Completeness and Regularization Techniques for Wiener-Hopf problems with Discontinuous Layers

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***Abstract*—Scattering and radiation by buried multiple objects in multilayered media is of great interest in a vast variety of electromagnetic applications. In this work, we present a general methodology to analyze the complex scattering problems with the capability of semi-analytical method in spectral domain, which allows physical interpretation of the problem and asymptotics estimation of field behavior. The method is based on the combination of the Wiener-Hopf technique, completeness and regularization techniques.**

***Index Terms*— Multilayered structures, buried objects, Wiener-Hopf technique, electromagnetic scattering, Green's function, integral equations, radar technologies, wave propagation, diffraction.**

I. INTRODUCTION

In computational electromagnetics, full numerical techniques share with semi-analytical ones the difficulty of correct modeling near-field interaction among close penetrable and impenetrable structures with edges in addition to possible immersion in multilayered materials.

Of course, closed-form analytical solution of such problems are not available, however, in this work, we present a general and effective semi-analytical spectral technique based on the generalization of the Wiener-Hopf method.

One of the main benefits of this technique is that it allows physical interpretation of the solution as in problems with closed form solutions.

The technique is based on the combination of multiple mathematical tools: 1) Wiener-Hopf (WH) technique in spectral domain [1-3], 2) characteristic Green's function procedure [4-5], 3) completeness of WH equations [5] and Fredholm factorization technique [1-7].

In this work, we first present the basic steps of the procedure, while, in the second part and at the conference, we formulate a specific problem: the scattering by a rectangular penetrable cylinder. This canonical tricky problem is unsolved in literature via analytical/semi-analytical techniques and demonstrate the effectiveness of the method. In the framework of Wiener-Hopf method, the literature presents solutions only in case of impenetrable rectangular cylinders, see in particular [8-14].

While presenting the theory and the equations we use extensively network equivalence of the formulated equations

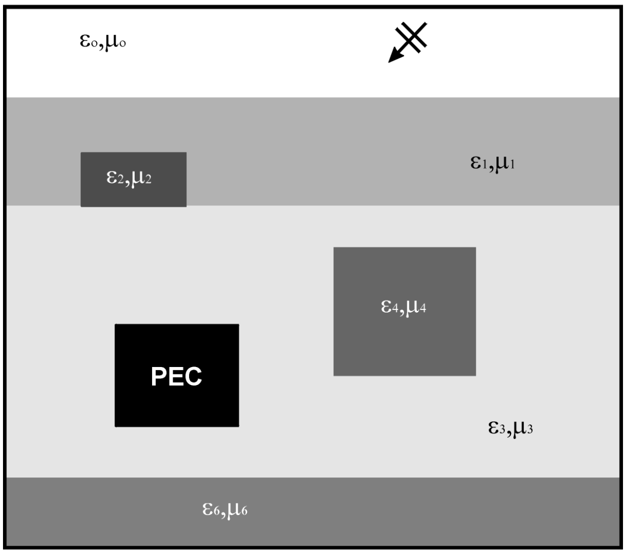

Fig. 1. Multiple buried objects in multilayered media illuminated by a plane wave

to provide the systematization of the procedure as effectively demonstrated in [1],[5],[7].

II. THE CLASS OF PROBLEMS

The aim of this work is to provide a general method to allow the analysis of complex scattering problems constituted of multiple buried objects in multilayered media. Fig. 1 provides an example of these structures where we have multilayered regions characterized by permittivities and permeabilities with immersed rectangular cylindrical structures made by penetrable or perfect electric conducting (PEC) materials.

Fig. 2 provides a domain decomposition useful for the implementation of the method as described in the next sections. The partitioning of regions is performed considering layers that are homogenous in the vertical direction. The example of Fig. 2 identifies nine layers with respect to the physical model reported in Fig. 1.

For each layer k we identify the thickness d_k and a set of widths s_{kj} related to one d_k due to the possible multiple discontinuities on the media. Each couple (d_k,s_{kj}) defines

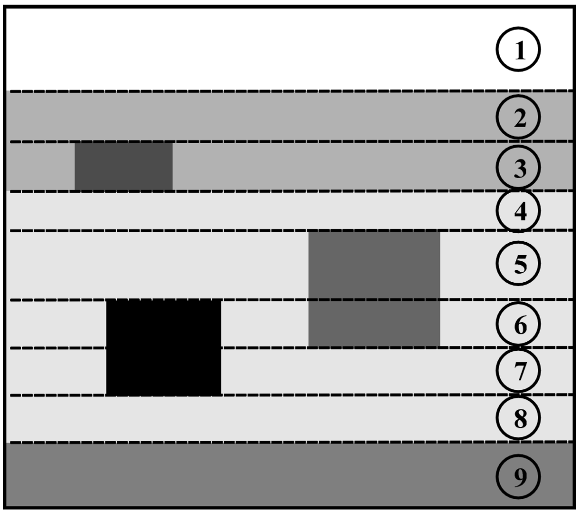

Fig. 2. Partitioning of regions is performed considering layers that are homogenous in the vertical direction. For each layer k=1..9 of the figure we identify the thickness d_k and a set of widths s_{kj} related to one d_k.

rectangular homogenous regions with finite, semi-infinite or infinite width and fixed constitutive parameters.

III. The Method in General

With the help of a flow chart to systemize the procedure, see Fig. 3, we provide the basic steps of the procedure to deal with the class of problems presented in Section II using the Wiener-Hof technique completed with several other mathematical tools.

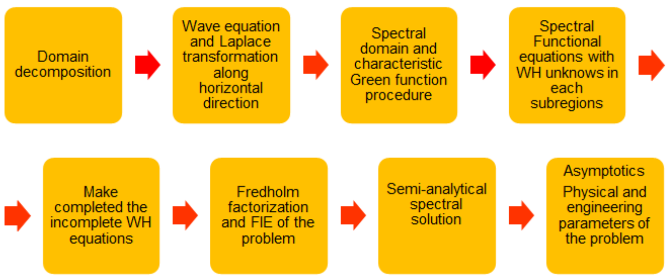

Fig. 3. Flow chart of the fundamental steps of the proposed method

After performing the domain decomposition as illustrated in Fig. 2, the basic structure to be modelled is each layer k of thickness d_k constituted by several blocks of width s_{kj}, see Fig. 4.

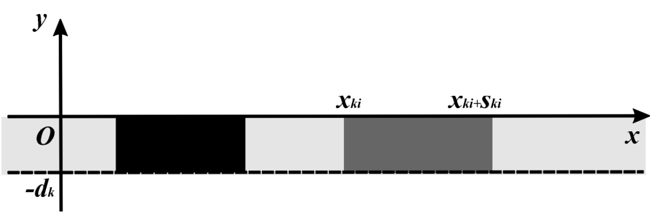

Fig. 4. Geometrical definitions for a general layer k of thickness d_k composed by different blocks of width s_{kj} and different material properties.

For illustrative purpose, in the following, we consider only Ez polarization. Let us consider the block defined by $(x_{kj},x_{kj}+s_{kj})$ and use a local coordinate system with $X=x- x_{kj}$.

We note that while applying Laplace transformation along horizontal direction to get Wiener-Hopf equations we encounter two kind of problems: 1) boundary conditions along different sub regions, 2) Laplace transforms along finite regions yield entire functions. Both these properties are not classical properties of WH formulations where we are accustomed to use plus and minus unknowns.

Focusing on finite bricks, we introduce the following WH unknowns defined on the planes y=0 and y=-d:

$$\begin{aligned} V_{1o}(\eta) &= \int_0^s E_z(X,0)e^{j\eta X}dx, \quad V_{2o}(\eta) = \int_0^s E_z(X,-d)e^{j\eta X}dX \\ I_{1o}(\eta) &= \int_0^s H_x(X,0)e^{j\eta X}dx, \quad I_{2o}(\eta) = \int_0^s H_x(X,-d)e^{j\eta X}dX \end{aligned} \quad (1)$$

While considering a finite homogenous brick (finite s) we have that (1) are entire functions. The application of Laplace transform to the wave equation at Ez polarization yields an ordinary differential equation with a forcing term constituted of the boundary conditions (in case of finite brick, we have both side of the sub-region). This problem can be dealt with a general form of the characteristic Green's function procedure, see [4] and [5] for theory and implementation ([5] reports all steps of implementation for a semi-infinite layer/brick). Evaluating the solution at y=0,-d we obtain, for a finite brick, a system of WH equations that can be interpreted as a two-port network model:

$$\begin{aligned} -I_{s1o}(\eta) - I_{1o}(\eta) &= Y_{11}(\eta)V_{1o}(\eta) + Y_{12}(\eta)V_{2o}(\eta) \\ I_{s2o}(\eta) + I_{2o}(\eta) &= Y_{21}(\eta)V_{1o}(\eta) + Y_{22}(\eta)V_{2o}(\eta) \end{aligned} \quad (2)$$

where I_{s1o} and I_{s2o} are particular integrals of the Green's function procedure (see [5]) in terms of the boundary conditions of both sides of the brick. The admittances defined in (2) depends on the material properties of the considered brick through the propagation constant k_d and the impedance Z_d:

$$Y_{11}(\eta) = Y_{22}(\eta) = -jY_{cd}(\eta)\cot[\xi_d(\eta)d], \quad Y_{12}(\eta) = Y_{21}(\eta) = \frac{jY_{cd}(\eta)}{\sin[\xi_d(\eta)d]},$$

$$\xi_d(\eta) = \sqrt{k_d^2 - \eta^2}, Y_{cd}(\eta) = \frac{\xi_d(\eta)}{k_d Z_d}.$$

The presence of I_{s1o} and I_{s2o} in (2) makes them incomplete WH equations, i.e. not explicitly written in terms of WH unknowns. However, exploiting analytical properties of I_{s1o}, I_{s2o} and (2) in general we represent them in terms of samples of the WH unknowns defined in the entire layer with the help of Mittag-Leffler theorem (see [5]). To complete the list of equations we double the unknowns used in (2)

$$\begin{aligned} V_{1\pi o}(\eta) &= e^{j\eta s}V_{1o}(-\eta), V_{2\pi o}(\eta) = e^{j\eta s}V_{2o}(-\eta) \\ I_{1\pi o}(\eta) &= -e^{j\eta s}I_{1o}(-\eta), I_{2\pi o}(\eta) = -e^{j\eta s}I_{2o}(-\eta) \end{aligned} \quad (3)$$

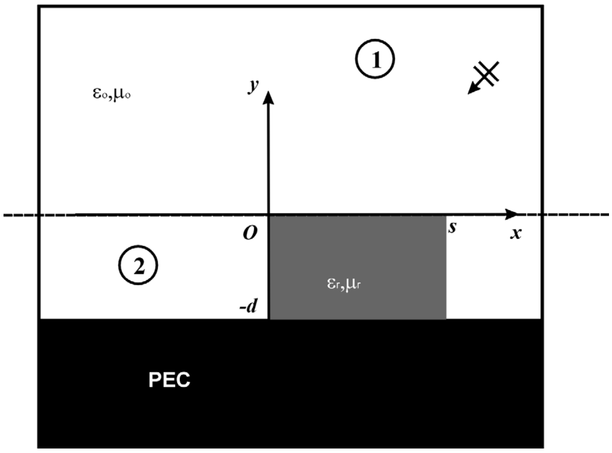

Fig. 5. Rectangular penetrale cylinder/brick grounded by PEC and immersed in free space of thickness d and width s illuminated by a plane wave. The figure shows also two layers according to the methodology reported in the previous sections: layer 1 is a half space region in vaccum while layer 2 is a layer of thickness d where three brick regions are present (a semi-infinite subregion on the left in vaccum, a semi-infinite subregion on the right in vacuum and a finite brick made by penetrable material of width s in the center).

Now (2) and the new equations with (3) are complete WH equations since they are written in terms of spectral unknowns (1) and (3) and their samples.

To complete the set of equations of layer k we repeat the procedure to the other bricks of the layer, also to the ones at the end that present semi-infinite dimensions. For semi-infinite bricks the procedure is already available in [5].

The procedure then is repeated in the other layers to obtain the set of equations of the entire problem.

The solution of the set of completed WH equations is performed via Fredholm factorization techniques [1-7] which reduces the system of spectral WH equations to integral representations of second kind with compact kernels by eliminating one kind of unknowns (plus/minus or entire o/entire π, see (1) and (3)) after a regularization procedure.

However, completed equations such as (2), shows sample of spectral unknowns that do not lie on the integration path. To overcome this problem we resort to Cauchy representation formulas, see for instance [5], that allows the evaluation of spectrum in a regularity point using integral representation along a contour:

$$V_+(-\alpha_n) = \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{V_+(\eta')}{\eta'+\alpha_n} d\eta' - V_+^{n.s.}(-\alpha_n) \qquad (4)$$

The procedure yields a system of Fredholm integral equations of second kind that is effectively solvable by simple quadrature techniques (i.e. sample and hold), as demonstrated in a variety of scattering problems, see [1-3], [5-7] and references therein.

IV. The Grounded Rectangular Penetrable Cylinder

In this section, we formulate a canonical tricky problem that is unsolved in literature using analytical/semi-analytical method. Furthermore, it demonstrates the capability of the proposed general method in practice.

The structure is composed by a penetrable cylinder/brick immersed in free space and grounded by PEC as shown in Fig. 5.

Making reference to the caption of Fig. 5, and for compactness of presentation, we let the reader retrieve all the equations of layer 1 in [5].

Let us focus the attention on layer 2, where we observe that, due to the PEC grounding, the WH unknowns are defined only at y=0 and no equation is written at y=-d, thus the models reduce to one port network models for each sub region (center (5), right (6), left (7)):

$$\begin{aligned} -I_o(\eta) &= Y_{d3}(\eta)V_o(\eta) + I_{do}(\eta) \\ -I_{\pi o}(\eta) &= Y_{d3}(\eta)V_{\pi o}(\eta) + I_{d\pi o}(\eta) \end{aligned} \qquad (5)$$

$$-I_+(\eta) = Y_d(\eta)V_+(\eta) + I_d(\eta) \qquad (6)$$

$$I_{\pi+}(\eta) = Y_d(\eta)V_{\pi+}(\eta) + I_{d\pi}(\eta) \qquad (7)$$

with WH unknowns defined in (1) and (3) for (5) and, respectively for (6) and (7)

$$V_+(\eta) = \int_0^\infty E_z(X,0)e^{j\eta X}dx, \quad I_+(\eta) = \int_0^\infty H_x(X,0)e^{j\eta X}dx \qquad (8)$$

$$V_{\pi+}(\eta) = \int_{-\infty}^0 E_z(x,0)e^{-j\eta x}dx, \quad I_{\pi+}(\eta) = -\int_{-\infty}^0 H_x(x,0)e^{-j\eta x}dx \qquad (9)$$

with X=x-s. Note that I_{do}, $I_{d\pi o}$, I_d, $I_{d\pi}$, as explained in the previous section, are derived for the application of the Green's function procedure and constitute an incompleteness of WH equations. The completeness technique allows to define these terms as a function of samples of all spectral WH unknowns of the layer, i.e. (1), (8), (9). For this property (5), (6), (7) results to be complete WH equation amenable of solution via Fredholm factorization techniques in combination of Cauchy representation formulas, see previous section.

Conclusion

The paper shows the fundamental steps of a new effective semi-analytical method based on WH technique for the analysis of scattering by complex structures as multiple buried objects in multilayered media illuminated by plane waves. The effectiveness of the technique is presented illustrating the formulation of a canonical tricky problem that is unsolved in literature using analytical/semi-analytical method: scattering by a penetrable cylinder/brick immersed in free space and grounded by a PEC plane.

The effectiveness of the method will be demonstrated by solving the relevant system of Fredholm integral equations presented in abstract form in Section IV with simple quadrature rule. The semi-analytical spectral solution of the problem allows physical interpretation of the solution in terms of field by applying asymptotics.

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