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A Review of New Canonical Scattering Problems Solved Using a Spectral Approach Integrating the Wiener–Hopf Method with Fredholm Factorization and Network Formalism

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***Abstract*— The Wiener–Hopf technique is a fundamental analytical and semi-analytical method in applied mathematics and mathematical physics for solving PDE and integral-equation boundary-value problems. Using spectral domain, it enables rigorous solutions and with the Fredholm factorization very accurate approximate solutions in computational physics and in particular in electromagnetics allowing efficient analysis of propagation, diffraction, guided-wave discontinuities, antenna radiation, and modern applications. Recent advances, as the introduction of the novel direct Fredholm factorization for arbitrary generalized Wiener Hopf equations, extend the method to more complex geometries and arbitrary media beyond rectangular, isotropic media, while preserving strong physical insight through spectral-domain interpretation. Wiener-Hopf formulations can be interpreted through equivalent networks to avoid redundancy for a practical and systematic application of the methodology.**

Keywords— Wiener-Hopf method, applied mathematics, computational electromagnetics, integral equations, Green's function, spectral domain, network representations, regularization, Fredholm factorization, wave motion, diffraction, complex materials.

I. Introduction

The Wiener–Hopf (WH) technique was introduced by Wiener and Hopf in 1931 [1] to solve boundary-value problems formulated in terms of semi-convolutional integral equations in physical domain that take origin from problems with semi-infinite geometries/supports. Nowadays the Wiener-Hopf technique in electromagnetics has been successfully applied to wave propagation in waveguide devices and junctions, open waveguides, scattering and diffraction by sharp structures, slot antennas and other interesting applications. Noble in his systematic review book [2] reports spectral domain implementation of the techniques by the use of Fourier and Laplace transforms exploiting also the well-known Jones's method [3]. The literature reports fundamental contributions for the development of the technique in electromagnetics by famous scientists like Noble, Jones, Mittra, Lee, Hashimoto, Idemen, Tretyakov, Kobayashi, Buyukaksoy, Serbest, Luneburg and Daniele.

In particular, Daniele expanded the capability of Wiener-Hopf method by introducing Wiener-Hopf explicit/exact factorization techniques for matrices of arbitrary order for particular classes of matrices. The monograph with R. Zich [4] on WH technique also presents the state of the art on analytical procedures for the solution of matrix and modified Wiener-Hopf equations. He carried on the development of the theory for the analysis of structure of non-rectangular shapes (wedges) with the definition of Generalized Wiener-Hopf equations (GWHEs) and a technique to reduce them to Classical Wiener-Hopf equations by a spectral transformation [5]. Inspired by Vekua's work [6], the Fredholm factorization method was developed in the early 2000s as an approximate, flexible, and effective way to factorize and solve matrix Wiener–Hopf equations [7-8]. Applications to practical wedge scattering problems within the Generalized Wiener–Hopf framework was presented in 2006 [9]. Daniele and Lombardi's 2020 two-volume monographs [10-11] presented theoretical foundations and application on wedge-scattering using the Generalized Wiener–Hopf method, with detailed analytical/approximate factorization tools and practical insights. They emphasized that Fredholm factorization is a key advance on WH technique: it preserves the problem's physical spectral properties and provides accurate solutions by reducing factorization to Fredholm integral equations solvable with simple quadratures avoiding problem-specific strategies as in classical Wiener–Hopf solution procedures like Jones's one. Spectral methods are fundamental in physical interpretation of problems. Recent work extends the Wiener–Hopf technique with Fredholm factorization to more complex arbitrary linear media and to scattering objects of different shapes. This short paper has the intent to introduce a review that covers traditional and new fundamentals of the WH technique, then recent advances and applications.

II. Formulation's and Solution's Procedures

Classical Wiener-Hopf equations (CWHEs) in spectral domain by means of Laplace transform show plus and minus unknowns that are defined on semi-infinite supports and appear in equal number, thus they constitute a closed problem. Beyond the case of unclosed WH equations that require special procedures for their solution, we introduce GWHEs where the plus and minus WH unknowns are defined into different complex planes but related together. For example, this is the case of wedge problems [10-11] that, when modelled by impenetrable material and immersed in a isotropic medium, show GWHEs with unknowns defined into two complex planes that by special mapping can be reduced to CWHEs, where analytical solution and approximate

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solutions can be reached with the help of Fredholm factorization method.

$$G(\eta)F_+(\eta) = X_+(-m(\eta)) \qquad (1)$$

If we consider more general media (i.e. bi-anisotropic) or penetrable multiple structures, the complex planes to be used are more than two although still related together [12-13]. In this case the direct Fredholm factorization method that uses Cauchy representation (2) yields integral equations in spectral domain defined only in one complex plane. The reduction of the representation in one complex plane allows a direct Fredholm factorization of arbitrary GWHEs without the necessity to reduce them to CWHEs.

$$F_+(-m_i(\eta)) = \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{F_+(t)}{t+m_i(\eta)}dt + F_+^{ns}(-m_i(\eta)) \qquad (2)$$

In summary the methodology to formulate WH problems is reported in Fig. 1 and starting from Maxwell's equation and the decomposition of the problem into canonical subregions, the WH equations of the problems are obtained by using trasversalization, the characteristic Green's function procedure, the weighting of the expression towards the reciprocal eigenvector and the application of boundary conditions. The spectral representations we obtain are constituted mainly by equations that link spectral physical quantities defined in two faces of the canonical subdomain and they can be interpreted as network for a systematic procedure of formulation. Fig. 2 shows the solution procedures in the classic case and in the generalized form using direct Fredholm factorization as outlined before. Other issues can be derived with incomplete WH equations due to finiteness of sub-regions that yields integral contribution not directly expressed in terms of WH unknowns. In this case we make complete the equations resorting to Mittag-Leffler theorem and (2) again, before applying Fredholm factorization for the solution [14-15]. Finally, from approximate spectra we get physical quantities by inversion of Laplace transform and using asymptotics. In Fig.3 we list some of the problems analyzed in literature composed of canonical subregions.

III. Conclusions

We propose a comprehensive methodology in spectral domain based on WH technique for the analysis of complex scattering problems that allows physical interpretation.

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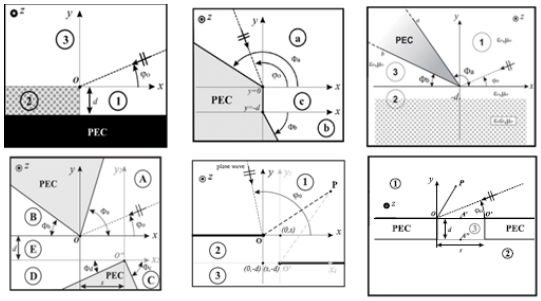


Fig. 3 From top left to bottom right: semi-infinite gounded dielectric slab, connected double wedge, wedge over a semi-infinite dielectric, separated double wedge, staggered half-planes, thick slot.



Fig.1 Flowchart to formulate the problem with Generalized Wiener-Hopf equations

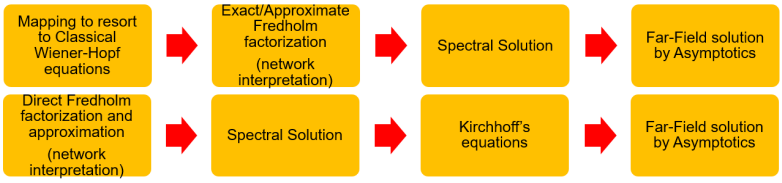


Fig.2 Solution procedures. Top: standard procedure applicable to WH problems (with CWHEs and/or GWHEs) that can be reduced to CWHEs. Bottom: alternative procedure applicable to problem with GWHEs with spectral unknowns defined in more than two spectral domains, i.e. that cannot be reduced to CWHEs.