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Raj B. Bharati, M. Fillipi, Prashanta K. Mahato, E. Carrera

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# Flutter analysis of laminated composite structures using Carrera Unified Formulation

Raj B. Bharati<sup>a,b,1</sup>, M. Fillipi<sup>a,2</sup>, Prashanta K. Mahato<sup>b,3,\*</sup>, E. Carrera<sup>a,4</sup>

<sup>a</sup>*MUL<sup>2</sup> Group, Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Turin, Italy*

<sup>b</sup>*Department of Mechanical Engineering, Indian Institute of Technology (Indian School of Mines), Dhanbad, India*

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## Abstract

In present work, the flutter analysis of laminated composite structures has been performed using the  $p$ - $k$  method in Carrera Unified Formulation (CUF). In the framework of CUF, a hierarchical kinematic finite element model is used to compute the flutter condition of laminated composite plate and box-beam structures as it is very accurate and computationally efficient. The CUF refined theories are based on the Lagrange and Taylor-like cross-sectional displacement fields. In CUF, the order of the expansion can be chosen arbitrary, which is an independent parameter in the formulation. The governing equation is based on the principle of virtual displacement and defined in the form of “fundamental nuclei” using CUF. Theodorsens theory was used to define the aerodynamics loading conditions and the  $p$ - $k$  method was used to compute the flutter conditions. Flutter conditions of different types of laminated composite structures with Lagrange and Taylor expansion were performed. A similar model was developed in MSC-Nastran and computed results were compared with literature and CUF model. The results indicate that the analyzed model has good agreement with reference

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\*Corresponding author

*Email addresses:* raj.bharati@polito.it (Raj B. Bharati),  
matteo.filippi@polito.it (M. Fillipi), pkmahato@iitism.ac.in (Prashanta K.  
Mahato), erasmo.carrera@polito.it (E. Carrera)

<sup>1</sup>PhD. Student

<sup>2</sup>Assistant Professor

<sup>3</sup>Associate Professor

<sup>4</sup>Professor of Aerospace Structures and Aeroelasticity

and MSC-Nastran. The study suggests that the CUF models can produce accurate results with a low computational cost.

*Keywords:* Flutter, Unified Formulation,  $p$ - $k$  method, Composites

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## 1. Introduction

Flutter is a dynamic instability found in a different kind of flexible structure like air-vehicles, bridges, and blades. It can introduce the catastrophic failure of the structure. It is a devastating reason for failure, and most anxiety subject to the designer. So, the flutter should be considered and thoroughly analyzed during the design process of the aircraft structure. Several authors have analyzed the aeroelastic behavior of aircraft structure since Theodorsen developed a mechanism for the flutter analysis and demonstrated the flutter problem theoretically and experimentally [1, 2]. Later many methods have been developed to solve the flutter problem and improve the solution methodology such as  $v$ - $g$  method,  $k$ -method and  $p$ - $k$  method [3, 4].

Now-a-days, the laminated composite structures are broadly used in the design process of modern air vehicles, bridges and blades, owing to their high structural efficiency, specific strength and potential benefits. The wings can be modeled as the isotropic and thin-walled composite beam [5]. It can also be modeled as the laminated composite beam for the finite element solution to performed the free vibration analysis [6]. For further extension, the laminated composite beam can be considered as the thin- and thick-walled box-beam model for experimental, analytical and numerical solutions in order to determine the elastic stiffness, tailoring effect and torsional warping in finite element approach [7, 8, 9].

The laminated composite structures are very much flexible and aeroelastic behavior of these flexible structures should be analyzed to avoid flutter failure. For the prediction of flutter condition and divergence behavior of these structures, the combination of strip theory with simplified structural box-beam models was analyzed [10]. Various parameters such as aspect ratio, stacking sequences and sweep angle are considered for aeroelastic tailoring [11]. The analytical approach has been used to find the flutter condition for the wing made of the composite material [12]. In the aeroelastic investigation of a structure made with anisotropic composite, directionality property played a complex role in predicting the flutter and divergence behavior [13]. The analytical approach also used for the box-beam model and aeroelas-

33 tic optimization have been studied [14, 15]. The aeroelastic characteristics of  
34 laminated plate have been observed for various lamination parameters, aspect  
35 ratio, material properties and sweep angle and found different aeroelastic pa-  
36 rameters (flutter/divergence) at the same speed of air (free stream) [16]. The  
37 Theodorsen theory can be used for quasi-steady and unsteady aerodynamics  
38 to compute the flutter. Several authors predicted the flutter condition using  
39 the Theodorsen theory along with various approaches (such as strip theory  
40 and panel method) [17, 18, 19, 20]. To improve the flutter solution, Hassig  
41 [4] proposed the  $p$ - $k$  method that can provide better approximation than the  
42 other methods [21]. This method also can be used for the smart laminated  
43 plate in hygrothermal environment [22].

44 The aim of this work is to develop a finite element model of laminated  
45 composite structures (plate and box-beam) using Carrera Unified Formulation  
46 (CUF) and to perform flutter analysis by the  $p$ - $k$  method. The dif-  
47 ferent kinds of complex laminated composite structural models have been  
48 considered and those types of the model required proper description of the  
49 kinematics, which are defined accurately and computationally efficient man-  
50 ner in CUF. Initially, CUF was developed to deal with a plate and shells  
51 [23, 24, 25] later, it was extended and to deal with the beam model [26, 27].  
52 The CUF has a unique capability such as the order of Taylor expansion can  
53 be chosen arbitrarily to define the cross-sectional displacement fields. These  
54 capabilities give us freedom to choose the order of structural models with-  
55 out making changes in matrices or equations and without the need of any  
56 ad hoc formulation, it can deal with arbitrary geometry, different material  
57 characteristics, and boundary conditions. This present approach is compu-  
58 tationally efficient for different kinds of structures such as thin-walled [28],  
59 laminated and sandwich structure [29]. It can be used for rotating blades  
60 [30] and spinning blades [31] in the rotor dynamics fields. In the field of fluid  
61 interaction, CUF has been used for vortex [32, 33, 34] and double lattice  
62 method [35, 36], and piston theory [37, 38] for supersonic flows. Recently,  
63 [39] the coupling of CUF with the Theodorsen theory has been done to pre-  
64 dict the flutter condition by the v-g method. In this work, the  $p$ - $k$  method  
65 has been implemented in CUF framework to analyze the flutter condition.

## 66 2. Structural Model: Carrera Unified Formulation

67 The present structural model formulated within the popular Carrera Uni-  
68 fied Formulation (CUF) framework [40, 41, 42, 43, 44]. In accordance with

69 CUF,  $u$  is defined as the displacement fields which can be stated as a com-  
 70 bination of the  $F_\tau(x, z)$  (function of cross-section) and  $u_\tau(y)$  displacement  
 71 vector (generalized), and can unify as:

$$72 \quad u(x, y, z, t) = F_\tau(x, z)u_\tau(y), \quad \tau = 1, 2, \dots, T \quad (1)$$

73 where subscript  $\tau$  and  $T$  stands for summation (i.e. Einstein notation) and  
 74 the number of terms in the expansion, respectively. The details of Taylor  
 75 expansion (TE) and displacement fields are given in Table 1.

Table 1: Compact form and displacement fields of Taylor expansion.

Order	T	$F_\tau$	Displacement fields
0	1	$F_1 = 1$	Second-order (TE2)
1	3	$F_2 = x \quad F_3 = z$	$u_x = u_{x1} + x u_{x2} + z u_{x3} + x^2 u_{x4} + xz u_{x5} + z^2 u_{x6}$
2	6	$F_4 = x^2 \quad F_5 = xz \quad F_6 = z^2$	$u_y = u_{y1} + x u_{y2} + z u_{y3} + x^2 u_{y4} + xz u_{y5} + z^2 u_{y6}$
3	10	$F_7 = x^3 \quad F_8 = x^2z \quad F_9 = xz^2 \quad F_{10} = z^3$	$u_z = u_{z1} + x u_{z2} + z u_{z3} + x^2 u_{z4} + xz u_{z5} + z^2 u_{z6}$
$\vdots$	$\vdots$	$\vdots$	
N	$(N+1)(N+2)/2$	$F_{(N+1)(N+2)/2} = x^N \quad F_{(N+1)(N+2)/2} = z^N$	

76 Two types of CUF models are used in this present work: Taylor expansion  
 77 (TE) and Lagrange expansion (LE). The TE model is refined by increasing  
 78 the order of expansion from second-order (TE2) to fourth-order (TE4). Sim-  
 79 ilarly, LE is refined by adding the elements in the cross-section, i.e., one  
 80 nine-noded (1L9) and two nine-noded (2L9) along the chord. The details  
 81 of the displacement fields of the LE model can be found in literature [44].  
 82 Cross-sectional elements are extended along the y-axis with four-noded beam  
 83 elements (B4) to create the structure (Figure 1).

### 84 3. Aeroelastic Model: Steady and Unsteady Theories

85 A complete solution of a thin airfoil in the incompressible fluid, which is  
 86 associated with the harmonic oscillations laterally, Theodorsen's presented  
 87 the lift distribution function by going beyond the quasi-steady model. He  
 88 considered a control surface of a plate that was assumed as flat, which can  
 89 rotate with regard to an axis at distance  $x = b_c a$  via the angle of attack  $\Lambda(t)$   
 90 and move vertically  $h(t)$ . Theodorsen's unsteady lift prediction expression is

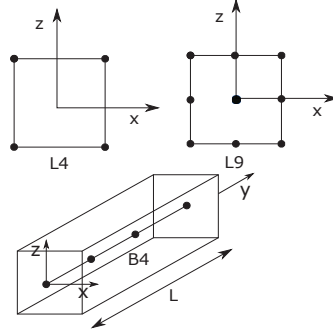


Figure 1: Four- and nine-nodes Lagrange elements and four-nodes beam element.

91 [45]:

$$L_a = \pi\rho_a b_c^2 \left[ \ddot{h} + V_\infty \dot{\alpha} - b_c a \ddot{\alpha} \right] + 2\pi\rho_a b_c C(k) V_\infty \left[ \dot{h} + V_\infty \alpha + b_c \left( \frac{1}{2} - a \right) \dot{\alpha} \right] \quad (2)$$

92 where  $\rho_a$  is air density,  $b_c$  is semi-chord,  $V_\infty$  is free-stream velocity and  
 93 Theodorsen function  $C(k)$  is the relating the reduced frequency  $k = \frac{\omega b_c}{V_\infty}$ .  
 94 The position of the axis of rotation with regard to the center of section denoted as  $a$ , which depends on the load applied, used lamination scheme and support condition. The first term of equation (2) is the non-circulatory (i.e., added mass term), whereas the second term is the circulatory. The second term is called “quasi-steady” model when Theodorsen function  $\{C(k) = 1\}$  and “unsteady aerodynamics” when Theodorsen function is a complex function  $C(k) = F(k) + iG(k)$ . The simplified expression of  $C(k)$  has been presented by Jones [46] by considering the solution of Wagners indicial (1925), which was concerned to exponential approximation:

$$104 \quad C(k) \equiv 1 - \frac{0.165}{1 - \left( \frac{0.0455}{k} \right) i} - \frac{0.335}{1 - \left( \frac{0.3}{k} \right) i} \quad (3)$$

105 The first term of equation (2) can be neglected due to mass properties of the  
 106 structure are small and it is related to single and double differentiated terms  
 107 and can be written as:

$$108 \quad L_a \equiv 2\pi\rho_a V_\infty b_c \left[ \dot{h} + V_\infty \alpha \right] \quad (4)$$

109 For correcting the coefficient of  $C_L$  (sectional lift) associated with an aspect  
 110 ratio of the wing ( $AR_w$ ) and sweep angle ( $\Lambda$ ) effects, by using Diederich's  
 111 approximation, the expression becomes:

$$112 \quad C_{l\alpha} = \frac{dC_L}{d\alpha} = \frac{\pi AR_w}{\pi AR_w + C_{l\alpha 0} \cos(\Lambda)} C_{l\alpha 0} \cos(\Lambda) \quad (5)$$

113 where,  $(AR_w) = \frac{2 L_w}{C_m}$ , ( $L_w$  = wing length and  $C_m$  = mean chord) [Figure 2],  
 114 and  $C_{l\alpha 0}$  is stated as slope of lift-curve ( $\approx 2\pi$ ). In order to reproduce the  
 115 pressure distribution on atop of airfoil, which is thin, slightly inclined and  
 116 uncambered. For capturing the effect of pressure distribution on concerned  
 117 geometry of the wing model, the quantity of  $b_c \pi$  has been approximated by  
 118  $\int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} dx$ , and equations (2) and (4) can be rewritten as:

$$120 \quad L_a = \frac{2\pi AR_w \cos(\Lambda)}{\pi AR_w + 2\pi \cos(\Lambda)} \int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} dx \rho_a V_\infty c(k) \left[ \dot{h} + V_\infty \alpha + b_c \left( \frac{1}{2} - a \right) \dot{\alpha} \right] \quad (6)$$

$$121 \quad L_a \equiv \frac{2\pi AR_w \cos(\Lambda)}{\pi AR_w + 2\pi \cos(\Lambda)} \int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} dx \rho_a V_\infty c(k) [\dot{h} + V_\infty \alpha] \quad (7)$$

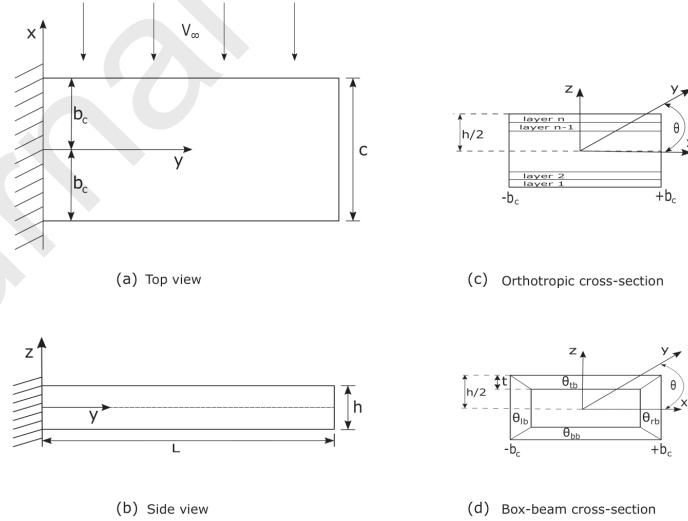


Figure 2: Sketch of beam model and coordinates reference system.

#### 123 4. Formulation of equation of motion: CUF framework

124 The equation of motion for aeroelastic model has been derived using the  
125 principle of virtual displacement (PVD), stated as below:

$$126 \quad \delta L_{int} = \delta L_{ext} + \delta L_{ine} \quad (8)$$

127 where  $\delta L_{int}$  (internal work),  $\delta L_{ext}$  (external work),  $\delta L_{ine}$  (inertial work) and  
128  $\delta$  stands for virtual variation.

129 The strain and kinetic energy can be write in the form of fundamental nuclei  
130 as follow:

$$131 \quad \delta L_{int} = \delta q_{\tau i}^T K_s^{ij\tau s} q_{sj}, \quad (9)$$

$$132 \quad \delta L_{ine} = \delta q_{\tau i}^T M_s^{ij\tau s} \ddot{q}_{sj} \quad (10)$$

134 where  $K_s^{ij\tau s}$  and  $M_s^{ij\tau s}$  defined as a fundamental nucleus for the stiffness and  
135 mass matrix, respectively and components are found in literature [47].

136 The generalized form of the work produced by the lift is:

$$137 \quad \delta L_{ext} = \int_y \int_x \delta u_z(x, y, z_{top}) L_a(x, y, z_{top}) dx dy \quad (11)$$

138 where  $z_{top}$  is associated with upper coordinate of a cross-section in  $z$  and  
139  $L$  is lift given in equation (7). In Carrera Unified Formulation framework,  
140 external work can be written as:

$$141 \quad \delta L_{ext} = \delta q_{\tau i}^T D_L^{ij\tau s} \dot{q}_{sj} + \delta q_{\tau i}^T K_L^{ij\tau s} q_{sj} \quad (12)$$

142 where  $D_L^{ij\tau s}$  (damping) and  $K_L^{ij\tau s}$  (stiffness) are contributions due to the  
143 aerodynamic force, which are defined in fundamental nuclei form:

$$144 \quad D_L^{ij\tau s} = cost I_l^{ij} \int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} F_{\tau}(x, z_{top}) I_L F_s(x, z_{top}) dx \quad (13)$$

$$K_L^{ij\tau s} = cost I_l^{ij} \int_{-b_c}^{b_c} \sqrt{\frac{b_c - x}{b_c + x}} F_{\tau,x}(x, z_{top}) I_L F_s(x, z_{top}) dx$$

145 where

$$146 \quad cost = \frac{2\pi AR_w \cos(\Lambda)}{\pi AR_w + 2\pi \cos(\Lambda)} \rho_a V_{\infty} c(k) \quad (14)$$

147

148

$$I_L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15)$$

149

150

$$I_l^{ij} = \int_l N_i N_j dy \quad (16)$$

151 After assembly of global finite element matrices by assuming the periodic  
152 solution  $a_q = \bar{a}_q e^{i\omega t}$  in the form of quadratic eigenvalue problem (QEP) and  
153 transforming into classical liner system of  $2 \times R$  can we write as:

$$\begin{cases} [M_s]\ddot{a}_q + [D_L]\dot{a}_q + ([K_s] + [K_L])a_q = 0 \\ -\dot{a}_q + \dot{a}_q = 0 \end{cases} \quad (17)$$

155 and by presenting:

156

$$a = \begin{Bmatrix} a_q \\ \dot{a}_q \end{Bmatrix}, \quad \dot{a} = \begin{Bmatrix} \dot{a}_q \\ \ddot{a}_q \end{Bmatrix} \quad (18)$$

157 equation of motion assumed in the following form:

158

$$\frac{R}{T} - \frac{1}{i\omega} I = 0, \quad (19)$$

159 where

160

$$I^{-1}R = \begin{bmatrix} (K + K_L)^{-1}D_L & (K + K_L)^{-1}M \\ -I & 0 \end{bmatrix} \quad (20)$$

161 An iterative procedure was required to calculate the flutter condition because  
162 of aerodynamic contribution matrices, which have a dependency on the re-  
163 duced frequency ( $k$ ). The  $p$ - $k$  method proposed by Hassig [4] is used for  
164 solving the flutter problem. The General form of a proposed equation given  
165 as:

$$\left[ \left( \frac{V_\infty}{c_m} \right)^2 [M_s]p^2 + [K_s] - \frac{1}{2}\rho V_\infty^2 [A(p)] \right] \{a_q\} = 0 \quad (21)$$

167 where,  $[A(p)] =$  unsteady aerodynamic forces. The Simplified fundamental  
168 equation for modal flutter analysis presented in [48]:

169

$$\left[ [[M_s]p^2 + \left( [B_s] - \frac{1}{4}\rho c V_\infty \frac{[Q_a^I]}{k} \right) p + [K_s] - \frac{1}{2}\rho V_\infty^2 [Q_a^R] \right] \{a_q\} = 0 \quad (22)$$

170 where,  $[Q_a^I], [Q_a^R]$  are imaginary and real part of aerodynamic force matrix,  $c$   
 171 is reference length,  $p$  is eigenvalue,  $k$  reduced frequency and  $[B_s] = 0$  because  
 172 structural damping is not considered here. The circular frequency  $\omega$  and  
 173 reduced frequency  $k$  are not independent since  $k = \frac{\omega b_c}{V_\infty}$ ,

$$174 \quad k = \frac{b}{V_\infty} im(p) \quad (23)$$

175 and

$$176 \quad p = \gamma\omega + i\omega \quad (24)$$

177 where,  $\gamma$  is transient decay rate coefficient ( $2\gamma = g$ ).

## 178 5. Results And Discussion

179 Based on the above formulation, a FORTRAN finite element model is  
 180 developed to analyze the flutter condition of laminated composite structures  
 181 (plate and box-beam) model. Carrera Unified Formulation (CUF) has a  
 182 hierarchical finite element model, which is very accurate and economically  
 183 efficient. In CUF, we can choose any order of Taylor's expansion with-  
 184 out changing in the formulation. Plate elements (CQUAD4) based on the  
 185 Mindlin-Reissner shell theory are used to develop the MSC-Nastran model  
 186 (i.e., isotropic, orthotropic and box-beam) for the comparison of results ob-  
 187 tained by CUF. Numerical examples of isotropic and laminated composite  
 188 structures are presented in subsequent sections. The flutter condition of both  
 189 models is computed by using the  $p$ - $k$  method and flutter condition (flutter ve-  
 190 locity) is defined at the point where the real part of the eigenvalue (damping)  
 191 is null.

### 192 5.1. Isotropic plate

193 The first numerical example is based on the work of Petrolo [49]. Where  
 194 a cantilever straight plate with the geometrical data: length  $L = 0.305$  m,  
 195 thickness  $t = 0.001$  m, and chord  $c = 0.076$  m, having isotropic material prop-  
 196 erties:  $E = 73.8$  GPa,  $\nu = 0.3$  and  $\rho = 2768$  Kg/m<sup>3</sup> has been analyzed. The  
 197 flutter analysis of the plate model using CUF based finite element method  
 198 (FEM) and MSC-Nastran model with sweep angle  $\lambda = 0^\circ$  has been performed.  
 199 The different orders of Taylor expansion (TE) and Lagrange expansion (LE)  
 200 with nine-noded (L9) cross-sectional elements have been considered for CUF

201 model to compute the results. The first five modes (i.e., bending and tor-  
 202 sion) of natural frequency ( $f_n$ ), flutter velocity ( $V_F$ ), and flutter frequency  
 203 ( $f_F$ ) of CUF and MSC-Nastran model are reported in Table 2. MSC-Nastran  
 204 model was created using the plate elements (CQUAD4) with 8x20 grids. The  
 205 computed flutter velocity and frequency using the  $p$ - $k$  method are compared  
 206 with MSC-Nastran and results obtained using the doubled-lattice method of  
 207 literature. It is observed that the modal frequencies are matching well with  
 208 MSC-Nastran. Also, observed that modal frequencies and flutter conditions  
 209 of third-order Taylor expansion (TE3), fourth-order (TE4), and two nine-  
 210 noded LE model are more accurate for the present isotropic plate and the  
 desired result is achieved with a very low computational cost.

Table 2: Natural frequencies (Hz) and flutter conditions.

	Model	Mesh	DOFs	$f_{n_1}$	$f_{n_2}$	$f_{n_3}$	$f_{n_4}$	$f_{n_5}$	$V_F$ (m/s)	$f_F$ (Hz)
Reference [49]	TE4	20B4		9.40	58.84	74.21	165.10	230.81	72.75	59.77
Present	TE2	12B4	666	9.14	57.19	73.71	160.60	227.94	71.60	42.05
	TE3	12B4	1110	9.14	57.16	73.70	160.52	227.77	69.80	39.15
	TE4	12B4	1665	9.14	57.17	73.72	160.54	227.97	74.90	40.47
	LE (1L9)	12B4	999	9.14	57.17	73.70	160.53	227.74	70.70	39.41
	LE (2L9)	12B4	1665	9.14	57.17	73.70	160.53	227.74	70.70	39.41
	MSC-Nastran	8x20	945	9.09	56.74	72.12	158.95	222.07	66.51	39.52

211

## 212 5.2. Orthotropic plate

213 A six-layered laminated composite plate is considered in this section with  
 214 symmetric laminate  $[30_2/0]_s$ . The cantilever plate with length ( $L = 0.305$   
 215 m), chord ( $c = 0.0762$  m) and thickness ( $t = 0.000804$  m) having material  
 216 properties  $E_L = 98.00$  GPa,  $E_T = 7.90$  GPa,  $G_{LT} = 5.60$  GPa,  $\nu = 0.280$ , and  
 217  $\rho = 1520.00$  kg/m<sup>3</sup> has been analyzed to find flutter conditions. The different  
 218 orders of Taylor expansions, e.g., second-order (TE2), third-order (TE3) and  
 219 fourth-order (TE4) used to describe the cross-section of a plate. Similarly,  
 220 two types of Lagrange elements, one nine-noded (1L9) and two nine-noded  
 221 (2L9), along with the chord used to defined the cross-section of a single layer.  
 222 The first five modes of natural frequency and flutter condition for the  $[30_2/0]_s$   
 223 plate are reported in Table 3. The mode shapes of bending and torsion for  
 224 LE (2L9) model are shown in Figure 3. Computed results indicate that TE3,

225 TE4 and 2L9 produce more accurate results as compared to TE2 and 1L9.  
 226 The first five modes of frequencies and damping with respect to functions  
 227 of speed for the present CUF (TE3) and MSC-Nastran model are plots in  
 228 Figure 4. The various stacking sequence  $[0_2/90]_s$ ,  $[45/-45/0]_s$  and  $[45_2/0]_s$   
 229 are used to compute flutter conditions for TE3 and 2L9 models, which are  
 230 reported in Table 4. Here results show that different stacking sequences can  
 influence both natural frequency and flutter conditions.

Table 3: Natural frequencies (Hz) and flutter conditions for a laminated plate (6-layers)  $[30_2/0]_s$ .

	Model	Mesh	DOFs	$f_{n_1}$	$f_{n_2}$	$f_{n_3}$	$f_{n_4}$	$f_{n_5}$	$V_F$ (m/s)	$f_F$ (Hz)
Reference [36]	TE4	15B4	2070	6.05	35.91	56.51	100.03	172.23	25.86	26.66
Present	TE2	10B4	558	6.34	37.91	69.43	107.43	213.96	31.90	28.38
	TE3	10B4	930	6.31	37.49	57.73	104.65	178.90	27.80	27.90
	TE4	10B4	1395	6.21	37.25	56.94	103.76	173.82	27.60	27.22
	LE (1L9)	10B4	3627	6.31	37.52	57.77	104.72	179.11	29.50	27.59
	LE (2L9)	10B4	6045	6.30	37.33	57.11	104.04	174.68	28.10	27.10
	MSC-Nastran	10x30	1705	6.26	37.05	55.97	103.15	170.42	25.40	27.39

231

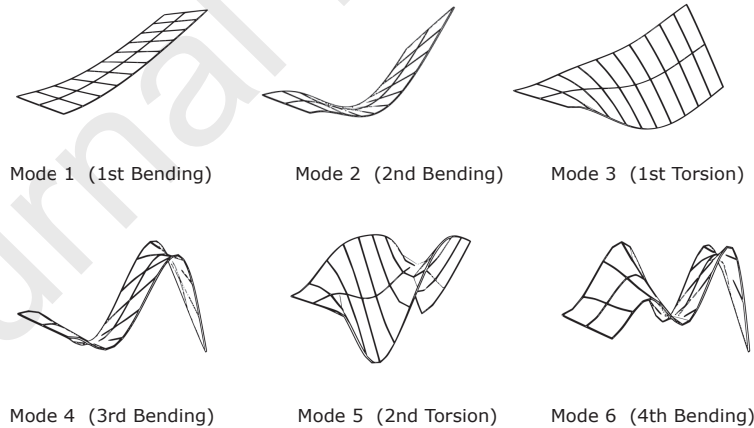


Figure 3: Mode shapes of six-layer plate  $[30_2/0]_s$  for LE (2L9) model using CUF.

232 Another plate model was considered from the Kameyama *et al.* [16] work  
 233 having eight-layer symmetric laminates  $[-22.5/67.5/22.5/-67.5]_s$  with a total  
 234 thickness of the laminate 0.804 mm. The thicknesses of the plies were 0.037,

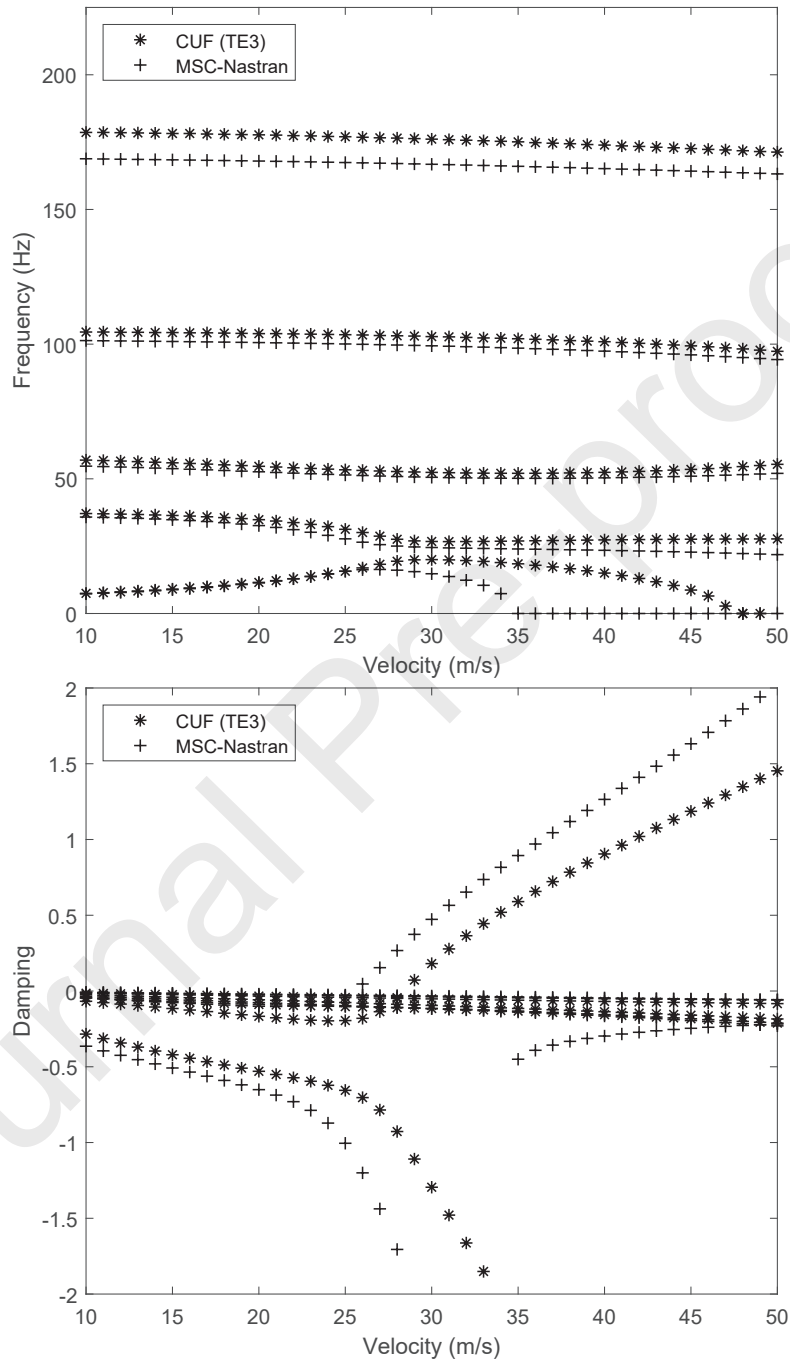


Figure 4: Flutter diagram of orthotropic six-layer plate  $[30_2/0]_s$ .

Table 4: Natural frequencies (Hz) and flutter conditions for 6-layers plate.

Lamination	Model	Present						Reference		
		$f_{n_1}$	$f_{n_2}$	$f_{n_3}$	$f_{n_4}$	$f_{n_5}$	$V_F$ (m/s)	$f_F$ (Hz)	[16] (CLT)	[50] (Exp)
[0 <sub>2</sub> /90] <sub>s</sub>	TE3	11.04	39.55	69.16	133.08	193.62	23.20	27.33	23.0	25
	LE (2L9)	11.04	39.51	69.16	132.50	193.62	22.90	27.23		
[45/−45/0] <sub>s</sub>	TE3	4.88	30.22	50.74	84.59	157.83	47.30	31.48	40.1	>32
	LE (2L9)	4.88	30.10	49.93	84.09	152.84	46.80	31.44		
[45 <sub>2</sub> /0] <sub>s</sub>	TE3	5.78	36.60	69.08	103.38	207.51	29.60	25.04	27.5	28
	LE (2L9)	5.78	35.89	68.98	102.78	206.20	29.00	24.93		
[30 <sub>2</sub> /0] <sub>s</sub>	TE3	6.31	37.49	57.73	104.65	178.90	27.60	27.22	27.1	27
	LE (2L9)	6.30	37.33	57.11	104.04	174.68	28.10	27.10		

235 0.048, 0.064 and 0.253 mm, respectively. The first five modes associated  
 236 with bending and torsion of natural frequency and flutter velocities for CUF  
 237 model and MSC-Nastran are reported in Table 5. The present results have  
 238 a good agreement with reference. Similar trends observed like the previous  
 239 6-layer plate models. All the TE2, TE3, TE4, 1L9, and 2L9 are suitable, but  
 240 TE4 and 2L9 are most accurate. Flutter velocity has been found in order  
 241 with reference results obtained by the generic algorithm for flutter solution,  
 whereas in the present analysis  $p$ - $k$  method has been used.

Table 5: Natural frequencies (Hz) and flutter condition (velocity [m/s]) for 8-layers plate.

	Model	Mesh	DOFs	$f_{n_1}$	$f_{n_2}$	$f_{n_3}$	$f_{n_4}$	$f_{n_5}$	$V_f$ (m/s)
Reference [16]	CLT			7.2	45.4	59.1	127.7	182.3	38.8
Present	TE2	10B4	558	7.3	46.2	59.1	129.8	182.7	38.60
	TE3	10B4	930	7.2	45.1	59.0	126.8	182.5	37.20
	TE4	10B4	1395	7.2	45.1	59.0	126.7	182.3	36.30
	LE (1L9)	10B4	4743	7.2	45.1	59.0	126.8	182.5	38.90
	LE (2L9)	10B4	7905	7.2	45.1	59.0	126.8	182.2	36.80
	MSC-Nastran	10x30	1705	7.2	45.0	58.1	126.2	179.0	35.01

242

### 243 5.3. Box-beam structure

244 A prismatic thin-walled beam wing model considered for flutter anal-  
 245 ysis as the main frame of the wing was a box-type structure. A similar

246 type of analysis on the box-beam was discussed in the introduction section.  
 247 The dimensions and material properties associated with the composite beam  
 248 (box-beam) structure are listed in Table 6. The box-beam configuration is  
 249 shown in Figure 2, where the lamination sequence of four walls, i.e., bottom  
 250 wall, right wall, top wall and left wall are defined as  $[\theta_{bb}/\theta_{rb}/\theta_{tb}/\theta_{lb}]$ . The  
 251 arbitrary lamination sequence  $[\theta/0.1\theta/2\theta/10\theta]$  of the walls is considered to  
 252 avoid the bending-torsion coupling of the structure. In previous sections, it  
 253 is already observed that TE4 and 2L9 are the more accurate model and the  
 254 same model in general, is taken here for flutter analysis. In the first case  
 255  $\theta = 30^\circ$  is considered and the flutter conditions have been computed using  
 256 CUF and MSC-Nastran model. The first five modes of natural frequencies  
 257 and flutter conditions (frequency and velocity) are reported in Table 7. The  
 258 present TE4 and 8L9 results are matching well with the MSC-Nastran. First  
 259 four modes of frequencies and damping with respect to the function of speed  
 260 for present TE4 and MSC-Nastran models are shown in Figure 5. The fre-  
 261 quency and damping variation with velocity also matching well but with a  
 262 low computational cost. So, it can be commented that the present models  
 263 are efficient and accurate and CUF models can be used for flutter analysis  
 264 of laminated composite box-beam structure. Now, the detailed flutter anal-  
 265 ysis of various composite box-beam model has been performed by the  $p$ - $k$   
 266 method. Flutter velocities are computed with variations in  $\theta$  for different  
 267 cross-sectional elements and are listed in Table 8. The polar plot for these  
 268 models is also shown in Figure 6. The polar plot shows that the flutter veloc-  
 269 ity is increasing between  $\theta = 0^\circ$  to  $\theta = 30^\circ$  for LE but in TE model case up  
 270 to  $\theta = 60^\circ$ . The flutter velocities are reaching a minimum for all the models  
 271 at  $\theta = 90^\circ$ . The lower order TE models (i.e., TE2 and TE3) are less effec-  
 272 tive in predicting flutter conditions compared to the LE models. In addition,  
 273 the variations in the computed flutter velocity of the TE model indicate that  
 274 the higher-order model (TE4) is more effective than lower-order models in  
 275 determining flutter conditions. It can be observed from results that the TE4  
 276 and 8L9 models predict the most accurate flutter conditions as compared to  
 277 other models.

## 278 6. Conclusion

279 Flutter condition of plates and box-beam structures made of isotropic and  
 280 orthotropic materials has been analyzed in this work. Carrera Unified Formu-  
 281 lation (CUF) has been used to define the structural model, one-dimensional

Table 6: Material and dimensional parameter for composite box beam structure.

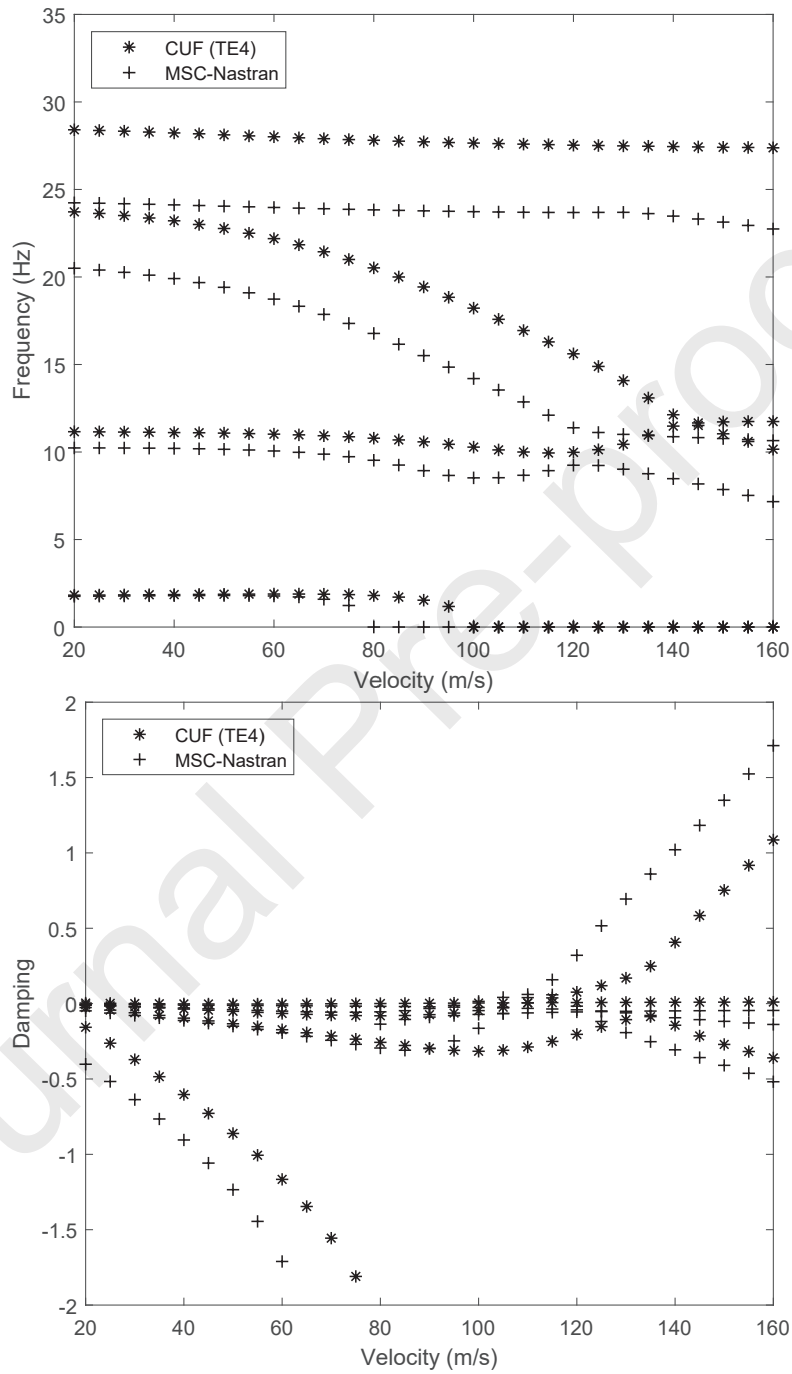
Material Properties	GPa, kg/m <sup>3</sup>
$E_{11}$	206.8
$E_{22}$	5.17
$G_{23} = G_{31}$	2.55
$G_{12}$	3.10
$\nu_{12}$	0.25
$\rho$	1528.5
Dimensions	m
chord ( $c$ )	0.5
height ( $h$ )	$c / 15$
thickness of wall ( $t$ )	$c / 150$
length ( $L$ )	3.5

Table 7: Natural frequencies (Hz) and flutter conditions for box-beam ( $\theta = 30^\circ$ ).

Model	Mesh	DOFs	$f_{n_1}$	$f_{n_2}$	$f_{n_3}$	$f_{n_4}$	$f_{n_5}$	$V_F$ (m/s)	$f_F$ (Hz)
TE4	10B4	1395	1.80	11.16	23.84	28.46	30.83	105.09	17.52
LE (8L9)	10B4	4464	1.75	10.62	21.55	25.40	28.81	109.10	15.84
MSC-Nastran	5x30(Flange) 3x30(Web)	2480	1.74	10.41	20.73	24.38	26.03	95.09	14.82

Table 8: Flutter condition {velocity (m/s)} for box-beam model.

Model	DOFs	$\theta=0^\circ$	$\theta=30^\circ$	$\theta=60^\circ$	$\theta=90^\circ$	$\theta=120^\circ$	$\theta=150^\circ$	$\theta=180^\circ$
TE2	558	105.4	120.3	122.8	98.9	121.8	118.9	104.5
TE3	930	100.6	114.7	116.2	89.4	105.0	113.0	100.7
TE4	1395	93.0	105.0	109.9	90.2	106.6	110.7	96.6
LE (4L9)	1584	109.5	115.6	110.8	90.8	112.3	114.8	111.0
LE (8L9)	4464	95.4	109.1	100.2	91.9	95.9	106.1	100.5

Figure 5: Flutter diagram of box-beam with  $\theta = 30^\circ$ .

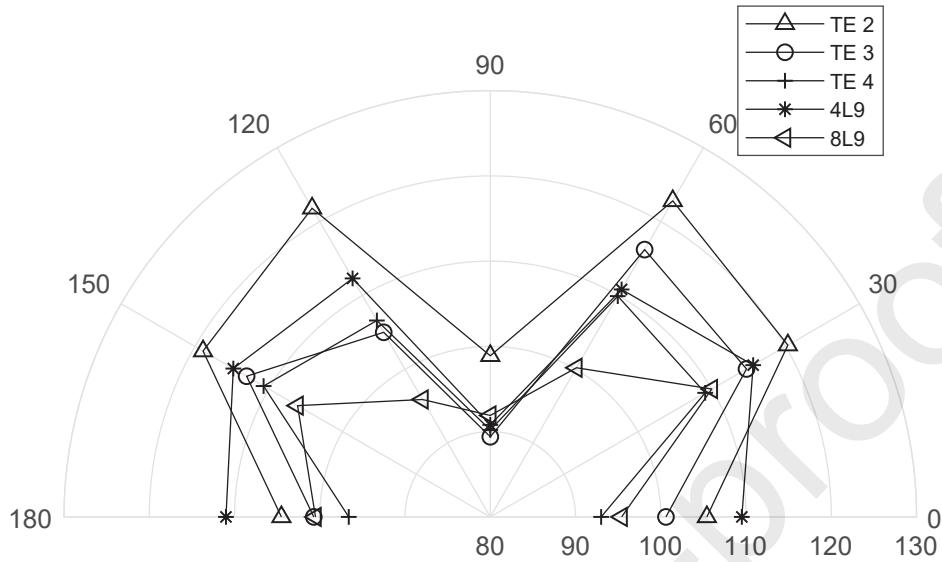


Figure 6: Polar plot of flutter velocities for angle ply of box-beam.

282 models exploit with Lagrange- and Taylor-like expansions to describe the  
 283 displacement field of cross-section accurately. The order of Taylor expansion  
 284 can be chosen arbitrarily in input; it is an independent parameter of the  
 285 formulation. The governing equation is derived using the principle of virtual  
 286 displacement and aerodynamic loading conditions are defined in the form of  
 287 fundamental nuclei in CUF. Furthermore, the  $p$ - $k$  method has been imple-  
 288 mented and flutter conditions are computed for the unsteady aerodynamics  
 289 in CUF framework and results have been compared with literature and MSC-  
 290 Nastran; they are in good agreement. The computed results indicate that for  
 291 isotropic case, all the TE and LE model can produce good results. Similarly,  
 292 for the orthotropic plate the TE3, TE4, and 2L9 are preferable to find the  
 293 flutter conditions. For the composite box-beam, TE4 and 8L9 are the most  
 294 accurate models. It is also observed that the present TE and LE aeroelastic  
 295 1D models are very accurate, efficient with a low computational cost. In  
 296 the future investigation, aeroelastic analysis of the fixed and rotating-beam  
 297 models with a quasi-steady and unsteady aerodynamics and gust analysis  
 298 can be performed in the CUF framework.

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