An introduction to TWG18: Building bridges between different perspectives: Emerging themes from international research into mathematics teacher education and professional

An introduction to TWG18: Building bridges between different perspectives: Emerging themes from international research into mathematics teacher education and professional development

Marita Friesen[1], Tracy Helliwell[2], Raffaele Casi[3], Andreas Ebbelind[4], Janne Fauskanger[5], Pere Ivars[6], Macarena Larrain[7] and Libuše Samková[8]

[1]University of Education Heidelberg, Germany; friesen@ph-heidelberg.de

[2]University of Bristol, School of Education, UK

[3]University of Turin, Department of Mathematics, Italy

[4]Linnaeus University, Faculty of Technology, Växjö, Sweden

[5]University of Stavanger, Faculty of Arts and Education, Norway

[6]University of Alicante, Faculty of Education, Alicante

[7]University of Education Freiburg, Germany

[8]University of South Bohemia in České Budějovice, Faculty of Education, Czech Republic

In this paper, we report of the major themes that emerged from the presentations and discussions of the paper and poster contributions in TWG18a+b. We identified the following overarching themes as key to the research and conference conversations in the field of pre-service mathematics teacher education and the professional development of in-service mathematics teachers and asked in particular: How can we (1) generalise from research whilst maintaining complexity? (2) best reflect our roles as mathematics teacher educators and researchers? (3) research the relationship between teacher education, professional development and classroom practice? (4) productively cross boundaries in our research? (5) better reflect the role of context? (6) make sure that our research makes mathematics sufficiently visible? Finally, we discuss how we can build on one another's work to further develop the field and suggest possible future developments and research directions.

Keywords: Educational research, mathematics teacher education, professional development.

Introduction to TWG18

Thematic Working Group 18 (TWG18) is devoted to research into the professional growth of mathematics teachers across all phases (Fauskanger et al., 2022): from initial teacher education (TE) of pre-service teachers (PSTs) to further professional development (PD) of in-service teachers (ISTs). The working group's main research interests include various aspects of mathematics teachers' professional knowledge, beliefs, reflection, and noticing related to different mathematical content areas and aspects of instructional quality. Frameworks, models, and practices of TE and PD programmes are also central to the discussions between members of TWG18, particularly the content, methods, tools, and related impacts. In accordance with the CERME spirit, we offer a communicative, collegial, and critical forum for discussions on research topics related to the long-standing activities of the working group. We attract and assemble educational research around a diverse range of

perspectives and approaches, and so we aim at contributing to our understanding of our role as researchers, educators and practitioners in wider contexts.

TWG18 is traditionally one of the working groups at CERME conferences with a large number of submissions and this time was no different: With 64 paper proposals and 6 poster proposals entering the review process, we again faced the necessity to divide the group into two subgroups. Unlike in previous conferences, where numbers of submissions relating to TE and PD were similar, at CERME13 there were significantly more proposals relating to TE than to PD. Thus, rather than separating research relating to TE and PD into distinct subgroups, we instead found several overarching topics to group the proposals around. In TWG18a, we presented and discussed research addressing the professional growth of PSTs and ISTs in their teaching of mathematical content, such as fractions, functions, mathematical pattern and structure or early numeracy. Another key topic in TWG18a was how teaching mathematical core activities (e.g., problem-solving and problem-posing, inquiry-based learning, reasoning and proof) could be fostered in TE and PD. TWG18a also focused on current issues and challenges in the field, such as teaching online, the PD of teachers without mathematics as a subject or PSTs' learning to use information and communications technology (ICT) in teaching. In TWG18b, we presented and discussed research addressing teachers' professional growth in core classroom practices such as noticing and student assessment/diagnosing, and also related to the planning and professional reflection of lessons and classroom activities. Other key topics in TWG18b included the development of future teachers' mathematical subject matter/content knowledge (CK) and pedagogical content knowledge (PCK) in university education, and the interrelatedness of different aspects of professional knowledge and their role in the teaching of mathematics. TWG18b also focused on promoting teachers' professional growth using particular methods and formats in TE and PD, such as role plays, microteaching or online formats.

We finished the review process with a total of 42 papers and 6 posters accepted for the presentation at the conference. 19 papers and 4 posters were allocated to TWG18a, 23 papers and 2 posters to TWG18b. In general, each session consisted of two or three paper/poster presentations and subsequent reactions by pairs of discussants. Small group discussions then followed. To conclude each session, discussants reported back to the wider group on the content of the discussions and a short reflection by each author. During the conference, we also devoted time to topic specific discussions. Topics were identified and agreed by participants based on themes that had emerged across the various conversations. From across these topic specific discussions, we identified six overarching themes emerging from the international perspectives of the TWG18 participants, namely: How can we (1) generalise from research whilst maintaining complexity? (2) best reflect our roles as mathematics teacher educators and researchers? (3) research the relationship between teacher education, professional development and classroom practice? (4) productively cross boundaries in our research, e.g., between different frameworks? (5) better reflect the role of context, e.g., related to different cultural perspectives? (6) make sure that our research makes mathematics as subject matter sufficiently visible? In this review paper, we shortly introduce each of these themes by connecting them to current strands of research in the field of mathematics TE and PD and presenting exemplary paper and poster contributions from across both TWG18 subgroups.

How can we generalise from research whilst maintaining complexity?

Research into mathematics TE and PD is a challenging endeavour. Like mathematics classrooms, TE and PD settings are complex and multi-faceted presenting researchers with as many opportunities as dilemmas. What counts as evidence in mathematics education has been the topic of debate for several years (e.g., Wiliam & Lester, 2008) with comparisons having been drawn to research in the physical sciences. It is now generally acknowledged that the complexity of the objects of study within educational research far exceeds those of traditional scientific research. Unlike scientific findings that are generalisable across different contexts, research in mathematics education tends to generate results that hold in some situations but not others. Commonly, across the research presented in TWG18, researchers dealt with the issue of maintaining complexity by attending to the details of *particular* situations. Often this focus on the particular translated into modest numbers of participants. *Bråtalien*, for example, presented an in-depth study concerning four ISTs' conceptualisations of inquiry in teaching and learning mathematics before entering a particular PD programme. *Kuntze et al.*, on the other hand, examined how 56 PSTs used a pre-introduced framework for analysing representation registers. In both examples, as was the case for the vast majority of studies presented in TWG18, the research took place within a single context (e.g., one initial TE or PD programme). Whether the findings from these 'situated studies' are generalisable may depend on how thoroughly the phenomena of research are conceptualised.

The majority of research into mathematics TE and PD relates in some way to the study of teacher learning. It is widely acknowledged that learning is 'situated' in that it is "a product of the activity, context, and culture in which it is developed and used" (Brown et al., 1989, p. 32). From some perspectives, the learner is viewed as being situated *within* a particular environment or context, for others, the learner is considered *part of* the environment or context. How this relationship is conceptualised informs researchers' methodological decision making. During TWG18 discussions, participants grappled with the consequences of their own methodological decisions. A driving question that participants continued to return to throughout the topic study discussions was: How can we, as researchers, situate the specific focus of study within the bigger picture? How can we make more explicit the various and multiple aspects of the situated nature in which our research takes place? In relation to researching teacher noticing, for instance, it was the case that different frameworks and conceptualisations were used that had to be clarified and made transparent in order to interpret and compare presented findings. In relation to the generalisability of research findings, participants explored what it might mean if more central models were adopted by those conducting research into TE and PD.

How can we best reflect our roles as mathematics teacher educators and researchers?

The majority of participants from TWG18 were practicing mathematics teacher educators (MTEs). Given the topic of research, studies were often situated within researcher's own TE institutions or PD contexts. In some of the papers presented, this dual role (MTE-researcher) was discussed explicitly and discussions extended into how this dual role might affect the researcher's positioning considering the impact on how their research endeavours evolved. In the paper by *Lewis et al.,* for instance, the

close relationship between the researcher and the teachers can be viewed, a relationship that is a highly valued part of the research process. *Ebbelind and Helliwell's* paper also prompted discussions regarding the intimate relationship between researcher, researched, reader and the research context.

TWG18 discussions did not tend to focus on research ethics in the more traditional sense, regarding technical requirements such as data collection procedures, consent, confidentiality, rather the focus centred on the ethical and moral dimensions of the researcher's participation. Floyd and Arthur (2012) explore the researcher's moral concerns and internal matters relating to the research process, for example, how the researcher positions themselves in relation to the object of study. In relation to this issue, discussions during TWG18 touched upon the extent to which the researcher's positioning should be justified when reporting on research and how the lack of this justification can conceal certain ethical and moral dimensions. *Ebbelind et al.'s* paper, for example, explored how a research project might or might not support change when viewed from the teachers' perspectives. It was concluded that the teachers attached themselves to the project in different ways and provided teachers with opportunities to make changes for themselves.

How can we research the relationship between teacher education, professional development and classroom practice?

The chosen phenomena of study within mathematics TE and PD relates directly to the unit of analysis. At a micro-level, the unit of analysis might be the interactions that take place between ISTs during PD situations, or the detail of PSTs' written responses to a particular task. At the other end of the scale, at a more macro-level, the unit of analysis might be the overall design of a PD programme or existing policy documents relating to initial TE. One challenge for the research community to consider is how to study the relationships between these different aspects of mathematics TE and PD, i.e., what if the unit of analysis was the *relationship* between these different aspects, as opposed to the different aspects in isolation?

Efforts to research relations between different aspects of mathematics TE and PD were present across studies reported in TWG18. *Frey and Sproesser*, for example, considered how PSTs' conceptions of functional thinking changed *in relation to* them engaging in a teacher seminar on supporting students' functional thinking, by employing a pre-post design. Given the nature of research into mathematics TE and PD, research most commonly takes place within the boundaries of the TE or PD setting. One frequently discussed issue by TWG18 participants, was the *relationship* between what takes place during TE and PD settings and what happens in participating teachers' classrooms. Questions such as, "how do we know what we do as MTEs shapes how PSTs' teach?" and "which aspects of what teachers learn during their TE or PD becomes realised in their teaching of mathematics?" Examples of research that begin to consider the relationship between TE and PD and classroom practice include the work of *Taylan et al.* who investigated changes in PSTs' questioning skills as observed over a series of online lessons that the PSTs taught as part of their TE programme.

In topic group discussions, participants discussed the need to "follow up" on studies that focus on activity that takes place in TE and PD settings. Possibilities for following up included interviewing teachers about their experiences during PD programmes to establish their perceptions of how their practice had changed, or observing teachers in their classrooms. In either situation, conceptualising

and observing the relationship between what takes place during TE and PD settings and what happens in participating teachers' classrooms remains a challenging endeavour with many participants questioning the legitimacy of any findings that might suggest the existence of a *causal relationship*.

How can we productively cross boundaries in our research?

Akkerman and Bakker argue that boundaries are resources for learning, because "boundaries compel people to reconsider their assumptions and look beyond what is known and familiar" (Akkerman & Bakker, 2011, p. 2). Learning can be shaped by movement across boundaries since learning can occur when people interact with, move across or participate in different communities, move between different institutionalised practices and interact with people from different professions, disciplines and cultures. Learning by crossing boundaries was discussed in relation to several papers presented and discussed in TWG18 including boundaries between: mathematics and other school subjects; learning in the mathematics classroom and learning outside of the classroom (e.g., in museums); teachers' mathematics CK and PCK; different perspectives in the classroom such as the student, the teacher and the researcher; different phases in teachers' PD; and different frameworks used as theoretical lenses in the research presented.

As an example of boundary crossing, *Branchetti et al.* initiated a discussion about crossing boundaries between subjects. By exploring an initial TE sequence aimed at promoting development on interdisciplinarity (mathematics and physics), learning potentials emerged during class discussion. An example of boundary crossing between learning inside and outside of the mathematics classroom is *Casi and Sabena*'s paper, where the authors examined the boundary crossing between the mathematics learning in classrooms and museums. *Casi and Sabena* explored cultural spaces such as art and history museums to engage teachers in communities of practices to design informal mathematics workshops with the support of teacher educators and museum experts. *Fellenz and Schnell* investigated boundary crossing in the case of primary PSTs: the PSTs were supported in developing their noticing of children's mathematical thinking by being prompted to consider different perspectives (teachers' and students'). The findings showed that taking different perspectives can be useful for documenting, inquiring, advancing, and assessing student thinking in TE.

How can we better reflect the role of context?

The field of mathematics TE is profoundly influenced by culture and socio-political aspects (Valero, 2004). MTEs and researchers must recognise the importance of cultural sensitivity, inclusivity, and equity in shaping effective mathematics TE programs and in doing research that advances the field. Moreover, theories and frameworks developed to understand teachers' competencies and performance in the classroom should not only focus on cognitive aspects, but also on the cultural and ideological dimensions (Louie, 2018).

Since research methods and results shared in TWG18 came from a variety of countries and contexts, the influence of cultural and context-specific issues was discussed in relation to several papers. In particular, the need to attend to the particularities of each context when conducting research on TE was highlighted. Various factors influence the development of teaching competences: cultural aspects, political and social aspects, education reforms, organisation of schools and universities, and characteristics of the training institutions. This has various consequences, e.g., for the research

methods selected and for the interpretations researchers make of the data they collect and the results of their research. It emphasises the need to make explicit the multiple aspects of the contexts in which each research takes place. This issue came up numerous times during paper and poster presentations and related conversations. Very often additional explanations and clarifications were required for better understanding the context within which research took place. Relevant information about the culture, the national context and education system and the characteristics of the TE or PD programme was necessary for interpreting the results on how Turkish PSTs' participation in a video-based module supported their noticing skills (*Çaylan Ergene & Işıksal Bostan*), on what Chilean first-year teacher students noticed from a video segment of a mathematics lesson (*Martínez et al.*), on what Chinese teachers noticed in exemplary mathematics lessons (*Chen et al.*), or on what and how Norwegian mathematics ISTs noticed during co-planning instructional activities (*Bjuland & Fauskanger*).

At the same time, the TWG conversations pointed out the need to make transparent potential context-specific aspects of research findings when claiming to contribute to the field. Collaboration between research groups from different cultural or contextual backgrounds has been positively valued, as this is expected to enable mutual learning. An inclusive discussion would enrich points of view and allow for deeper insights into the issues being investigated, contributing to the development of the field and enabling advances in the theories and frameworks used internationally. For example, the paper by *Yıldırım and Karagöz Akar* examined how teachers' perspectives (i.e., their meaning making systems) influence their noticing skills. The group discussion evolved from focusing on the relations between teacher perspectives, knowledge, beliefs and noticing to incorporating cultural context as an additional relevant aspect. A related discussion, but from a different perspective, was initiated by *Raval and Österling*'s research. They proposed a methodology for analysing PSTs' geometry discourses in two different countries. This initiated a discussion about what is needed in cross-cultural comparison studies to distinguish between differences originated from culture and differences generated by other variables, such as participants' knowledge or noticing skills.

How can we make sure that our research makes mathematics sufficiently visible?

Since Shulman's Presidential Address at the 1985 annual meeting of the American Educational Research Association and its following publications (e.g., Shulman, 1986), research into the nature and relevance of teachers' professional knowledge has been placed firmly on the educational research agenda (see Adler & Venkat, 2020 for an overview). Key studies such as COACTIV showed, for example, that teachers' PCK has a substantial positive effect on students' learning gains. While teachers' PCK is inconceivable without sufficient CK, CK cannot substitute for PCK (Baumert et al., 2010). There is also a broad consensus that pre-service TE is paramount to the development and integration of different aspects of professional knowledge for teaching mathematics. Correspondingly, several papers were presented in TWG18 focusing on research into the development of future teachers' mathematical CK in university education, its interrelatedness with PCK and its role and relevance for the teaching of mathematics.

The papers of *Dellori and Wessel* and *Sebök* both addressed the discontinuity between university mathematics and mathematics for teaching, a reason that PSTs often see little or no relevance in the mathematics subject matter they are taught at university for their later profession. In their theoretical

paper, *Dellori and Wessel* introduced design principles for tasks that may strengthen connections between abstract algebra and school algebra and elaborated samples for such tasks alongside an exemplary learning trajectory. *Sebök* based her empirical study on the hypothesis that PSTs often underestimate the mathematical demands of their future profession. She designed a situated learning intervention for improving the PSTs' noticing of subject matter in analysis alongside with views on subject matter knowledge as a teacher resource. Other TWG contributions highlighted in particular the important link between teachers' profound knowledge of fundamental mathematical concepts and high-quality teaching: *Götz et al.* presented an intervention study from a university course addressing basic ideas of fundamental analysis (e.g., continuity) and the question of how to teach these in a mathematically correct and didactically appropriate way. The study by *Ableitinger et al.* investigated ISTs' subject knowledge of basic ideas related to the limit concept before and after an intervention in a didactically oriented analysis course. They found that explicitly addressing such basic ideas lead to a significant increase in the quality of students' written responses.

During the group discussions, the role of mathematics subject matter evolved into an important overarching perspective for reflecting research into TE and PD. Emerging questions explored by the TWG attendees were, e.g., where does the subject matter become visible in TE and PD curricula and programmes? What is specific about planning, reflecting on, and noticing mathematics lessons and what role do different mathematical content areas play? Which of our considerations in TE, PD and related research are applicable to other subject disciplines and which are deeply related to the subject of mathematics?

Possible future developments and research directions

Each of the themes explored within this paper present the mathematics education community with potential further developments and future research directions. In this final section we particularly highlight two of these areas, namely, how TE and PD activities/outcomes can be linked more closely to teachers' classroom practice; and how innovative online formats can help to scale up PD activities.

In relation to researching the links between TE and PD activities and classroom practice, we see this as a significant challenge for the mathematics education community to grapple with. In several studies presented in TWG18, implications for classroom practice were discussed, for instance, in the work of *Guiñez et al.* who explored the perceptions of PSTs regarding the nature, learning and teaching of mathematics, revealing the nuanced differences in their discourse and potential implications regarding the PSTs' future practices. Other researchers tackled the problem of linking to classroom practices in different ways. *Samková*, for instance, researched the use of concept cartoons, that is "individual cartoon pictures with children discussing a content related situation", as a way of both developing and assessing PSTs' professional knowledge. The hypothetical classroom situations, what could be referred to as approximations of practice, allowed *Samková* to study PSTs' responses to such situations and in doing so began to bridge the gap between TE and classroom situations. What was lacking from across the set of research reports in TWG18 was the development of methods that allowed the *relationship* between TE and PD and current and future classroom practices to be studied and hence theorised. We suggest this particular research direction to have significant potential in the communities' quest to support teachers in developing themselves as classroom practitioners.

In relation to the challenge of scaling up effective TE and PD programmes by making them available beyond small groups of participants, the study by *Friesen et al.* described online PD programmes as a more flexible way to access in-service support. The paper presented design elements that were found to be motivating and effective in self-learning modules for in-service teachers and described how an asynchronous online course on teaching algebra can be designed accordingly. The potential of online environments for systematic research into teachers' professional learning (e.g., by allowing for the controlled implementation of learning materials and the investigation of usage behaviour) should be considered another important future research direction in the field.

References

Adler, J., & Venkat, H. (2020). Subject matter knowledge within "mathematical knowledge for teaching". In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Springer. https://doi.org/10.1007/978-3-030-15789-0_98

Akkerman, S. F., & Bakker, A. (2011). Learning at the boundary: An introduction. *International Journal of Educational Research, 50*(1), 1–5. https://doi.org/10.1016/j.ijer.2011.04.002

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan A., ... et al. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133–180. https://doi.org/10.3102/0002831209345157

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32–42. https://doi.org/10.3102/0013189x018001032

Fauskanger, J., Ebbelind, A., Friesen, M., Samkova, L., Helliwell, T., & Larrain, M. (2022). International perspectives on mathematics teacher education and professional development: Current and emerging research. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)* (pp. 3051–3059). Free University of Bozen-Bolzano and ERME. https://hal.archives-ouvertes.fr/hal-03744589v1

Floyd, A., & Arthur, L. (2012). Researching from within: External and internal ethical engagement. *International Journal of Research & Method in Education, 35*(2), 171–180. https://doi.org/10.1080/1743727X.2012.670481

Louie, N. L. (2018). Culture and ideology in mathematics teacher noticing. *Educational Studies in Mathematics, 97*, 55–69. https://doi.org/10.1007/s10649-017-9775-2

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14. https://doi.org/10.3102/0013189X015002004

Valero, P. (2004). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education* (Vol. 35, pp. 5–23). Springer. https://doi.org/10.1007/1-4020-7914-1_2

Wiliam, D., & Lester, F. K. (2008). On the purpose of mathematics education research: Making productive contributions to policy and practice. In L. English & D. Kirshner (Eds.), *Handbook of International Research in Mathematics Education* (2nd Ed., pp. 32–48). Routledge. https://doi.org/10.4324/9780203930236