Abstract

This thesis explores different topics arising from the field of Diophantine approximation.

The first part is devoted to an old problem in number theory: the explicit construction of new transcendental numbers. Classical continued fractions have long served as a powerful tool in this direction, exploiting fundamental results in Diophantine approximation, such as Roth's theorem. By contrast, their multidimensional generalization—introduced by Jacobi and later developed by Perron in response to a question of Hermite—remains largely unexplored.

In a joint work with Murru and Romeo, we establish new transcendence criteria for multidimensional continued fractions generated by the Jacobi–Perron algorithm. We prove that expansions characterized by either sufficiently rapid growth or specific quasi-periodic structures in their partial quotients are transcendental. These results extend classical one-dimensional constructions due to Liouville and to Maillet–Baker, to the multidimensional setting.

This part of the thesis provides a self-contained exposition of these original contributions, while reviewing the necessary background on Diophantine approximation and the theory of classical and multidimensional continued fractions.

The second part concerns the distribution of real sequences modulo 1. A seminal result by Weyl states that for an irrational real number $\alpha$, the sequence $\alpha, 2\alpha, 3\alpha, \dots$ is uniformly distributed modulo 1, meaning the proportion of terms $(\{na\})_{n\geq 0}$ falling within any subinterval of $[0, 1)$ is equal to the length of that subinterval, where $\{\cdot\}$ denotes the fractional part. Many other sequences are known to enjoy the same property. For instance, geometric progressions of the form $(\lambda\alpha^n)_{n\geq 0}$ are uniformly distributed modulo 1 for fixed $\lambda > 0$ and almost every $\alpha > 1$, or conversely, for fixed $\alpha > 1$ and almost every $\lambda > 0$. However, when both $\alpha$ and $\lambda$ are fixed, the problem becomes significantly more challenging. A theorem of Pisot and Vijayaraghavan states that, if $\alpha$ is algebraic, then such a sequence has finitely many limit points modulo 1 if and only if $\alpha$ is a Pisot number and $\lambda \in \mathbb{Q}(\alpha)$.

A problem arising from this context is whether, given a Pisot number $\alpha$, one can find $\lambda$ such that the geometric progression modulo 1 has a prescribed number of limit points. This question is closely related to the problem of identifying linear recurrences with a fixed characteristic polynomial that exhibit a predetermined number of residues modulo an integer.

The aim of the second part is to examine these questions in greater depth and to present recent results obtained in joint work with Sanna.