A Combined Analytical and Numerical Approach for the Evaluation of Radial Loads on the Lining of Vertical Shafts

A combined analytical and numerical approach for the evaluation of radial loads on the lining of vertical shafts

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ABSTRACT

The evaluation of the load acting on a shaft support is of fundamental importance for the correct dimensioning of the structure. The load acting on the support can appear somewhat complex. One approach may be to use the convergence-confinement method (CCM) normally used in the tunneling design. This process involves intersecting the convergence-confinement (CC) curve with the support reaction line. However, in order to be able to adopt this technique, it is necessary to know the radial displacement of the shaft wall at the point in which the support is to be installed. Using the equations of Vlachopoulos and Diederichs (2009) the reaction line of the support can be calculated. Numerical models developed with Flac 2D v.6.0 considering the Mohr-Coulomb criterion and an ideal elastic-plastic behavior simulating stepwise excavation and support installation were developed. The relation between applied internal stress and radial displacement of the wall shaft, obtained by the numerical simulation was compared with the CC curve obtained by the CCM and it showed a good match between the two methods. However, an iterative procedure has also been used to insert the reaction line in the CC graph. The result shows lower initial displacements (and therefore greater radial stress) when compared with the values obtained by numerical calculation with the axisymmetric model. It is therefore recommended the combined use of the

CCM (analytical method) and the axisymmetric numerical model (step by step simulation) to obtain the values of the final load on the lining and the final plastic radius, necessary for the correct design of supporting structures on the shaft wall.

Key words: shaft, lining, convergence-confinement method, FLAC, weak rocks, numerical modelling, axisimmetric model, wall radial displacement profile.

Introduction

Underground mining methods invariably rely on tunneling networks to gain access to the zones of valuable minerals (Carter et al. 2011). Underground mines can be reached via ramps, inclined or vertical shafts or adits (Bullock 2011). To achieve great depths, vertical shafts are created. The method used to date for the construction of shaft is the shaft sinking (drill and blast), however over the past recent years, the mechanical excavation has become remarkably common, especially in underground mining (Rostami 2011). Depths up to 1,000m can be achieved through the conventional method whereas deep shafts up to 2,000m can be reached through mechanical method. The mechanical method has besides several advantages over the conventional method such as (Bullock 2011):

- Improved personal safety;
- Minimal ground disturbance;
- Less material to move;
- Uniform muck size;
- Continuous operations;
- Conducive to automation of system;
- Higher production rates.

Research into shaft construction is now becoming of increasing importance. Because ore bodies are becoming depleted, shafts are being constructed ever deeper and in ever more challenging geologies such as weak rocks. Studies carried out on these rocks show a rock intumescent, which can easily be damaged (Guo et al. 2012). Stress testing and analysis of structural geology must be taken into account for a correct mechanical characterization of the rock. Design methods for shaft lining are therefore very important. Jia et al. (2013) described the case of the Boulby Potash Mine where shaft linings installed in Marl suffered considerable radial pressure from the Marl stratum together with vertical compression from the upper part of the shafts resulting from subsidence of the host surrounding rock.

The main objectives of the linings are to maintain stability and preserve the ability of stress-carrying rocks near the boundaries of underground excavations. Different types of reinforcement may be used for this purpose: shotcrete, mesh, steel, concrete sets, and reinforced concrete linings. In literature many design methods for shafts exist. The calculation of the lining thickness for circular wells is based on the assumption that the pressure on the contact rock-lining is known (Öztürk and Ünal 2001). In general, according to Wong and Kaiser (1988), the design of a shaft consists of:

1. The design of the shaft lining to prevent instability of shaft wall and;
2. Estimation of the soil movement associated with shaft construction.

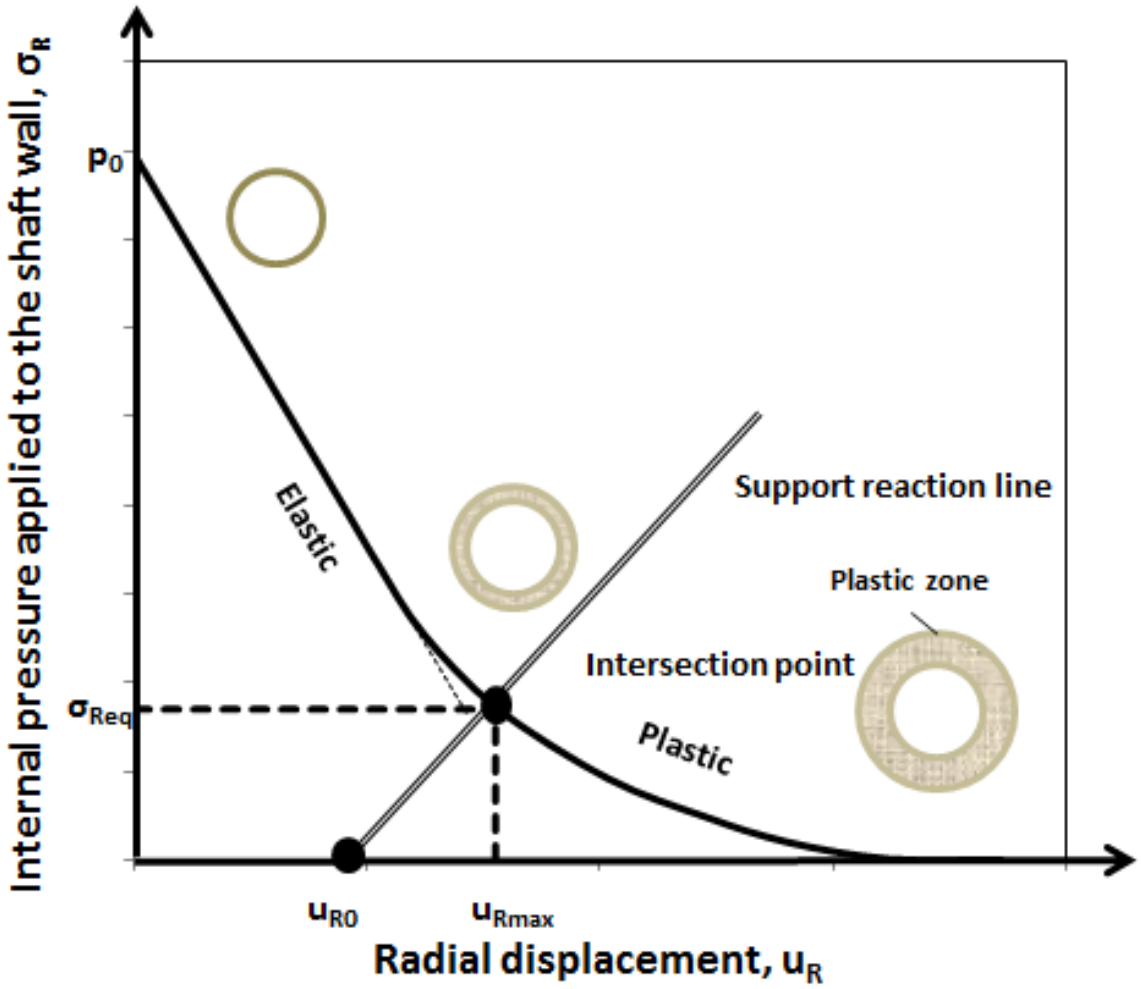
Although these two tasks are interrelated, they are usually handled separately. According to Wong and Kaiser (1988) many design approaches are based on soil plasticity considering the stress redistribution around a circular shaft opening (see Terzaghi 1943). Berezantzev (1958) used the Mohr-Coulomb failure criterion as a condition of plastic equilibrium and also made the assumption of equal principal stresses to render the problem statically determinate. The yield zone considered by Berezantzev (1958) is bounded by Rankine slip lines. Alternatively Coulomb-type analysis with a conical sliding surface are also used (e.g. Pratere 1977). Both shaft design model consider gravity or the influence of the vertical principal stress. Another method is the one treating a shaft as a two dimensional *hole-in-a-plate* model in order to calculate the extent of yielding, the equilibrium support pressure, and the related deformations for circular openings in a uniform stress field and in perfectly plastic or strain-weakening ground (e.g. Abel et al. 1979; Brown et al. 1986). The actually expected and observed pressures depend, however, on such factors as ground deformation, in situ stress, and ground strength-deformation properties. However, as stated by Wong and Kaiser (1988) the methods described above describe the actual shaft behavior. These methods do not indicate when gravity effects are relevant or when the limits of applicability of the "hole-in-a-plate" approach have been reached. McCreath (1980) found that the convergence-

confinement method (CCM) could be applied to explain the shaft performance of deep shaft in yielding rocks.

In this research the main analytical methods used in the calculation of the loads acting on the shaft lining and two-dimensional numerical models of the cross section and longitudinal section (axisymmetric modelling) are presented. The comparison of the calculation results for a specific case will permit to obtain useful indications on the optimal calculation technique in order to correctly assess the radial load on the lining and to design it.

The convergence-confinement method and the equation of Vlachopoulos and Diederichs for the evaluation of the radial displacement profile

The CCM method is one of the most commonly used analytical methods in the field of tunneling (Fig. 1). It permits to analyze the stresses and strains that develop around a deep circular cavity. The extent of the yield (or plastic) zone can also be estimated by this method under well-defined conditions (e.g. Fenner 1939; Pacher 1964; Rechsteiner and Lombardi 1974). This method requires the intersection of the convergence-confinement (CC) curve with the support reaction line. The value of the radial stress acting at the extrados of the support structure is an important result that can be obtained (Oreste 2005a; 2005b). The CCM was adopted and proposed by Wong and Kaiser (1988) as a rational approach to predict shaft behavior. In this manner in situ stress, rock strength, and deformation properties as well as many construction details can be included in the analysis.



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106 Fig. 1 Convergence-confinement curve of an underground opening and the reaction line of
107 the support (modified after Vlachopoulos and Diederichs 2009). Key: p_0 is the lithostatic
108 stress, u_{R0} is the radial wall displacement in the point along the cavity axis where the lining is
109 constructed, u_{Rmax} is the final radial wall displacement of the cavity, σ_{Req} is the final radial load
110 on the lining.

111 The assumptions used in the development of the CCM are:

112 • Circular and deep shaft;
113 • Homogeneous and isotropic rock around the shaft;
114 • Isotropic lithostatic stress around the shaft, with the horizontal lithostatic stresses
115 equal in the two main directions in the case of a vertical shaft (e.g. Panet 1995;
116 Oreste 2009a).

In order to proceed with the correct evaluation of the load acting on the support with the CC curve, it is necessary to know the radial displacements of the shaft walls at the moment in which the support structure is installed (u_{R0}). The formulation presented by Vlachopoulos and Diederichs (2009), which allows an estimation of the radial displacements in function of the final radial displacement of the shaft, at a long distance from the excavation face, appears to be particularly interesting. This equation allows estimating the radial movement u_{R0} of the shaft wall where the support structure of the shaft is installed (generally near to the excavation bottom). For Vlachopoulos and Diederichs (2009) u_R is described by the following equation in function of the distance x from the shaft bottom:

$$\frac{u_R}{u_{Rmax}} = 1 - \left(1 - \frac{1}{3} \cdot e^{-0.15 \cdot \frac{R_{pl}}{R}}\right) \cdot e^{\left[\frac{-3 \cdot x}{2 \cdot R_{pl}}\right]} \quad (1)$$

Where: u_{Rmax} is the maximum radial displacement of the shaft (for very elevated x);

R_{pl} is the final plastic radius of the shaft (for very elevated x);

The iterative procedure to design the linings using the convergence confinement method

In order to insert the reaction line of the support in the CCM, the formulation of Vlachopoulos and Diederichs (2009) can be used; this formulation permits to estimate the wall displacement, u_{R0}, of the shaft at the point where the supports are installed. Because the radial displacement and the plastic radius at a great distance from the excavation bottom are influenced by u_{R0} (which is unknown) an iterative procedure can be adopted (Oreste 2009b) This procedure involves the following steps:

1. Inserting the reaction line in the graph of the curve obtained by the CCM (Fig. 1), initially considering $u_{R0} = 0$; the support reaction line has a slope given by its stiffness k_{sup} (Hoek and Brown 1980):

$$k_{sup} = \frac{E_{sup}}{(1+\nu_{sup})} \cdot \frac{R^2 - (R - t_{sup})^2}{(1-2 \cdot \nu_{sup}) \cdot R^2 + (R - t_{sup})^2} \cdot \frac{1}{R} \quad (2)$$

where: R is the shaft radius;

t_{sup} is the lining thickness;

E_{sup} e ν_{sup} are the elastic modulus and Poisson ratio of the lining material respectively.

2. Determination of the intersection point of the convergence-confinement curve with the reaction line of the lining, obtaining the first value of the final shaft wall displacement (u_{Rmax}) and determining the final pressure applied to the lining (σ_{Req}), at a great distance from the temporary bottom.
3. Calculation of the plastic radius, R_{pl}, which develops at a great distance from the temporary excavation bottom, starting from σ_R as determined above.
4. Estimation of u_{R0} through Equation 1 knowing u_{Rmax} and R_{pl} obtained from steps 2 and 3. The distance from the shaft bottom, x, is replaced by the actual distance from the temporary shaft bottom, where the support is installed.
5. The new value of u_{R0} is used to re-calculate the position of the reaction line, by repeating steps 2 to 4.

The iterative procedure rapidly converges and can be stopped when the difference between two consecutive values of u_{R0} or σ_{Req} is below a certain predetermined tolerance. After this procedure it is possible to obtain in the convergence-confinement curve graph, the position of the reaction line according to the equation of Vlachopoulos and Diederichs (2009). The intersection between the two curves allows calculating both the final displacement of the shaft wall and the final radial load acting on the lining. This in turn, quickly permits the verification about the ability of the lining to sustain the rock pressure, without using any numerical model. This approach could be useful in the determination of the shaft lining type and consistency, until reaching a final configuration that guarantees not only shaft walls stability but also the optimization of the available economic resources.

Numerical simulations of the shaft installation

The shaft installation can be studied in great detail by the two-dimensional numerical modeling when the lithostatic state has equivalent horizontal stress in the two main directions and when the section of the shaft is assumed to be circular. In fact, in this case, it is possible to study the cross section with a plane model (plane strain state condition) and the longitudinal section (vertical section) with an axisymmetric model. The combined use of these two numerical models permits to have all the necessary information on the stress and strain developing in the rock mass around the shaft and in the lining.

This paragraph analyzes, with the combined use of the two-dimensional modeling, the stress and strain at the shaft contour considering a circular section with 3m radius. Assuming the Mohr-Coulomb strength criterion and an ideal elastic-plastic behavior, the following mechanical properties of the rock mass were considered in the studied case:

- cohesion (c): 0.9MPa;
- friction angle (φ): 31°;
- elastic modulus (E): 8000MPa;
- Poisson ratio (ν): 0.3.

Lithostatic stress was assumed to be equal to 15MPa, in both vertical and in two main horizontal directions (isotropic stress conditions). This stress state refers to conditions that may be encountered at a depth of about 650-700m.

The numerical models were developed with the numerical code FLAC 2D v.6.0 (Flac 2008), which uses a finite differences numerical solution. The two-dimensional numerical model of the plane section considers 2745 quadrilateral elements and represents a rock portion at the shaft contour up to the distance of 60m (Fig. 2).

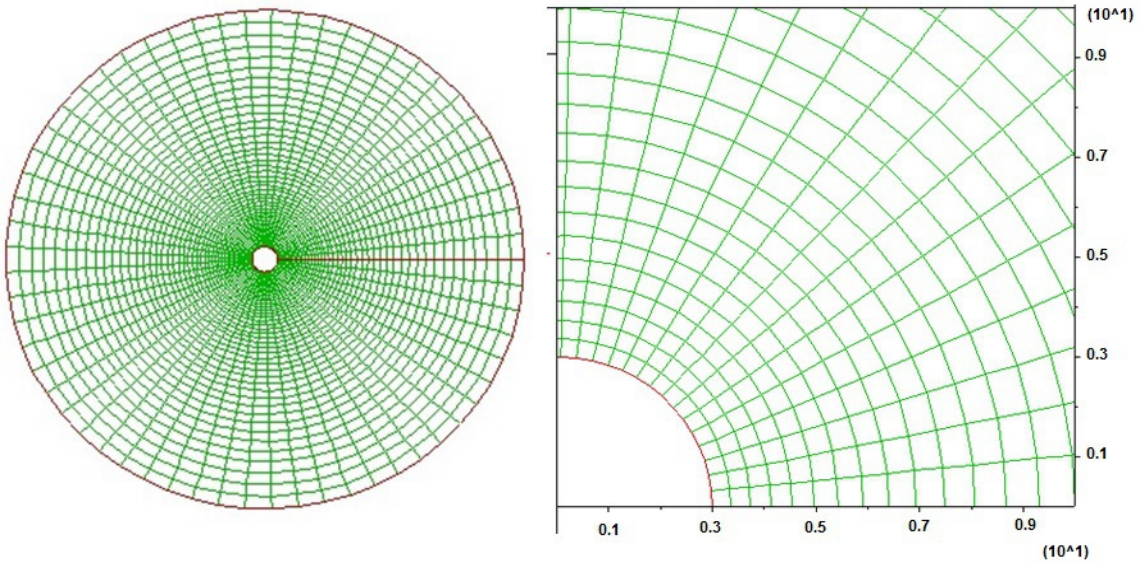
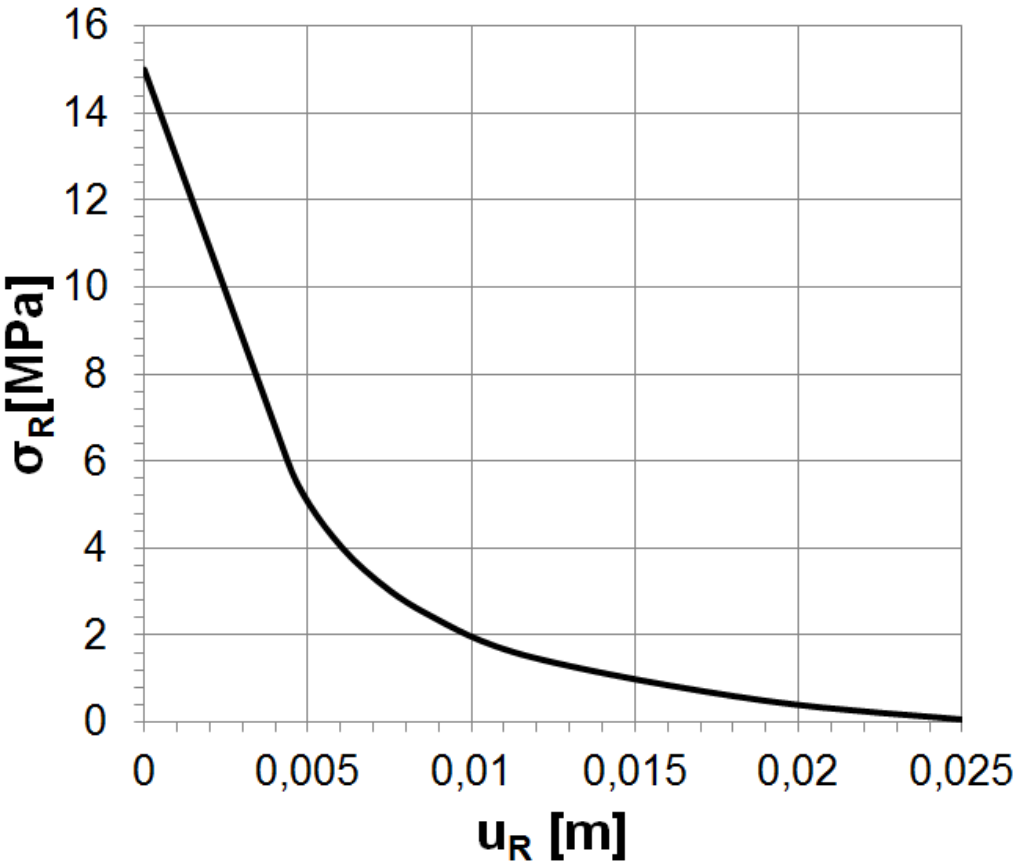


Fig. 2. Two-dimensional numerical model of the plane section. Left: the whole model; Right: a detail of the zone close to the shaft wall.

The internal pressure at shaft has been reduced from the lithostatic value of 15MPa to 0MPa, for 0.5MPa intervals (to simulate the excavation process) in order to obtain a detailed relationship between the radial displacement of the shaft wall and the internal pressure (Fig. 3).



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198 Fig 3. Relationship between the radial displacement of the shaft wall u_R and the internal

199 pressure for the studied case σ_R obtained by the numerical modeling of the cross-section

200

201 The axisymmetric numerical model simulated a vertical longitudinal section of half shaft, for

202 48m depth and 24m width (i.e. 16 times the shaft radius) (Fig. 4). The quadrilateral elements

203 used for the analysis (about 12,800 items) have dimension 0.3m side and square in shape.

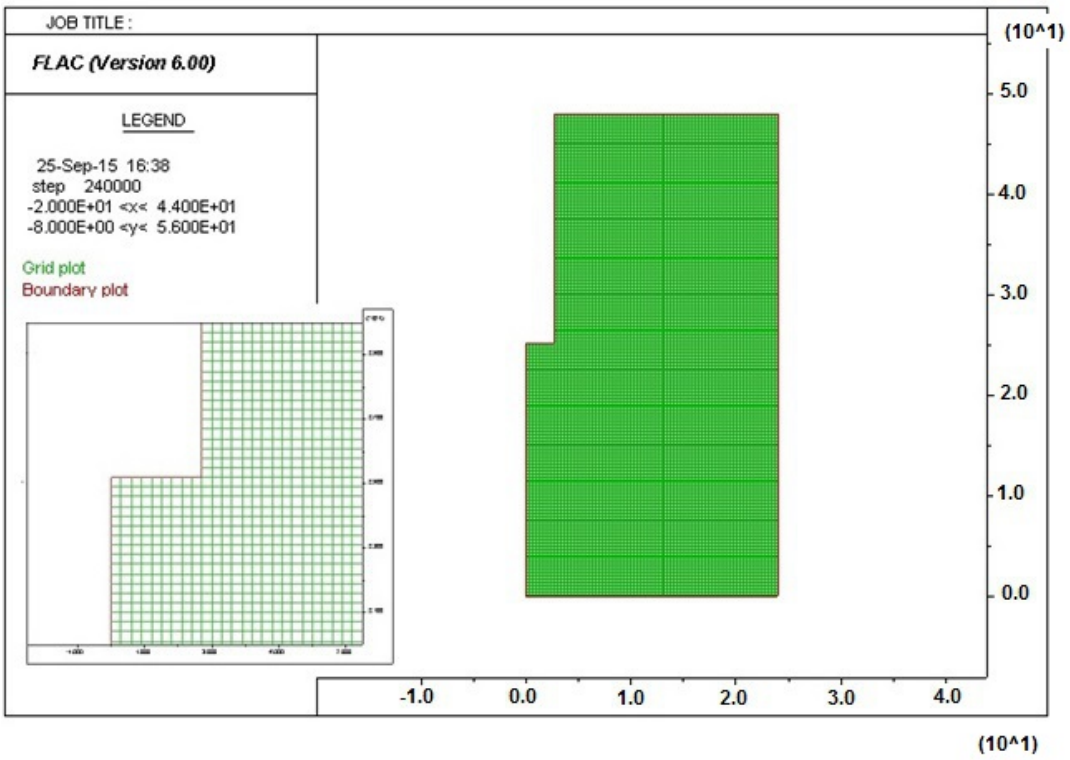

Fig 4. Element mesh of the axisymmetric numerical model of the half shaft longitudinal section: the global view and a detail of the zone of the model close to the temporary shaft bottom.

Through the axisymmetric numerical model, it was possible to simulate both excavation operations and lining installation, along the entire depth of the model, proceeding from the upper edge until reaching the bottom edge. Different excavation and support step lengths were considered: 0.3, 0.6 and 1.2m.

The lining was considered immediately active after the completion of the excavation. The excavation was simulated through the cancellation of model elements in the shaft zone. The lining considered in the calculation consists of concrete (with E=25000MPa and ν=0.15) and it was simulated by the same elements of the numerical model, reactivated after the cancellation with the mechanical characteristics of the concrete and with a zero initial stress state.

The situation obtained by the calculation with the bottom of the excavation positioned at half depth of the model, was analyzed with great detail. It was possible to detect the trend of the radial displacement of the wall shaft, obtained by the numerical calculation at different dis-

tances from the excavation bottom. This trend seems to be of great interest because it represents the deformation condition of the shaft in the radial direction, with the presence of lining in the section already excavated (Fig. 5). From the analysis shown in Fig. 5 it is possible to observe how the radial displacement from the shaft wall is less influenced by the excavation steps, s. In addition to the conditions of deformation, also the plastic zones, observed during the calculation, were analyzed: more specifically a final plastic radius of 3.9 m was observed for each analyzed excavation step (Fig. 6).

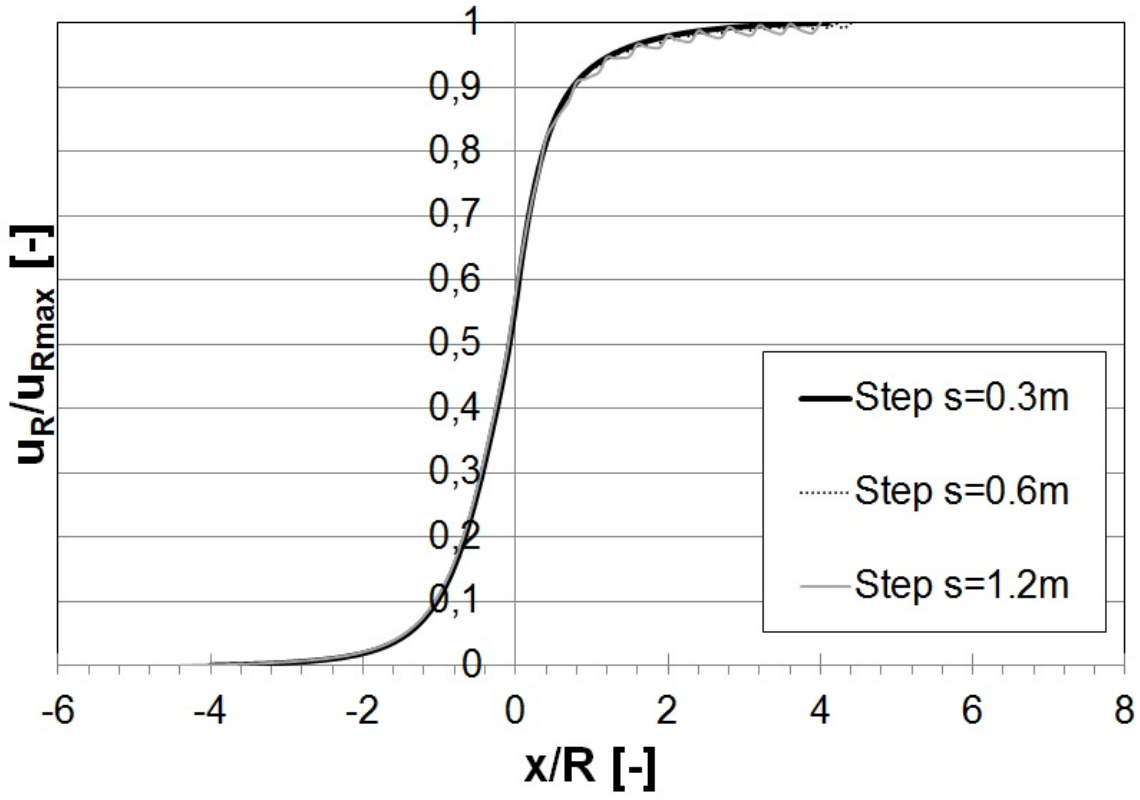

Fig 5 Deformation condition of the shaft in the radial direction, with the presence of lining, in the section already excavated with different step values, s. Key: u_R is the radial displacement of the shaft wall, u_{Rmax} is the maximum radial displacement at a great distance from the shaft bottom, R is the shaft radius.

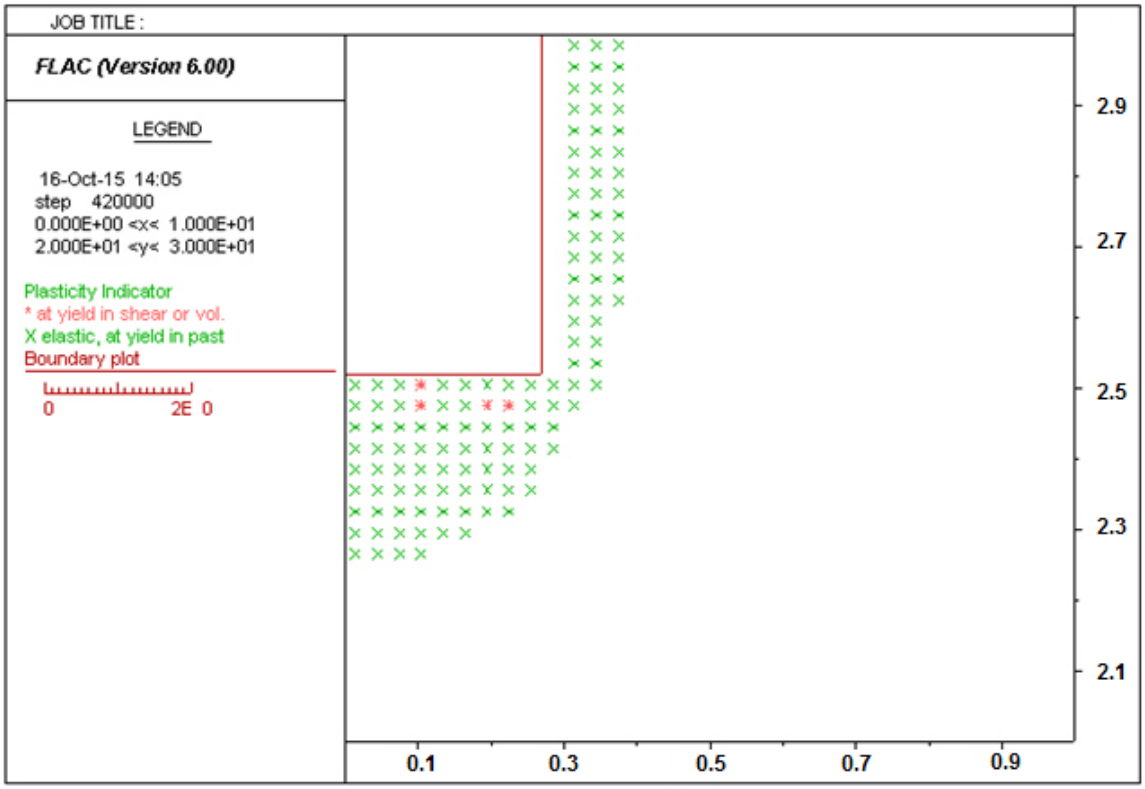

Fig 6. Plastic zones around the shaft and below the shaft bottom during the axisymmetric numerical calculation: a final plastic radius R_{pl} in the interval between 3.75 m and 4.05 m can be estimated by the results.

Fig. 7 shows the comparison of the curves shown in Fig. 5 (for s = 0.6m) and the result from the simplified excavation simulation, obtained by instantaneously eliminating all the elements until reaching the half of the model, with the simultaneous activation of the support along the excavated section. The instantaneous shaft excavation for half depth, without simulating the excavation and support installation steps, performed in the previous analyses, was therefore simulated. In Fig. 7 also the radial displacements of the shaft walls obtained using the equation of Vlachoupoulos and Diederichs (2009) are shown, considering the displacement u_{Rmax} and two different values of the final plastic radius R_{pl}, all obtained by the numerical simulation. The two extreme values of plastic radius (i.e. 3.75 and 4.05m) obtained by the numerical modelling, were considered. Analyzing the data of Fig. 7 we can observe:

1. The simplified shaft excavation and support simulation ("LDP simplified" curve in Fig. 7) shows a trend for the ratio between radial displacement u_R and the maximum displacement u_{Rmax}, which practically corresponds to the trend obtained with the procedure considering both excavation and support installation by steps (step by step procedure: "LDP s=0.6" curve).
2. The equation of Vlachoupulos and Diederichs (2009), which is very widespread in geomechanical design practice, for R_{pl} 3.75 (black continuous line) and R_{pl} 4.05 (grey continuous line) creates radial displacements of shaft walls, which are different from the ones obtained by the numerical axisymmetric calculation, even if in the Vlachoupulos and Diederichs equation (2009), values of u_{Rmax} and R_{pl} obtained by the numerical simulation, are used;
3. In the numerical simulation, the radial wall displacement in correspondence of the temporary excavation bottom (where the support is activated) is about 60% of the final displacement; for Vlachoupulos and Diederichs (2009) this value is about 28%, i.e. near the half obtained by the numerical simulation.

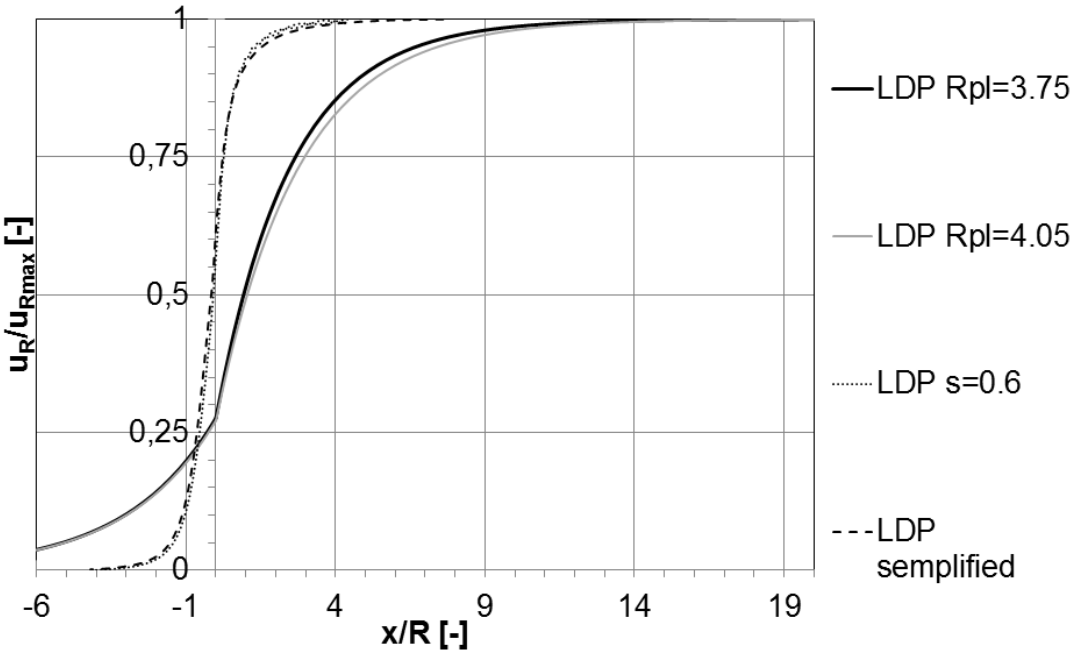

Fig. 7. Comparison of the trends of radial displacements of shaft wall with the distance x from the temporary shaft bottom, obtained by the numerical simulation and Vlachoupulos and

Diederichs formulation. Key: LDP: longitudinal displacement profile; LDP R_{pl} 3.75: displacement profile obtained by the Vlachoupulos and Diederichs formulation considering a plastic radius of 3.75 m; LDP R_{pl} 4.05: displacement profile obtained by the Vlachoupulos and Diederichs formulation considering a plastic radius of 4.05 m; LDP s=0.6: displacement profile obtained by the step by step axisymmetric numerical calculation for a step length of 0.6m; LDP simplified: displacement profile obtained by the simplified axisymmetric numerical calculation.

Comparison between the analytical iterative procedure and the numerical simulation

The relation between the applied internal pressure and the radial displacement of the shaft wall, obtained by the numerical simulation of the cross-section (see Fig. 3) was compared with the CC curve of the circular cavity obtained by the CCM (Fig. 8). It is possible to observe a very good match between the two methods. The CCM represents therefore an interesting alternative tool to the numerical modeling of the shaft cross-section. The CCM permits to quickly calculate the CC curve of the circular cavity, in comparison with the numerical modelling.

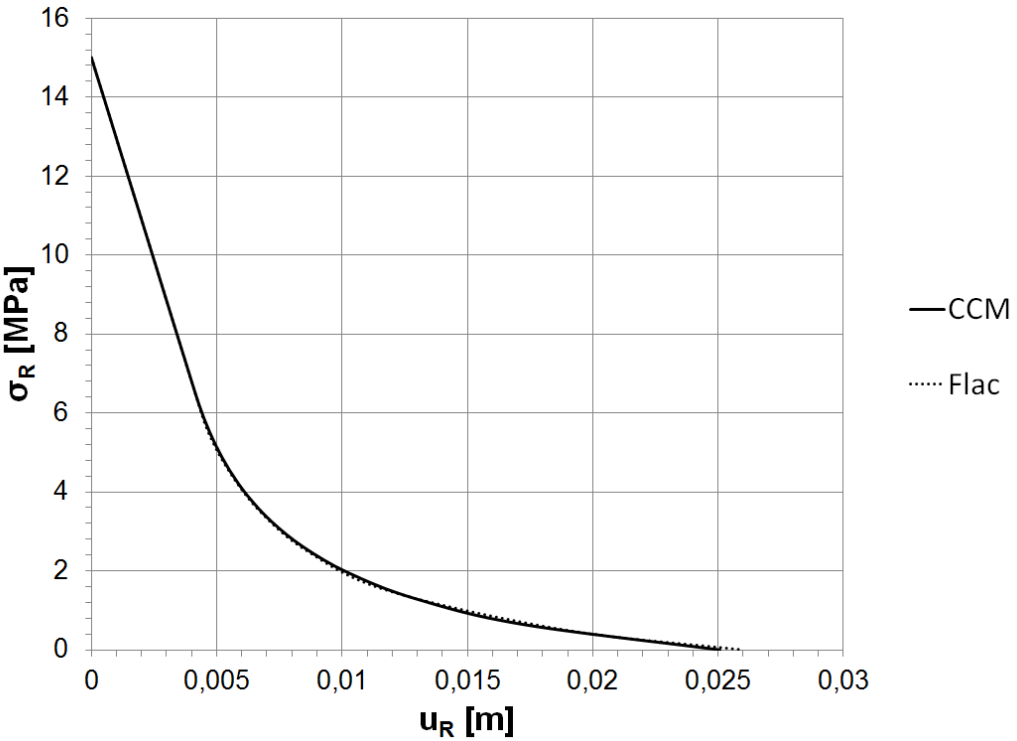

Fig. 8. Comparison of the applied internal pressure-radial displacement of the wall shaft curve, obtained with FLAC numerical modelling (dotted line) and with the CCM (continuous line).

In the CC graph, the reaction line of the support has been inserted as previously described in paragraph 3 (see Fig. 9). The results coming from the iterative procedure give an initial displacement, u_{R0}, of 1.7mm and a final radial stress, σ_{Req}, of 4MPa. In Fig. 9 the reaction lines of the support obtained from the u_{R0} values of the axisymmetric models for the following conditions are also shown:

- excavation and support installation simulation for steps with length of 0.3, 0.6 and 1.2m (step by step procedure);
- Simplified excavation and support installation simulation ("Flac simplified" reaction line);

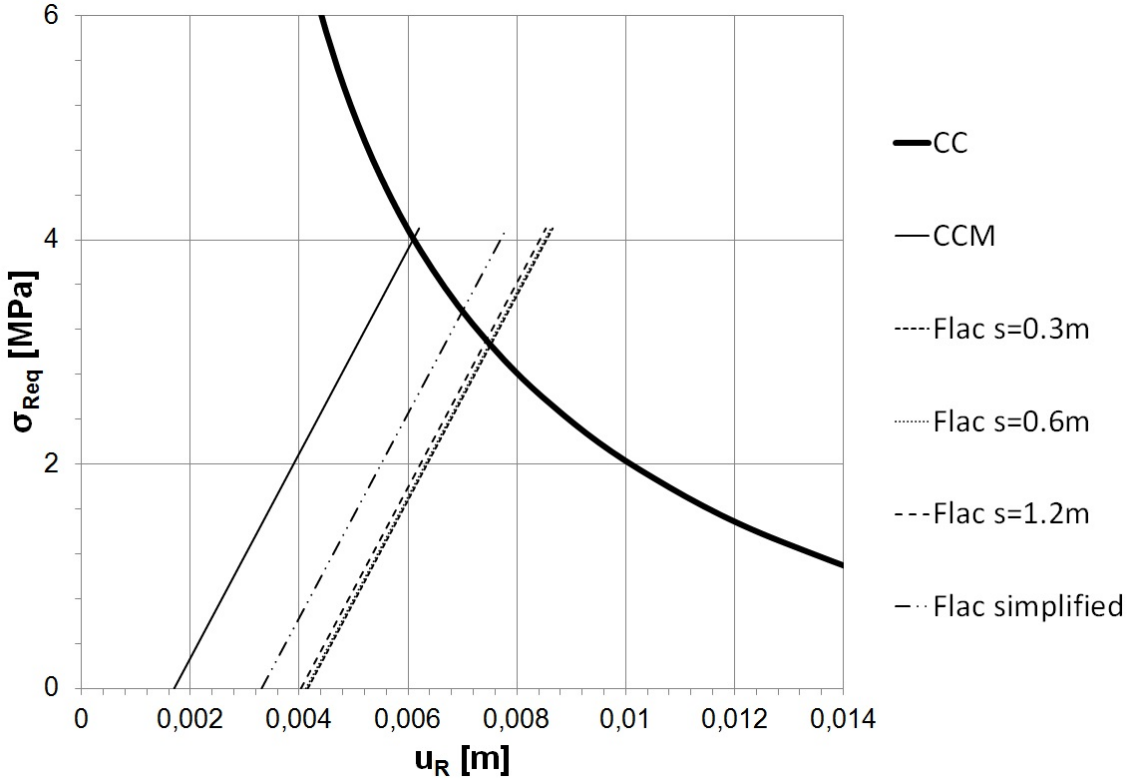

Fig. 9. Comparison of the support reaction lines obtained with different calculation procedures. Key: CC: convergence-confinement curve obtained by CCM; CCM: support reaction line located on the basis of the iterative procedure of section 3; "Flac s=0.3 m": support reac-

tion line located on the basis of step by step procedure using the axisymmetric numerical model and a step length of 0.3 m; “Flac s=0.6 m”: step length of 0.6 m; “Flac s=1.2 m”: step length of 1.2 m; “Flac simplified”: support reaction line located on the basis of the simplified procedure using the axisymmetric numerical model.

Comparing the data it is possible to observe how the iterative procedure previously described in paragraph 3 gives a u_{R0} value less than 54% and a σ_{Req} value higher than 30% with respect to the three analyzed cases with the axisymmetric numerical model and by considering the excavation step by step ($u_{R0} \cong 4.1$mm; $\sigma_{Req} \cong 3.1$MPa). The different excavation step values do not appreciably influence u_{R0} neither does the final stress on the lining, σ_R. The simplified axisymmetric numerical simulation give intermediate values of u_{R0} and σ_R with respect to those described above. The value of u_{R0} is in fact less than 20% and σ_R is higher than 10% if compared with the step by step simulations. Regarding the final plastic radius, considered at a long distance from the temporary excavation bottom, we can observe how the value obtained from the analytical iterative procedure is about 8% less than the one obtained by the axisymmetric numerical simulations (Fig. 10). The value of the plastic radius calculated by the step by step axisymmetric numerical simulation agrees with the one obtained by the relation R_{pl}-σ_{Req} calculated by means of the CCM. In this case, too, the final plastic radius is 2% smaller than the one obtained by the step by step axisymmetric numerical simulation.

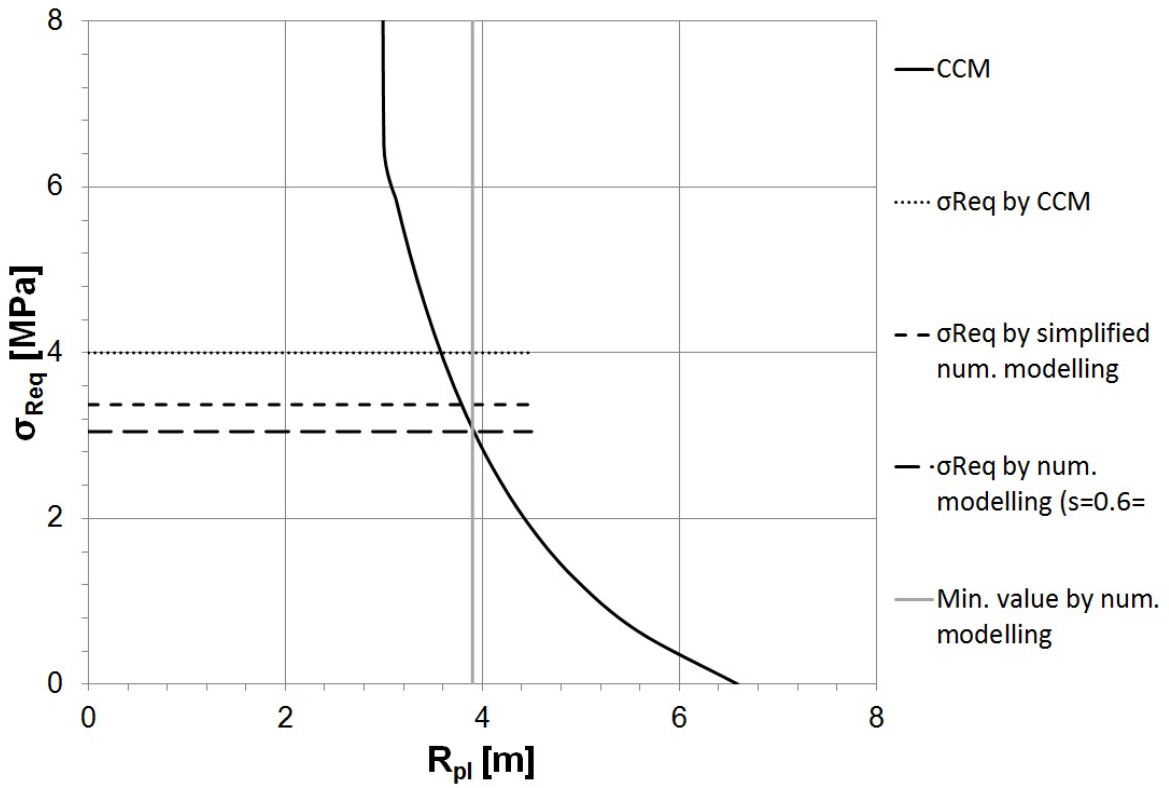

Fig. 10. Calculation of the final plastic radius R_{pl} obtained with different calculation procedures. Key: "CCM": plastic radius vs internal pressure curve obtained by CCM; "σ_{Req} by-CCM": plastic radius obtained by the internal pressure derived by the analytical iterative procedure; "σ_{Req} by simplified num. modelling": plastic radius obtained by the internal pressure derived by the simplified axisimmetric numerical model; "σ_{Req} by simplified num. modelling (s=0.6)": plastic radius obtained by the internal pressure derived by the step by step axisimmetric numerical model; "Min. value by num. modelling": the plastic radius value obtained by the axisymmetric numerical analyses.

Conclusions

The design of the shaft lining is a very important stage in the shaft design. From the results coming from the research presented in the paper, the CCM seem to be an interesting tool for describing the relation between the applied internal stress and the radial displacement of the shaft walls. The CCM could be an alternative calculation method to the 2D numerical model for the horizontal cross-section of the shaft.

The simplified iterative procedure, used for positioning the reaction line of the support on the characteristic curve graph of the circular cavity, gives, however, final stress values on the lining higher (about 30%) with respect to the values obtained with the axisymmetric numerical model step by step, i.e. simulating the progressive shaft excavation and support installation. For this reason, a step by step axisymmetric numerical model, in order to obtain the radial displacement of the shaft wall (u_{R0}) in the zone where the lining is installed, was developed. The simplified axisymmetric numerical model, although it correctly estimates the relation between u_{R0} and u_{max}, it over-estimates the value of the final stress on the lining, giving higher value than the one obtained with the step by step axisymmetric numerical model.

The combined use of the CCM and the step by step axisymmetric numerical model gave very good estimation of the final stress on the lining and final plastic radius values, both of them necessary to correctly design the support structure of the shaft, in order to guarantee its stability.

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Figure caption

Fig. 1 Convergence-confinement curve of an underground opening and the reaction line of the support (modified after [19]). Key: p_0 is the lithostatic stress, u_{R0} is the radial wall displacement in the point along the cavity axis where the lining is constructed, u_{Rmax} is the final radial wall displacement of the cavity, σ_{Req} is the final radial load on the lining.

Fig. 2. Two-dimensional numerical model of the plane section. Left: the whole model; Right: a detail of the zone close to the shaft wall.

Fig 3. Relationship between the radial displacement of the shaft wall u_R and the internal pressure for the studied case σ_R obtained by the numerical modeling of the cross-section

Fig 4. Element mesh of the axisymmetric numerical model of the half shaft longitudinal section: the global view and a detail of the zone of the model close to the temporary shaft bottom.

Fig 5 Deformation condition of the shaft in the radial direction, with the presence of lining, in the section already excavated with different step values, s. Key: u_R is the radial displacement of the shaft wall, u_{Rmax} is the maximum radial displacement at a great distance from the shaft bottom, R is the shaft radius.

Fig 6. Plastic zones around the shaft and below the shaft bottom during the axisymmetric numerical calculation: a final plastic radius R_{pl} in the interval between 3.75 m and 4.05 m can be estimated by the results.

Fig. 7. Comparison of the trends of radial displacements of shaft wall with the distance x from the temporary shaft bottom, obtained by the numerical simulation and Vlachoupulos and Diederichs formulation. Key: LDP: longitudinal displacement profile; LDP R_{pl} 3.75: displacement profile obtained by the Vlachoupulos and Diederichs formulation considering a plastic radius of 3.75 m; LDP R_{pl} 4.05: displacement profile obtained by the Vlachoupulos and Diederichs formulation considering a plastic radius of 4.05 m; LDP s=0.6: displacement profile obtained by the step by step axisymmetric numerical calculation for a step length of 0.6m;

LDP simplified: displacement profile obtained by the simplified axisymmetric numerical calculation.

Fig. 8. Comparison of the applied internal pressure-radial displacement of the wall shaft curve, obtained with FLAC numerical modelling (dotted line) and with the CCM (continuous line).

Fig. 9. Comparison of the support reaction lines obtained with different calculation procedures. Key: CC: convergence-confinement curve obtained by CCM; CCM: support reaction line located on the basis of the iterative procedure of section 3; “Flac s=0.3 m”: support reaction line located on the basis of step by step procedure using the axisymmetric numerical model and a step length of 0.3 m; “Flac s=0.6 m”: step length of 0.6 m; “Flac s=1.2 m”: step length of 1.2 m; “Flac simplified”: support reaction line located on the basis of the semplified procedure using the axisymmetric numerical model.

Fig. 10. Calculation of the final plastic radius R_{pl} obtained with different calculation procedures. Key: “CCM”: plastic radius vs internal pressure curve obtained by CCM; “σ_{Req} by-CCM”: plastic radius obtained by the internal pressure derived by the analytical iterative procedure; “σ_{Req} by simplified num. modelling”: plastic radius obtained by the internal pressure derived by the simplified axisimmetric numerical model; “σ_{Req} by simplified num. modelling (s=0.6)”: plastic radius obtained by the internal pressure derived by the step by step axisimmetric numerical model; “Min. value by num. modelling”: the plastic radius value obtained by the axisymmetric numerical analyses.