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The Regularized Wiener-Hopf Method Applied to EM Scattering Problems Involving Entire Unknowns

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***Abstract*— This work presents a robust approach for addressing Wiener–Hopf equation problems involving entire unknown functions with exponential phase factors. The proposed technique deals with the factorization problem using a system of regularized integral equations. The effectiveness of the method is demonstrated through a practical electromagnetic application: the scattering of a plane wave by a thick slot that can be arbitrary filled by stratified media.**

***Index Terms*— Wiener–Hopf method, integral equations, Green's function, spectral domain, entire functions, exponential phase factors, regularization, Fredholm factorization, wave motion, diffraction, electromagnetism**

I. INTRODUCTION

SOLUTION of Wiener-Hopf (WH) equations in spectral domain involving entire unknown functions with exponential phase factors are still considered a challenging and cumbersome problem in Applied Mathematics. The literature presents several studies, most of them are specialized to specific cases. Current approaches often build upon the fundamental work of Jones on modified Wiener–Hopf equations [1] where, in case of availability of pre-factorization of WH equations, it yields to integral equations formulations.
Alternative techniques are presented [2] and [3-5] respectively with reduction of modified WH equations to classical matrix WH equations and with the use of sampled representation of entire functions in terms of unknown coefficients.
The present work is based on [6-9] and allows to have a general method to effectively extend the Wiener-Hopf techniques effectively applied in isotropic media isotropic media [2],[10],[11].
In particular, this generalization allows to handle wedge problems immersed in or made of arbitrary linear media [7] and now problems with finite and staggered penetrable/impenetrable regions, such as strips, slots, slits, bricks [9].
The proposed method extends the GWHT furthermore with respect to the effective analysis of staggered semi-infinite structures reported in [12], [13].

II. THE METHOD

In this work, we propose a revisited Generalized Wiener–Hopf technique (GWHT) capable to handle new scattering problems. This advance in the technique is useful in multiple problems that looks very different from each other's from a physical point of view, such as the two categories reported at the bottom of previous section.

In Fig. 1 we report in summary the standard GWHT methodology used to deal with wedge diffraction problems immersed in isotropic media [10-11], that is based on:

1) derivation of spectral functional equations obtained applying transversalization to Maxwell equations and characteristic Green's function procedure
2) application of boundary conditions to get GWHEs
3) spectral mapping for isotropic regions to transform GWHEs into CWHEs
4) exact factorization or Fredholm factorization to get respectively analytical spectral solution or semi-analytical ones
5) inverse Laplace transform and asymptotic evaluation at far field.

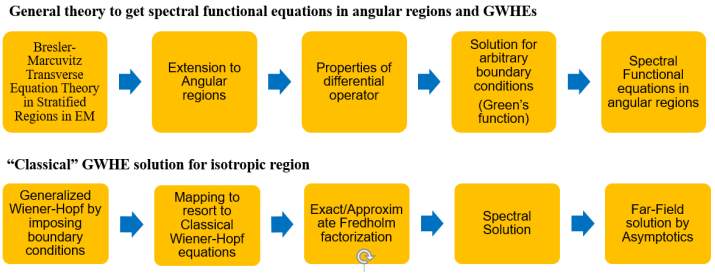

Fig. 1. Flowchart of GWHT used to deal with wedge diffraction problems immersed in isotropic media [10-11.

Points 1 to 2 can be implemented also in wedge problems with arbitrary linear media yielding unknowns defined in different complex planes and, in finite region problems made by penetrable/impenetrable materials yielding incomplete WH equations, i.e. with terms involving boundary values under integral form beyond classical WH unknowns.

The technique cannot continue with point 3 and subsequent. In particular for arbitrary media there are not available mappings that transforms GWHEs to CWHEs.

The key mathematical tool to develop a solution in both categories of problems is the representation of all "weird" unknowns present in the GWHEs by means of Cauchy representation formula (1).

$$F_+(m)=\frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{F_+(t)}{t+m}dt+F_+^{ns}(m) \qquad (1)$$

In case of finite region problems, before (1), the technique needs to represent the implicit integral terms via the Mittag-Leffler theorem [9], thus *m* is a sample in the spectral domain. Moreover, if (1) is applied to an entire function, the ns on right (non-standard term) is null, otherwise it is related to offending poles due to sources [2],[10].

Before applying Fredholm factorization [8] to the set of equations we symmetrize and duplicate them. The set of equations might be erroneously considered redundant; on the contrary, the regularity properties of the WH unknowns in the set of equations are fundamental to get stable convergent solutions through this generalized form of the Fredholm factorization.

The procedure yields a set of Fredholm integral equations (FIEs) of second kind with fast convergent kernel, that are amenable of semi-analytical solution with sample and hold discretization and reconstruction formula.

As usual, Fredholm factorization with this semi-analytical discretization accurately preserves the spectral features of the problem allowing asymptotic analysis.

For a validation test case see [9] with reprocessed data about Geometrical Theory of Diffraction field component in Fig. 2 and new tests will be shown during the conference presentation about the scattering of a plane wave by a thick slot that can be arbitrary filled by stratified media.

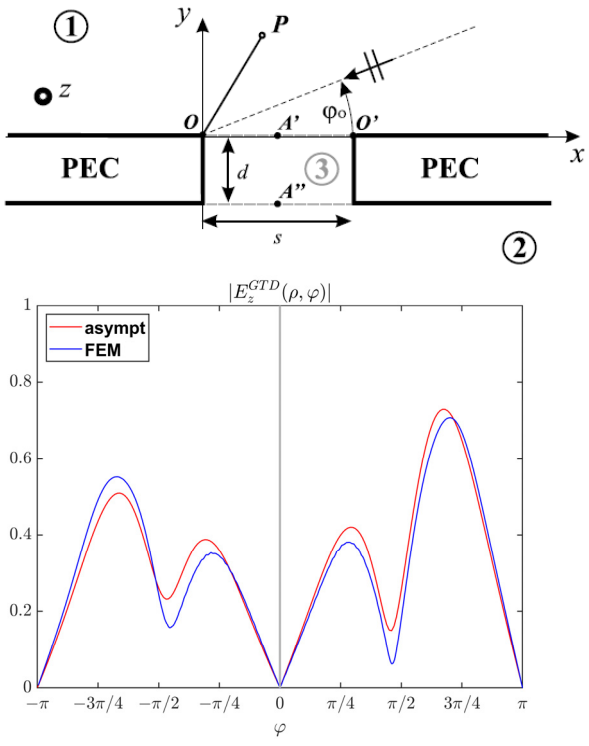

Fig. 2. Thick slot in free space (on the top) illuminated by an Ez plane wave with Eo = 1V/m, k = kr – jki (ki=kr/10000), $\varphi o = 2\pi/9$ rad, kr d = 2, kr s = 7 (we assume for simplicity kr = 1). GTD field (on the bottom) for the test problem under consideration respectively for region 1 at $k\rho = 10$ from A′ and for region 2 at kr $\rho = 10$ from A″. Comparison between our asymptotic solution and the one obtained with FEM with PML boundary condition around the structure at $\rho = 30\ \lambda$.

IV. Conclusion

The work introduces a comprehensive self-consistent theory that allows the analysis of diffraction problems constituted by impenetrable/penetrable finite regions based on the use of GWHT, Mittag-Leffler theorem and the Cauchy representation formula to solve incompleteness of the formulation.

The spectral solution is then accomplished by means of Fredholm factorization obtained also in presence of entire functions that allows asymptotic evaluation of far-fields in terms of Geometrical Optics, Geometrical Theory of Diffraction and total far field.

Acknowledgment

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