates, were found for the different fiber path rotation angles considered. Gürald et al. [4] solved numerically the set of partial differential equations that govern the buckling problem and that rely on the CLPT. Raju et al. [5] also utilized the CLPT for modeling VAT structures and solved the resulting differential equations by means of the differential quadrature method (DQM) to study the prebuckling and buckling of VSC structures with general boundary conditions. Apart from CLPT, shear deformation theories based on those by Reissner [6] and Mindlin [7] have been used to study the mechanical performance of VAT plates. Akhavan and Ribeiro [8] investigated the fundamental frequency by using the third-order shear deformation theory by Reddy [9]. Venkatachari et al. [10] analyzed VSC plates and shells considering different fiber orientations and shell shapes using the first-order shear deformation theory (FSDT). Hao et al. [11] employed FSDT to model VAT shells and coupled it with isogeometric analysis to calculate the buckling load of the analyzed structures. CLPT and FSDT are examples of equivalent-single-layer (ESL) models in which the properties are homogenized through the thickness. On the contrary, layerwise (LW) models consider each layer independently, and displacement continuity has to be imposed at the layer interface.

The optimization of composite structures has been of interest to tailor the mechanical performance of the final product, and several strategies have been proposed throughout the years. Seminal work

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by Haftka and Walsh [12] introduced integer programming for the stacking sequence optimization for buckling of straight-fiber laminates. Later, Le Riche and Haftka [14] then proposed an integer-valued genetic algorithm (GA) to maximize the buckling performance. Therein, $0$, $±45$, and $90^\circ$ plies were encoded in order to perform the genetic operators. Le Riche and Haftka [14] then proposed an improved version of GA for the minimum thickness design of composite laminates. The usage of GA eases the coding of manufacturing constraints gathered by Irisarri et al. [15]. However, this integer-valued GA leads to a nonconvex problem, which is cumbersome to face in structural optimization. To circumvent this issue, a strategy based on lamination parameters was derived by Fukunaga and Sekine [16]. The expressions of the lamination parameters impose constraints on the design space of the lamination parameters to determine the feasible convex region where laminate configurations exist. These expressions rely on the usage of CLPT and FSDT models. GA and lamination parameters were combined in the slice and swap method proposed by Silva et al. [17] for wing optimization. This strategy consists of two steps:

i) First, a continuous optimization provides a distribution of thickness and directional stiffness that satisfy the safety margins in a series of multidisciplinary criteria. In this step, the design variables are the shell thickness, the stringer dimensions, and the stacking sequences in terms of lamination parameters.

ii) A discrete optimization process is triggered in order to transform the previous stiffness distribution to one that satisfies all the design and manufacturing rules. For a more detailed description of the slice and swap method, the reader is referred to Ref. [17].

So far, the optimization strategies of straight-fiber laminates have been discussed. Nevertheless, these can also be applied to retrieve the optimal stacking sequence of VAT composites. Serhat and Basdogan [18] proposed a CLPT lamination parameter scheme in which the radius of curvature is calculated, ensuring the manufacturability of the plate. Because AFP machines are not restricted to generating a constant curvature of the AFP machine arm was studied by Nik et al. [19]. They utilized a surrogate model to mimic the in-plane stiffness and buckling load of the VAT plates and formed by Pagani et al. [31] to study the vibration around nonlinear equilibrium states. Last, LW stochastic analysis concerning the failure onset and the buckling of VAT laminates has been performed in [32,33]. In the former, in-plane waviness was accounted for, whereas in the latter both in-plane waviness and fiber volume fraction variability were considered. These uncertainty defects were modeled by means of stochastic fields.

This work proposes the optimization of VAT plates modeled with CUF-based LW models. Surrogate models based on polynomials are utilized to mimic the optimization’s objective function, namely, buckling load factor and fundamental frequency. The optimization problem is solved by GA. Additionally, the AFP machine turning radius is considered a manufacturing constraint. The paper is organized as follows: Section II provides the main features of VAT plates and the equations to calculate the curvature of the fiber path. Section III depicts the unified FEs used to model the laminated structures. The GA characteristics and how the surrogate model is generated are available in Sec. IV. Then, model verification and optimization results are available in Sec. V. Finally, conclusions are drawn in Sec. VI.

II. Variable-Stiffness Plates and Manufacturing Constraints

In VAT composites, a band of fibers, referred to as a course, is laid over a surface following a reference path. Different paths have been analyzed throughout the years; the most common ones are those with constant curvature and linear variation. In this work, the latter are the ones to be considered. The linear variation states that the fiber angle orientation varies along the $x'$ direction and reads as

$$\theta(x') = \phi + T_0 + \frac{T_1 - T_0}{d} |x'|$$

where $T_0$ is the fiber angle at $x' = 0$, and $T_1$ corresponds to the fiber orientation at $x' = d$, where $d$ is the length along which the fiber angle orientation varies and typically equals the semilength or seminewidth of the VAT plate. Note that $x'$ can be expressed in terms of the global reference system as $x' = x \cos \phi + y \sin \phi$. Last, $\phi$ is the fiber path rotation angle that defines along which axis, i.e., $x$, $y$, or a combination of both, the fiber orientation varies. These parameters can be appreciated in Fig. 1. In this paper, $\phi$ is set equal to zero. Therefore, the $x'$ direction coincides with the $x$ axis and $d = a/2$.

The AFP machine turning radius has a limitation on the curvature of the laid fiber path. Otherwise, an unfolded tow or a wrinkle will generate in the placed tape. Therefore, the turning radius of the AFP head limits the fiber angle distribution of each lamina and determines whether a laminate can be manufactured or not. In this regard, it is used as an optimization constraint when aiming to optimize a certain characteristic of a VAT laminate. The most commonly extended value of the AFP turning radius is $r_{\min} = 0.635$ m. Hence, the curvature is constrained as
The curvature at a specific position \((x, y)\) can be calculated as in the work by Brooks and Martins [34], by defining the unit tangent vector \(t(\theta)\) (see Fig. 2) of the fiber path as

\[
t(\theta) = \cos(\theta) \hat{i} + \sin(\theta) \hat{j}
\]

Then, performing the curl operator over vector field \(t\) and keeping the only nonzero vector component yields

\[
\kappa (x, y) = (\nabla \times t(\theta)) \cdot \hat{k} = \frac{\partial \theta}{\partial x} \cos(\theta) + \frac{\partial \theta}{\partial y} \sin(\theta)
\]

which can be evaluated to assess whether a design is feasible from the manufacturing point of view or not. The symbols \(\hat{i}, \hat{j}\), and \(\hat{k}\) denote the unitary vectors of a Cartesian reference frame. Note that for the case in which the fiber path varies along the \(x\)-direction, i.e., \(\theta = 0^\circ\), Eq. \((4)\) reads as

\[
\kappa (x) = \text{sgn}(x) \frac{T_x - T_0}{d} \cos \left( T_0 + \frac{T_1 - T_0}{d} |x| \right)
\]

where \(\text{sgn}(\cdot)\) indicates the sign function.

### III. Unified Finite Elements

Two-dimensional FEs are implemented by using the CUF formalism. According to [25], the 3D field of displacement can be expressed in terms of arbitrary through-the-thickness expansion functions \(F_i(z)\) of the 2D generalized unknowns laying over the \(x\)-\(y\) plane; i.e.,

\[
u (x, y, z) = F_i(z) u_i (x, y)
\]

where \(M\) is the number of expansion terms and \(u_i (x, y)\) is the vector containing the generalized displacements. Note that \(\tau\) denotes summation. Common approximations for the generalized shapes of multilayered structures are the ESL and LW approaches. In this paper, ESL models are obtained using Taylor polynomials as \(F_i\) along the thickness direction. On the other hand, LW makes use of Lagrange polynomials over the single layers and then imposes the displacement continuity at the interfaces; see [35,36]. In this regard, TE\(_n\) indicates a TE of \(n\)-th order, while LE\(_n\) represents the usage of an LE with \(n\)-th-order polynomials.

Moreover, X LE\(_n\) denotes the usage of \(X\) Lagrange polynomials of \(n\)-th order to describe each layer of the laminate.

Utilizing the FE and shape functions \(N_i(x, y)\), the displacement field becomes

\[
u_i (x, y) = N_i (x, y) F_i (z) q_i, \quad i = 1, \ldots, N_n
\]

where \(q_i\) denotes the unknown nodal variables, and \(N_n\) indicates the number of nodes per element. Two-dimensional nine-node quadratic, Q9, elements are employed as \(N_i\) for the \(x\)-\(y\) plane discretization.

The principle of virtual displacements (PVDs) is used to derive the governing equations of the FE model. The PVD states that the virtual work of the external forces \(\delta W_{\text{ext}}\) has to be equal to the virtual work of the external forces \(\delta W_{\text{int}}\); i.e.,

\[
\delta W_{\text{int}} = \delta W_{\text{ext}} - \delta W_{\text{ine}}
\]

which in the case of free vibration analysis becomes

\[
\delta W_{\text{int}} + \delta W_{\text{ine}} = 0
\]

The virtual variation of the strain energy can be calculated as

\[
\delta W_{\text{int}} = \int_V \delta \varepsilon^T \sigma \, dV
\]

while the virtual work of the inertia forces is computed as

\[
\delta W_{\text{ine}} = \int_V \rho \delta u^T \ddot{u} \, dV
\]

where \(\rho\) represents the mass density of the material. Equation \((10)\) can be rewritten using Eq. \((7)\), the constitutive law \(\sigma = C \varepsilon\), and the geometrical relations between strains and displacements, thus yielding

\[
\delta W_{\text{int}} = \delta q_i^T \left[ \int_V D^T (N_i F_i) \tilde{C} D(N_i F_i) \, dV \right] q_i = \delta q_i^T k_i^{\text{int}} q_i
\]

where \(k_i^{\text{int}}\) is the \(3 \times 3\) fundamental nucleus (FN) of the stiffness matrix, which is invariant to the order of the 2D shape functions and the through-the-thickness expansion, as shown in [25]. \(D()\) is the differential operator matrix containing the geometrical relations, and \(\tilde{C}\) is the material stiffness matrix expressed in the global reference frame, i.e., \(\tilde{C} = T(x, y) CT^T (x, y)\). Note the dependency of the rotation matrix \(T\) on the in-plane coordinates due to the VAT fiber paths; see [32].

The virtual work of the inertia forces can be expressed as

\[
\delta W_{\text{ine}} = \delta q_i^T \left[ \int_V \rho I F_i F_i N_i N_j \, dV \right] q_i = \delta q_i^T m_i^{\text{int}} q_i
\]

in which \(I\) is the \(3 \times 3\) identity matrix and \(m_i^{\text{int}}\) is the \(3 \times 3\) FN of the mass matrix. Note that \(m_i^{\text{int}}\) is a diagonal matrix.

The undamped free vibration problem can be written as follows:

\[
M \ddot{q} + K_0 q = 0
\]
respectively. If one imposes harmonic solutions \( q = \bar{q}e^{j\omega t} \), then Eq. (14) turns into the following eigenvalue problem:

\[
(K_\omega - \omega^2 M)\bar{q}_i = 0
\]

(15)

where \( \omega \) and \( \bar{q}_i \) are the \( i \)th natural frequency and eigenvector, respectively.

The buckling analysis consists in solving the equation

\[
|K_T| = 0
\]

(16)

where \( K_T \) is the tangent stiffness matrix of the structure. The formula for this matrix is derived by means of linearizing the virtual variation of the internal strain energy:

\[
\delta^2(L_{int}) = \int_V \left[ \delta(\sigma^T\varepsilon) - \delta^T \varepsilon \right] dV
\]

(17)

After introducing Eq. (7), the constitutive law, and the geometrical relations between strains and displacements, the previous equation adopts the following form:

\[
\delta^2(L_{int}) = \delta q_j^T (k^{ij}_{int} + k^{ij}_{str}) q_i
\]

(18)

This equation can be written in the case of linearized buckling problem as

\[
\delta^2(L_{int}) = \delta q_j^T (k^{ij}_{0} + k^{ij}_{str}) q_i
\]

(19)

where \( k^{ij}_{int} \approx k^{ij}_{0} + k^{ij}_{str} \). Therein, \( k^{ij}_{str} \) corresponds to the 3 × 3 FN of the geometric stiffness matrix, which strictly depends on the internal stress state of the structure. Note that the stress state will be dependent on the accuracy of the model. The equations that allow the calculation of the tangent stiffness matrix are not reported in the paper for the sake of brevity but can be found in [37]. Last, since the linear hypothesis holds, \( k^{ij}_{str} \) is supposed to be proportional to \( \lambda_{cr}^2 \), which is the solution to a linear eigenvalue problem and is proportional to the applied load in the case of linearized buckling. Therefore, after the FN’s are expanded and the elemental stiffness matrices assembled over the entire structural domain, Eq. (16) can be rewritten as follows:

\[
|K_s + \lambda_{cr}^2 K_g| = 0
\]

(20)

to calculate \( \lambda_{cr} \). Note that \( K_g \) denotes the assembled geometric stiffness matrix of the structure.

The assembly of \( K_0, M, \) and \( K_g \) differs whether an ESL or LW approach is chosen. In ESL, the homogenization of the properties of each layer is carried out and summed altogether when computing the stiffness matrix. As addressed by Carrera [38], ESLs do not fulfill the \( C^1 \) requirements. Conversely, LW considers each layer independently and expands the displacement field within each lamina. Consequently, the continuity of displacements has to be imposed at the interface (see [35,36]), thus guaranteeing the completion of the \( C^1 \) requirements. These two assembly approaches are displayed in Fig. 3.

IV. Optimization Problem

The VAT literature has demonstrated that this kind of laminates can tailor the in-plane stress resultants [22], fundamental frequencies [24], vertical deflections [39], and thermal buckling [40], among other mechanical characteristics. Based on these studies, it is observed that multiple local peaks of the mentioned characteristics exist in terms of the fiber path angles. In this context, an optimization algorithm that allows to explore the whole design space is needed. However, to perform a thorough exploration of the design space, a vast number of function evaluations are required. This need, coupled with the complexity of the VAT models, makes the optimization of these structures a computationally intensive problem. Thus, a strategy that permits exploring the design space as well as a quick evaluation of the objective, or constraint, functions is of utmost importance. The first is obtained with the usage of GA, while the latter is solved by creating response surfaces that mimic the mentioned objective or constraint functions. These two methods are explained next.

A. Genetic Algorithm

GAs are a group of evolutionary algorithms used to solve optimization problems. They are based on Darwin’s Theory of Evolution [41]. Genetic operators such as crossover, elitism, and mutation are used to generate better-performing offspring, in terms of the objective function, than their parents. A GA can be used to provide the solution to both unconstrained and constrained problems, like the one in the following equation:

\[
\min F(x) \text{ s.t. } \begin{cases} g_i(x) \leq 0 & i = 1, \ldots, r \\ h_j(x) = 0 & j = 1, \ldots, k \\ x_L \leq x \leq x_U \end{cases}
\]

(21)

where \( F(x) \) is the objective function, i.e., the function that one aims to minimize, while \( g_i(x) \) and \( h_j(x) \) are the inequality and equality constraints, respectively; \( x_L \) and \( x_U \) are the lower and upper bounds of the vector containing the design variables, \( x \), respectively. In this paper, an in-house developed GA based on that presented by Montemurro et al. [42] has been used. This GA allows the user to conceive individuals with a fixed or varying number of design variables and/or layers. For conducting the optimization of laminates in which the number of layers may vary, referred to as multispecies, additional genetic operators, such as specie crossover and layer addition/deletion, have to be incorporated. Despite these additional features, in this paper the number of design variables and layers of the laminates considered is kept constant throughout the optimization process. For further information about the aforementioned features, the reader is referred to [42].

Since the objective functions considered in this paper are expensive simulations, the computational cost becomes a major challenge. Therefore, the authors resort to a surrogate model that approximates the computationally expensive simulations through a multidimensional parametric surface [43].

B. Response Surface Modeling

Surrogate models can be used to accelerate the retrieval of an optimum solution. There exists a plethora of surrogate models that can be used, ranging from polynomial expressions up to ANNs [21] or passing by polynomial chaos expansion [44], radial basis functions [45], or kriging processes [46]. Because of the limited number of design variables involved in this document, a polynomial surrogate is
transverse displacements are restrained at form shortening.

vector can be calculated as which, in general, is not a square matrix. Hence, the coefficient

better fit of the numerical model. repeated several times, an average of randomly pick the 10 samples needed to calculate

variables. Therefore, a minimum of 10 samples (Ns = 10) are needed to construct the polynomial. One can generate a larger database and randomly pick the 10 samples needed to calculate \( \beta \). If this process is repeated several times, an average of \( \beta \) can be obtained, leading to a better fit of the numerical model.

\[ f(x) = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{ij} x_i x_j + \sum_{i=1}^{n} \beta_i x_i^2 \] (22)

where \( \beta_0, \beta_i, \) and \( \beta_{ij} \) are the polynomial coefficients; \( x_i \) are the independent variables; and \( n \) is the number of independent variables. To fit the surrogate model to the data set, the least-squares method is used to calculate the polynomial coefficients. If, for instance, a total of \( N_s \) samples are employed to construct the surrogate model and two design variables, \( n = 2 \), are considered, the regression problem reads as

\[ f = \Psi \beta \] (23)

in which \( f \) is an \( N_s \times 1 \) column vector containing the data samples of the function that one aims to mimic, \( \beta \) contains the coefficients of the polynomial, and \( \Psi \) is a matrix with the \( N_s \) values of the design variables, i.e.,

\[ \Psi = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{11} x_{12} & x_{11}^2 & x_{12}^2 & X_{11} & X_{12} & X_{11} X_{12} \\ 1 & x_{21} & x_{22} & x_{21} x_{22} & x_{21}^2 & x_{22}^2 & X_{21} & X_{22} & X_{21} X_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{N_s,1} & x_{N_s,2} & x_{N_s,1} x_{N_s,2} & x_{N_s,1}^2 & x_{N_s,2}^2 & X_{N_s,1} & X_{N_s,2} & X_{N_s,1} X_{N_s,2} \end{bmatrix} \] (24)

which, in general, is not a square matrix. Hence, the coefficient vector can be calculated as

\[ \beta = (\Psi^T \Psi)^{-1} \Psi^T f \] (25)

To improve the accuracy of the surrogate model, Eq. (25) is solved several times, and the coefficients are averaged. As an example, let us consider a third-order polynomial comprising two independent variables. This polynomial contains 10 terms combining those two variables. Therefore, a minimum of 10 samples (\( N_s = 10 \)) are needed to construct the polynomial. One can generate a larger database and randomly pick the 10 samples needed to calculate \( \beta \). If this process is repeated several times, an average of \( \beta \) can be obtained, leading to a better fit of the numerical model.

\section*{V. Results}

\subsection*{A. Buckling Verification}

The presented modeling approach is verified with a reference solution provided by Gürdal et al. [4]. It consists of a 12-layered VAT plate with stacking sequence [0 ± (0, 50)]\({}_b\). The width \( a \) and length \( b \) of the plate are \( a = b = 0.254 \) m, whereas the thickness of the individual ply is \( t = 0.127 \) mm. The material properties are listed in Table 1, and the boundary conditions are depicted in Fig. 4. The transverse displacements are restrained at \( y = \pm b/2 \), while a uniform shortening \( u_c = u_0 \) is exerted along \( x = \pm a/2 \).

The first step toward the verification is conducting a convergence analysis for the in-plane 2D mesh. The chosen elements consist of biquadratic Lagrange polynomials, referred to as Q9 elements hereinafter, since they comprise nine nodes per element. Concerning the through-the-thickness direction, a quadratic element LE2 is utilized per each layer, providing an LW model. Convergence results are available in Table 2. The outcomes are expressed in terms of the normalized buckling load, which is computed as

\[ K_{cr} = \frac{F_{cr} a^2}{E_b h^3 b} \] (26)

where \( h \) is the total thickness of the plate. It is observed that the 10 × 10 Q9 mesh provides a converged value of \( K_{cr} \). The difference between the reference value [4] and the one calculated with the present methodology stems from the models employed. Gürdal et al. [4] used the CLPT and the Rayleigh–Ritz method to calculate the buckling load, whereas the proposed model utilizes an LW description of the laminated VAT plate. Clearly, CLPT offers a stiffer and more conservative solution in this case. In addition, Fig. 5 provides the convergence of the first five buckling load factors and their respective modes. It is observed that the first four modes converged within 1% for the 10 × 10 Q9 mesh, while the fifth one shows roughly a 3% discrepancy.

The next step involves using different expansion functions in the through-the-thickness direction. A comparison between Taylor and Lagrange expansions is shown in Table 3. These models employ the

\begin{table}[h]
\centering
\caption{Material properties of the VAT plate considered for the linear buckling analysis, from [4]}
\begin{tabular}{|c|c|}
\hline
Parameter & Value \\
\hline
\( E_1 \), GPa & 181.00 \\
\( E_2 = E_1 \), GPa & 10.27 \\
\( G_{12}, \) GPa & 7.17 \\
\( G_{23}, \) GPa & 4.00 \\
\( \nu_{12} = \nu_{23} \) & 0.28 \\
\hline
\end{tabular}
\end{table}

The value of \( G_{23} \) was taken from [23].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Convergence of \( K_{cr} \) in terms of DOF. The first five buckling modes are illustrated.}
\end{figure}


10 × 10 Q9 mesh demonstrated to provide a converged solution for $K_{cr}$. Note that, in the current procedure, ESL models are obtained with TE polynomials. TE 1, which is similar to FSDT, provides the closest solution to the reference CLPT. Then, as the order of TE increases, lower values of $K_{cr}$ are predicted. Indeed, the TE 3 model can obtain the same value provided by the LE2 model with an 84% reduction in terms of degrees of freedom (DOF).

### B. Free Vibration Verification

The free vibration problem is verified against the results available in Akhavan and Ribeiro [8]. The study case concerns a three-layered squared plate with stacking sequence $(0°, 90°, 45°, −45°)$ and thickness ratio $a/b = 10$, having each ply the same thickness. The material elastic properties are reported in Table 4. The structure is fully clamped.

As performed in the previous section, a convergence of the FE mesh is done first, employing Q9 FE and LE2 in the thickness direction. The results are listed in Table 5 and illustrated in Fig. 6 along with the first five free vibration modes. A good agreement between Ref. [8] and the present results is observed. All the first five modes but the fourth one converge within 1% using the 10 × 10 Q9 grid. This mesh provides a higher value than the 16 × 16 Q9 one. However, the relative error between both solutions is 0.19%, presenting the converging within 1% using the present results is observed. All the first five modes but the fourth one are predicted. Indeed, the TE 3 model can obtain the same value provided by the LE2 model with an 84% reduction in terms of degrees of freedom (DOF).

### C. Buckling Optimization

Referring to the problem in Sec. V.A, the buckling optimization can be stated as

$$\min_x - F_{cr}(x)$$

in which $x = \{T_0, T_1\}$ are the design variables corresponding to a laminate stacking sequence $\theta = [0° \pm (T_0, T_1)]_s$, and $F_{cr}$ denotes the critical buckling load. The lower and upper bounds are $x_0 = -90°$ and $x_0 = 90°$, respectively, as in [23,24]. To solve the optimization problem, the response surface method explained in Sec. IV.B is used to mimic the distribution of $F_{cr}$ as a function of the variable orientation parameters $T_0$ and $T_1$. Therefore, $F_{cr}$ is approximated by $\tilde{F}_{cr}$, i.e., $F_{cr} \approx \tilde{F}_{cr}$, in Eq. (27). Note that $\tilde{F}_{cr}$ represents the response surface model of $F_{cr}$. Fifteen samples were generated by means of Latin hypercube sampling (LHS) [48] to construct the surrogate model for the LW-LE2 and ESL TE1 and TE3 structural models and width-to-thickness $a/h$ ratios. Figure 7 shows the constructed response surface and the sample data for the LW-LE2 and ESL-TE3 thin plate models, as well as their contour plot. Two local maxima are appreciated, from whom one is the

<table>
<thead>
<tr>
<th>Model</th>
<th>DOF</th>
<th>$f_{1}$, Hz</th>
<th>$f_{2}$, Hz</th>
<th>$f_{3}$, Hz</th>
<th>$f_{4}$, Hz</th>
<th>$f_{5}$, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. [8]</td>
<td>—</td>
<td>613.79</td>
<td>909.04</td>
<td>1231.65</td>
<td>1337.69</td>
<td>1484.53</td>
</tr>
<tr>
<td>6 × 6 Q9</td>
<td>3,549</td>
<td>614.82</td>
<td>916.09</td>
<td>1230.29</td>
<td>1361.72</td>
<td>1492.04</td>
</tr>
<tr>
<td>8 × 8 Q9</td>
<td>6,069</td>
<td>611.26</td>
<td>907.03</td>
<td>1219.97</td>
<td>1337.03</td>
<td>1475.20</td>
</tr>
<tr>
<td>10 × 10 Q9</td>
<td>9,261</td>
<td>609.91</td>
<td>903.95</td>
<td>1216.18</td>
<td>1328.88</td>
<td>1469.58</td>
</tr>
<tr>
<td>12 × 12 Q9</td>
<td>13,125</td>
<td>609.28</td>
<td>902.56</td>
<td>1214.47</td>
<td>1325.46</td>
<td>1467.15</td>
</tr>
<tr>
<td>16 × 16 Q9</td>
<td>22,869</td>
<td>608.74</td>
<td>901.46</td>
<td>1213.07</td>
<td>1322.85</td>
<td>1465.21</td>
</tr>
</tbody>
</table>

Each discretization employs 1 LE2 element per layer.
global maximum. As one can appreciate, both response surfaces from Figs. 7a and 7c, and contour plots in 7b and 7d, are practically identical, and just slight differences in the surrogate model’s coefficients are observed.

The optimization solution is obtained by employing the GA depicted in Sec. IV.A. The GA population comprises a total of 40 individuals, i.e., 20 per design variable. The crossover probability is set to 80%, whereas the mutation probability is equal to 5%. The coefficients are observed.

Table 6 Convergence analysis of the first five natural frequencies for the fully clamped [(0, 45), (−45, −60), (0, 45)] plate from [8]

<table>
<thead>
<tr>
<th>Model</th>
<th>DOF</th>
<th>$f_1$, Hz</th>
<th>$f_2$, Hz</th>
<th>$f_3$, Hz</th>
<th>$f_4$, Hz</th>
<th>$f_5$, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. [8]</td>
<td>——</td>
<td>613.79</td>
<td>909.04</td>
<td>1231.65</td>
<td>1337.69</td>
<td>1484.53</td>
</tr>
<tr>
<td>TE 1</td>
<td>2.646</td>
<td>638.87(4)%</td>
<td>955.51(1)%</td>
<td>1278.43(8)%</td>
<td>1419.71(8)%</td>
<td>1553.57(6)%</td>
</tr>
<tr>
<td>TE 2</td>
<td>3.969</td>
<td>634.39(3)%</td>
<td>943.64(8)%</td>
<td>1273.88(4)%</td>
<td>1399.97(4)%</td>
<td>1542.34(8)%</td>
</tr>
<tr>
<td>TE 3</td>
<td>5.292</td>
<td>611.17(4)%</td>
<td>908.11(0)%</td>
<td>1218.00(1)%</td>
<td>1338.39(0)%</td>
<td>1473.10(7)%</td>
</tr>
<tr>
<td>TE 4</td>
<td>6.615</td>
<td>611.04(5)%</td>
<td>907.83(1)%</td>
<td>1217.56(1)%</td>
<td>1337.69(0)%</td>
<td>1479.14(0)%</td>
</tr>
<tr>
<td>TE 5</td>
<td>7.938</td>
<td>609.49(0)%</td>
<td>903.63(0)%</td>
<td>1214.24(1)%</td>
<td>1328.60(0)%</td>
<td>1467.81(1)%</td>
</tr>
<tr>
<td>TE 6</td>
<td>9.261</td>
<td>609.49(0)%</td>
<td>903.63(0)%</td>
<td>1214.24(1)%</td>
<td>1328.59(0)%</td>
<td>1477.46(0)%</td>
</tr>
<tr>
<td>1 LE1</td>
<td>5.292</td>
<td>621.64(2)%</td>
<td>917.66(9)%</td>
<td>1244.85(1)%</td>
<td>1347.15(7)%</td>
<td>1499.61(2)%</td>
</tr>
<tr>
<td>1 LE2</td>
<td>9.261</td>
<td>609.91(0)%</td>
<td>903.93(0)%</td>
<td>1216.18(1)%</td>
<td>1328.88(0)%</td>
<td>1469.58(1)%</td>
</tr>
<tr>
<td>1 LE3</td>
<td>13.230</td>
<td>608.60(4)%</td>
<td>900.62(0)%</td>
<td>1213.16(1)%</td>
<td>1322.06(1)%</td>
<td>1464.94(1)%</td>
</tr>
</tbody>
</table>

Each model employs a 10 x 10 Q9 mesh. The relative difference between each model and the reference is reported in the superscript.

Table 7 Convergence analysis of the first five natural frequencies for the simply supported [(0, 45), (−45, −60), (0, 45)] plate from [8]

<table>
<thead>
<tr>
<th>Model</th>
<th>DOF</th>
<th>$f_1$, Hz</th>
<th>$f_2$, Hz</th>
<th>$f_3$, Hz</th>
<th>$f_4$, Hz</th>
<th>$f_5$, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. [8]</td>
<td>——</td>
<td>467.31</td>
<td>746.54</td>
<td>1141.80</td>
<td>1166.28</td>
<td>1349.00</td>
</tr>
<tr>
<td>TE 1</td>
<td>2.646</td>
<td>483.27(4)%</td>
<td>782.96(8)%</td>
<td>1167.42(7)%</td>
<td>1237.26(1)%</td>
<td>1555.62(4)%</td>
</tr>
<tr>
<td>TE 2</td>
<td>3.969</td>
<td>479.51(4)%</td>
<td>781.63(6)%</td>
<td>1162.71(3)%</td>
<td>1215.46(2)%</td>
<td>1339.97(1)%</td>
</tr>
<tr>
<td>TE 3</td>
<td>5.292</td>
<td>466.85(0)%</td>
<td>746.81(0)%</td>
<td>1141.73(0)%</td>
<td>1216.35(0)%</td>
<td>1348.81(0)%</td>
</tr>
<tr>
<td>TE 4</td>
<td>6.615</td>
<td>466.81(0)%</td>
<td>746.70(0)%</td>
<td>1141.51(0)%</td>
<td>1216.01(2)%</td>
<td>1320.75(2)%</td>
</tr>
<tr>
<td>TE 5</td>
<td>7.938</td>
<td>465.36(0)%</td>
<td>742.59(0)%</td>
<td>1142.11(0)%</td>
<td>1216.83(0)%</td>
<td>1320.60(2)%</td>
</tr>
<tr>
<td>TE 6</td>
<td>9.261</td>
<td>465.36(0)%</td>
<td>742.58(0)%</td>
<td>1142.20(0)%</td>
<td>1216.83(0)%</td>
<td>1317.28(2)%</td>
</tr>
<tr>
<td>1 LE1</td>
<td>5.292</td>
<td>431.91(5)%</td>
<td>691.34(7)%</td>
<td>1060.18(4)%</td>
<td>1076.65(2)%</td>
<td>1273.80(5)%</td>
</tr>
<tr>
<td>1 LE2</td>
<td>9.261</td>
<td>432.13(5)%</td>
<td>692.01(7)%</td>
<td>1061.63(4)%</td>
<td>1097.48(7)%</td>
<td>1276.17(4)%</td>
</tr>
<tr>
<td>1 LE3</td>
<td>13.230</td>
<td>429.63(8)%</td>
<td>686.85(4)%</td>
<td>1051.63(5)%</td>
<td>1098.96(5)%</td>
<td>1262.98(6)%</td>
</tr>
</tbody>
</table>

Each model employs a 10 x 10 Q9 mesh. The relative difference between each model and the reference is reported in the superscript.
variables similar to each other when thin plates \((a/h = 167\) case) are modeled. The optimal design variables for the thin plate are around \((T_0, T_1) \approx (20, 32)^\circ\). As occurred in the unconstrained problem, LW and ESL lead to the same solution when thick plates are analyzed. In this case, the optimal design variables are around \((T_0, T_1) \approx (12, 24)^\circ\). It is appreciated that, for each of the width-to-thickness ratios and structural theories, the optimal design lays in the constraint boundary.

Some differences are appreciated between the solutions to the unconstrained and the constrained problems for both width-to-thickness ratios. First, an abrupt sign change occurs between \(T_0\) and \(T_1\) in the unconstrained problem, as observed in Figs. 9a and 9c, whereas a smoother fiber path is appreciated in the constrained one (see Figs. 10a and 10c). Concerning the first buckling mode, a sharper form pointing toward the left inferior and right upper corners is observed for the constrained solution in Figs. 10b and 10d, while a rounder shape is presented in the unconstrained laminate in Figs. 9b and 9d. This difference is due to the larger stiffness close to the shortened edges in the thick \((T_0, T_1) = (12, 10, 23.83)^\circ\) and thin \((T_0, T_1) = (20.81, 33.08)^\circ\) design compared to the thick \((T_0, T_1) = (−1.90, 40.97)^\circ\) and thin \((T_0, T_1) = (−17, 52)^\circ\) unconstrained designs.

### D. Fundamental Frequency Optimization

The fundamental frequency optimization problem reads as

\[
\min_x f_3(x)
\]

where \(x = \{T_0, T_1\}\) are the design variables relative to a fully clamped plate whose stacking sequence is \(\theta = [0(T_0, T_1), 0(90 + T_0, 90 + T_1)]\), as in the work by Akhavan and Ribeiro \cite{8}. The lower and upper bounds are \(x_L = −90^\circ\) and \(x_U = 90^\circ\), respectively. The material properties are the ones used in Sec. V.B listed and in Table 4, while the width and length of the plate are \(a = b = 1\) m. Two width-to-thickness ratios are considered,

\[
\begin{array}{c|cccccc}
\hline
\text{Parameter} & \text{LW-LE2} & \text{ESL-TE 1} & \text{ESL-TE 3} & \text{LW-LE2} & \text{ESL-TE 1} & \text{ESL-TE 3} \\
\hline
\{T_0, T_1\} [^\circ] & (−1.90, −3.38, −3.37) & (40.97, 40.46, 40.97) & (−17, −17, −11.77) & (20.81, 33.08) & (−1.90, 40.97) & (−17, 52) & (51.88) \\
F_{cr}, N & 8.24 \cdot 10^6 & 8.37 \cdot 10^6 & 3.58 \cdot 10^3 & 3.58 \cdot 10^3 & 3.58 \cdot 10^3 & 3.33 \cdot 10^3 \\
F_{cr}, N & 8.28 \cdot 10^6 & 9.37 \cdot 10^6 & 3.48 \cdot 10^3 & 3.56 \cdot 10^3 & 3.33 \cdot 10^3 & 7.40 \\
\text{Error, %} & −0.51 & −0.31 & 0.06 & 1.90 & 3.38 & 1.00 \\
\hline
\end{array}
\]
Table 9  Optimal results of the constrained buckling optimization problem for the $\{0 \pm (T_0, T_1)\}_{3s}$ plate subjected to uniform end shortening and restrained transverse edges, comparison between surrogate model $F_{cr}$ and direct analysis $F_{cr}$ of the optimized stacking sequence, and maximum value of the steering curvature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$a/h = 10$</th>
<th>$a/h = 167$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(T_0, T_1)^\prime$</td>
<td>(12.10, 23.83)</td>
<td>(12.28, 24.01)</td>
</tr>
<tr>
<td>$\bar{F}_{cr}, N$</td>
<td>8.08 · $10^6$, 9.14 · $10^6$, 8.51 · $10^6$</td>
<td>3.05 · $10^3$, 3.05 · $10^3$, 3.15 · $10^3$</td>
</tr>
<tr>
<td>Error, %</td>
<td>0.07, 1.07, 0.49</td>
<td>3.97, 4.84, 0.59</td>
</tr>
<tr>
<td>$\kappa_{max}, m^{-1}$</td>
<td>1.57, 1.57, 1.57</td>
<td>1.57, 1.57, 1.57</td>
</tr>
</tbody>
</table>

Fig. 8  Distributions of $\sigma_{xx}$ and $\sigma_{zz}$ for the different width-to-thickness ratios and LW-LE2 and ESL-TE3 structural theories. For both width-to-thickness ratios, their respective LW-LE2 solution from Table 8 was used to calculate the stress distributions.

Fig. 9  Fiber paths and first buckling mode of the LW optimum solution for the unconstrained buckling load optimization problem for thick (a, b) and thin (c, d) laminates.
namely, \(a/h = 10\) and \(a/h = 100\), which represent the case of thick and thin laminates, respectively.

The effect of the structural theory on the fundamental frequency optimization results is analyzed for thick and thin laminates. The FE mesh comprises \(10 \times 10\) Q9 elements with different through-the-thickness expansion functions and modeling approaches, namely, ESL-TE 1, ESL-TE 3, and LW employing an LE2 discretization. The mesh convergence analysis has been omitted for the sake of brevity.

As in Sec. V.C, the optimization problem is solved by using a surrogate model that mimics \(f_1\) over the design space. Therefore, \(f_1\) is approximated by \(\tilde{f}_1\); i.e., \(f_1 \approx \tilde{f}_1\) in Eq. (29). Likewise, 15 samples were generated by means of LHS to construct the response surface. Note that the same \(T_0; T_1\) pairs are used to build the surrogate for the different structural theories and width-to-thickness ratio. All the structural models used a truncated fourth-order response surface to mimic the FE simulations. The LW response surface and its contour plot are represented in Fig. 11. Concerning the GA parameters, 40 individuals were considered per generation, with a crossover probability equal to 80 and 5% mutation probability.

Table 10 reports the optimum design variables achieved with the various structural theories and width-to-thickness ratios for the unconstrained optimization problem. One can observe that LW and ESL provide similar solutions for both thick (\(a/h = 10\)) and thin (\(a/h = 100\)) laminates. In all of the obtained laminations, \(T_1\) has a negative value as appreciated in Figs. 12a and 12c. In these edges, the fibers point toward the \(x\) edges with constant \(T_1\) orientation. Conversely, the fibers point toward the \(y\) edges at \(x = 0\), where they present the maximum transverse stiffness. This fiber pattern is in agreement with the optimization results for the fully clamped squared plate shown in Table 8 from [24]. Moreover, agreement in the negative sign of \(T_1\) between the proposed ESL solutions and those reported in [24] is found.
work [24] employed shell-like S4R Abaqus elements with similar capabilities as the ESL-TE 1 model used in this paper. The curvature-constrained fundamental frequency optimization reads as

\[
\min_x f_1(x) \text{ s.t. } 1/r_{\min} \leq \kappa(x) \leq 1/r_{\min} \tag{30}
\]

where \(\kappa(x)\) is calculated using Eq. (5). The optimization results are summarized in Table 11. Changes with respect to the unconstrained results are appreciated. Again, LW and ESL models provide similar results for both thick and thin laminates. The main difference is found in the prediction of the fundamental frequency for thick laminates, where ESL models lead to overestimated values of \(f_1\), especially

![Figure 12](image1.png)

**Table 10** Optimal results of the unconstrained first fundamental frequency optimization problem for the fully clamped \([0(T_0, T_1), 0(90 + T_0, 90 + T_1)]\) plate, and comparison between surrogate model \(\tilde{f}_1\) and direct analysis \(f_1\) of the optimized stacking sequence

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(a/h = 10)</th>
<th>(a/h = 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_0)</td>
<td>LW-LE2</td>
<td>ESL-TE 1</td>
</tr>
<tr>
<td>(T_1)</td>
<td>(-90, -84.68, -85.84, -89.73)</td>
<td>(-90, -84.68, -85.84, -89.73)</td>
</tr>
<tr>
<td>(f_1)</td>
<td>710.63 713.67 713.76 720.08</td>
<td>710.63 713.67 713.76 720.08</td>
</tr>
<tr>
<td>Error, %</td>
<td>-0.89 -1.67 -0.88 -0.64</td>
<td>-0.89 -1.67 -0.88 -0.64</td>
</tr>
<tr>
<td>(\kappa_{\text{max}}, \text{m}^{-1})</td>
<td>1.57 1.57 1.57</td>
<td>1.57 1.57 1.57</td>
</tr>
</tbody>
</table>

**Table 11** Optimal results of the constrained first fundamental frequency optimization problem for the \([0(T_0, T_1), 0(90 + T_0, 90 + T_1)]\) fully clamped plate, comparison between surrogate model \(\tilde{f}_1\) and direct analysis \(f_1\) of the optimized stacking sequence, and maximum value of the steering curvature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(a/h = 10)</th>
<th>(a/h = 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_0)</td>
<td>LW-LE2</td>
<td>ESL-TE 1</td>
</tr>
<tr>
<td>(T_1)</td>
<td>(-90, -84.68, -85.84, -89.73)</td>
<td>(-90, -84.68, -85.84, -89.73)</td>
</tr>
<tr>
<td>(f_1)</td>
<td>710.63 713.67 713.76 720.08</td>
<td>710.63 713.67 713.76 720.08</td>
</tr>
<tr>
<td>Error, %</td>
<td>-0.89 -1.67 -0.88 -0.64</td>
<td>-0.89 -1.67 -0.88 -0.64</td>
</tr>
<tr>
<td>(\kappa_{\text{max}}, \text{m}^{-1})</td>
<td>1.57 1.57 1.57</td>
<td>1.57 1.57 1.57</td>
</tr>
</tbody>
</table>
Fig. 13  Fiber paths and first vibration mode of the LW optimum solution for the constrained fundamental frequency optimization problem for thick (a, b) and thin (c, d) laminates.

Fig. 14  Histograms gathering the optimal design variables for the unconstrained and constrained buckling optimization problems for the different width-to-thickness ratios $a/h$ and structural theories.
It is worth noting that the solutions to the constrained problem lay on the constraint boundary, as occurred in the buckling optimization problem. Finally, the fiber patterns of the constrained LW solutions for thick and thin laminates are represented in Fig. 13 with their respective first vibration modes.

VI. Conclusions

This paper dealt with the unconstrained and constrained optimization of VAT plates. The VSC structures were modeled by means of ESL and LW CUF-based models. After the verification against literature results of both the buckling load and fundamental frequency, an LHS sampling strategy was used to generate the input that provided the aforementioned magnitudes as an outcome of the FE simulations. Surrogate models based on polynomial expressions were built using the problem input–output and employed to mimic the buckling load and fundamental frequency, which served as the objective function of the optimization problem. GA was utilized to provide the optimum design variables.

According to the results shown in this paper, the following comments can be made:

1) When optimizing the buckling load of VAT plates, ESL models led to similar results in terms of buckling load and design variables as those obtained with an LW theory in the case of thick and thin plates,
for both unconstrained and constrained optimization problems, as shown in Fig. 14. In this regard, ESL approaches should be preferable when optimizing the buckling load of VAT plates since they need lower DOF than its LW counterparts. Nevertheless, ESL will overestimate the buckling load since they lead to stiffer models. This is displayed in Fig. 15a, where larger circles are appreciated for the ESL models, especially ESL-TE 1.

2) When optimization of the fundamental frequency is faced, it has been shown that ESL models lead to similar optimal solutions for both unconstrained and constrained problems, as well as width-to-thickness ratios. This is appreciated in Fig. 16. In the case of thin plates, an ESL approach is advantageous compared to LW models since lower DOF are needed to obtain similar values of both the optimal design variables and fundamental frequency, as appreciated in Fig. 15b. However, despite providing similar, if not identical, optimal design variables, ESL-TE 1 models overestimate the frequency by roughly 40 Hz due to the higher stiffness this model leads to.

3) Surrogate models are helpful in solving optimization problems because of the quick evaluation of the objective function/constraints that they mimic. Moreover, if few design variables are considered, a graphical representation of the response surfaces can help predicting the region where the optimum might be located (see Figs. 7b, 7d, and 11b). However, errors might be committed if not enough samples are used to build such surrogates or if a lousy sampling strategy is followed; i.e., some regions of the design space are unexplored. This might represent an essential issue if failure constraints or uncertainty are considered in the optimization problem. In those cases, an optimization that uses the results from the actual simulation might be preferred.

Future investigations will account for the manufacturing defects that arise during the fabrication of VAT structures, such as gaps and overlaps, and will face the minimization of stress concentration factors in open-hole composite structures. In the latter case, LW models might lead to different optimal solutions when compared with ESL approaches. Because LW theories provide the kinematic variables of each independent layer, they can predict the 3D stress state of the laminated structure. Precisely, they can capture the shear and normal transverse stresses guaranteeing the C0 requirements. ESL cannot predict the aforementioned stresses, nor guarantee the C0 requirements unless very high order is employed. In that case, the increase in terms of DOF could be disadvantageous if compared to an LW approach.

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References

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R. Ohayon
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