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Diffraction in Presence of Angular Regions Filled by Arbitrary Linear Media

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Abstract—**In this work, we present a novel and general spectral theory to handle complex electromagnetic scattering problems. The technique is based on transverse equations for layered planar and angular structures, characteristic Green's function procedure, Wiener-Hopf technique. According to our opinion, for the first time, the proposed mathematical technique extends the possibilities of spectral analysis of EM problems in presence of angular regions filled by complex arbitrary linear media.**

I. INTRODUCTION

Spectral methods are of paramount importance in the analysis of electromagnetic diffraction problems. We recall in particular the Sommerfeld-Malyuzhinets (SM) method, the Kontorovich-Lebedev (KL) transform method and the Wiener-Hopf (WH) method with their impact in the study of diffraction problems by sharp discontinuities (see for instance respectively [1-3], [4], [5-6] and references therein).

One of the limits of the above techniques was at the same time the main advantage: the use of spectral complex angular plane born from the Sommerfeld integral theory. In fact, the definition of such complex plane is directly related to the physics of the problem, i.e. the necessary spectral transformations are related to the propagation constant. It means that the methodology is related to problems with one propagation constant such as isotropic media in EM and other particular problems with special configurations.

Different attempts were developed to extend the analysis to diffraction problems in more complex media as for example gyrotropic media and/or uniaxial media. In particular we recall the analysis of scattering by perfect electrically conducting (PEC) half-plane immersed in particular anisotropic media limited to the ones previously reported, see [7-12]. However, to the best of our knowledge, no spectral method has been developed for scattering problems with wedges in arbitrary linear media.

Given our experience in the analysis of EM complex scattering problems in isotropic media that takes origin from the fruitful collaboration with Prof. Zich [5-6,13-17], and with the help of the theory proposed in [18] for the analysis of structures embedded in layered media, we developed a new theory that allows to represent complex scattering problems immersed in arbitrary linear media of different shapes. In particular these new formulations are in spectral domain without introducing angular complex planes thus not limited to one propagation constant problems. In our contribution [19], we have developed the general theory to model angular regions filled by arbitrary linear media and we reported its implementation for isotropic media. With the present paper and the to-be-submitted contribution [20], we develop functional equations for arbitrary linear media of angular shapes and validate them through the few special cases reported in literature.

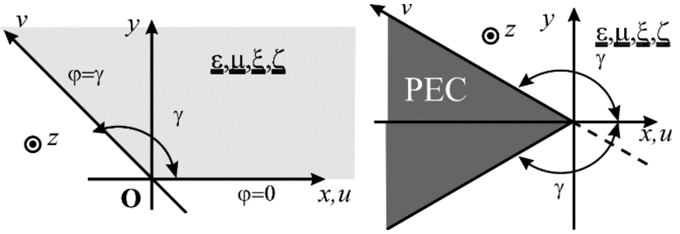

Figure 1: (right) Angular region (0<φ<γ) with oblique Cartesian coordinates (u,v) and (left) PEC wedge immersed in arbitrary linear media.

II. THE PROPOSED METHOD

The proposed method exploits the combination and the extension of powerful mathematical tools developed in different contexts. The first tool is the Bresler-Marcuvitz Transverse Equation Theory for layered media [21-22], the second is the characteristic Green's function procedure [10] and the third one is the Wiener-Hopf Technique [22-23].

We first revisited the Bresler-Marcuvitz Transverse Equation Theory for layered arbitrary linear media where the constitutive relations are of tensorial form:

$$\begin{aligned}\mathbf{D} &= \underline{\underline{\varepsilon}}\cdot\mathbf{E}+\underline{\underline{\xi}}\cdot\mathbf{H}\\ \mathbf{B} &= \underline{\underline{\zeta}}\cdot\mathbf{E}+\underline{\underline{\mu}}\cdot\mathbf{H}\end{aligned} \tag{1}$$

In particular, by assuming time harmonic field $e^{j\omega t}$ and invariant geometry along z with sources constituted of plane having z dependence $e^{j\alpha_o z}$, we study the operator properties of transverse w.r.t. y equations derived from Maxwell's equations:

$$-\frac{\partial}{\partial y}\boldsymbol{\psi}_y(x,y)=\mathbf{M}_{\mathbf{y}}(\frac{\partial}{\partial x})\cdot\boldsymbol{\psi}_y(x,y) \tag{2}$$

$$\boldsymbol{\psi}_y=\left|\mathbf{E}_{yt},\mathbf{H}_{yt}\right|,\ \mathbf{E}_{yt}=\left|E_z,E_x\right|^t,\mathbf{H}_{yt}=\left|H_z,H_x\right|^t \tag{3}$$

where the operator is of the following form

$$\mathbf{M}_y(\frac{\partial}{\partial x})=\mathbf{M}_{yo}+(\frac{\partial}{\partial x})\mathbf{M}_{y1}+(\frac{\partial}{\partial x})^2\mathbf{M}_{y2} \tag{4}$$

After the application of Fourier transform along x, we study the eigenvalues and the eigenvectors of transformed (4) $\mathbf{M}_y(\eta)$. For arbitrary linear media the expressions of $\mathbf{M}_{yi}$ i=0,1,2 and of the eigenvalues/eigenvectors are cumbersome to be reported, thus we omit them in this text. We introduce appropriate oblique Cartesian coordinates (5), see Fig.1, that modifies (2) into (6), preserving several important operator properties [19].

$$x = u + v\cos\gamma,\ y = v\sin\gamma \tag{5}$$

$$-\frac{\partial}{\partial v}\boldsymbol{\psi}_y(x,y) = \mathbf{M}_{\mathbf{e}}\left(\frac{\partial}{\partial u}\right)\cdot\boldsymbol{\psi}_y(u,v) \tag{6}$$

In particular, the Laplace transform of (6) along x≡u yields an operator $\mathbf{M}_{\mathbf{e}}(\eta)$ whose eigenvalues and eigenvectors are related to the ones of $\mathbf{M}_y(\eta)$: same eigenvectors while eigenvalues are

$$(\lambda_{ei}(\gamma,\eta) - j\eta\cos\gamma)/\sin\gamma = \lambda_i(\eta) \tag{7}$$

with $\lambda_i(\eta)$ eigenvalues for $\mathbf{M}_y(\eta)$.

The application of Green's function procedure to get solution of (6) in η spectral plane allows to represent the solution in terms of homogenous and particular solution, see [19, 24]. Applying [19], we get functional spectral equations that relate Laplace tangential field components along the two faces of the angular region (Fig. 1) defined into two different complex planes related together. We can repeat this procedure for any angular region with arbitrary orientation of the faces. Moreover, as in standard theory, (6) allows to get functional equations with transmission line formalism for layered regions. The application of boundary conditions on the faces of the angular regions yields a set of equations having the form of Generalized Wiener-Hopf equations (GWHEs), where "generalized" is added due to the definition of tangential field components in different complex planes but related together.

The main constraint in the present work resides in the complexity of the media that does not allow simple mappings between complex planes of GWHEs to transform them into classical Wiener-Hopf equations (CWHEs). For this reason, in arbitrary linear media, we need to resort to a general-purpose approximate method of factorization: the Fredholm factorization presented in [25] for CWHEs. Here we apply for the first time Fredholm factorization directly to GWHEs as no mapping is in general available in problems involving arbitrarily linear media [20]. We observe that the impossibility to map GWHEs to CWHEs in arbitrary linear media is similar to the impossibility to define an angular complex plane w for SM and KL methods, but the methodology proposed in this paper overcome this obstacle resorting to Fredholm factorizations specialized for GWHEs. Validation through examples will be provided at the conference, starting from the analysis of diffraction (GTD coeff) by PEC half-plane immersed in particular anisotropic media.

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