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Effect of rim and web interaction on crack propagation paths in gears by means of XFEM technique

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Abstract

Object of this work is to investigate the correlation between rim and web thickness on the crack propagation path in thin rim gears, referring to bending failures.

To this aim, numerical simulations have been performed, based on the 3D extended finite element method (XFEM).

Results related to gear models with different web and rim thickness have been interpreted in ISO Standard environment, relating the crack path to the so called gear blank factor C_R, useful in cases of mating gears consisting of rims and webs.

Results show that the interaction between web and rim thickness may influence the crack propagation and the corresponding safe or catastrophic failure mode.

Keywords: fracture mechanics; crack path; crack propagation, gears, XFEM

NOMENCLATURE

b: face width;

C_R: ISO blank factor;

F: force applied on the tooth at HPSTC;

m_n: normal gear modulus;

M_R: backup ratio;

M_W: web ratio;

t_R: rim thickness;

t_W: web thickness;

1. Introduction

One of the major causes of gear failure is the maximum tensile stress at the tooth root due to the bending fatigue.

Once initiated, generally on the tensile side, gears tooth root cracks may propagate in different ways, following different path directions. In particular in thin-rimmed gears, such as in aerospace applications, the direction of the crack propagation has to be seriously taken into account and, if possible, already predicted in the design phase.

As a matter of fact, cracks nucleating at the tooth root may propagate in two main directions: across the tooth, causing the whole tooth detachment, or across the rim, causing the detachment of big portions of the gear [1], [2]. The first failure mode may be classified as “fail safe” because the consequences of this breakage may be not so critical,

while the second one may be identified as “catastrophic failure”, because of the projection of big parts of the wheel may cause serious damage to the entire gearbox [3].
Crack path is influenced by many factors that may be classified in geometrical factors (i.e. rim thickness, web thickness, gear configuration), load factors (i.e. centrifugal load effect), and crack nucleation parameters (i.e. initial crack position and orientation).

In literature it is possible to find many studies dealing with the gear bending fatigue crack propagation but, if the topic concerns the thin-rimmed gear behavior, most of works originates from the researches of Lewicki et al. [1, 3, 4, 8, 11]. Particularly focused on geometrical parameters, Lewicki et al. deeply investigated the effect of rim thickness from both experimental and numerical point of view, considering slotted gears [3], [4], and relating the rim thickness to the tooth height and introducing the so called “backup ratio”.

On the basis of Lewicki works [1, 3, 4, 8, 11], it is possible to define three geometrical regions involving different bending fatigue behaviors: the first region, referring to gears with backup ratio values higher than 1 (following the definition given by the NASA [3]), identifies a crack propagation through the tooth, the second one refers to a crack propagation through the rim and involves backup ratio values lower than 0.5, the last one identifies an intermediate region, with backup ratio between 0.5 and 1, where the propagation path is not so defined, depending on different factors.

Some other papers deal with fatigue cracks in gears. As an example, Lalonde et al. [5] investigated by boundary element simulations the effect of six parameters (teeth number, speed, rim thickness, initial crack length, initial crack orientation and relative fillet position) influencing the crack path propagation and they predicted the crack paths by means of the factorial design approach.

Kramberger et al. [6] investigated by both finite element and boundary element methods the effect of rim thickness on bending fatigue life of a thin rim spur gear for truck gearbox. Pehan et al. [7] considered the effect of moving gear tooth load on the crack initiation life.

The most of the literature in crack propagation of thin rim gears used 2D models; only few works are available about 3D models. As some examples, Lewicki et al. [8] investigated crack growth in a split-tooth gear by means of 3D boundary elements models, Ural et al. [9] predicted crack shape and fatigue life of a spiral bevel pinion by 3D finite element method and Amiri et al. [10] calculated the fatigue life of a helical gear by means of the extended finite element method (XFEM).

The effect of centrifugal load has been also investigated by Lewicki at al. [11] from both experimental and numerical point of view.

Generally speaking it is possible, by analyzing the results presented in the literature, to point out that the global tooth stiffness may influence the crack propagation paths, being the global tooth stiffness identified as the sum of tooth, rim and web stiffness.

The main influences affecting the tooth stiffness, if surface parameters (as roughness and waviness) and mesh misalignment are disregarded, are those referred to both material (modulus of elasticity) and geometry. In particular, tooth data (number of teeth, addendum, dedendum, ⋯) and blank design (rim and web thickness) have to be carefully taken into account.

Considering geometrical parameters, the rim stiffness is mainly influenced by both rim thickness and face width, web stiffness by the web thickness and, finally, the tooth stiffness by module and face width.

As described above, the effect of rim stiffness, related to the tooth stiffness (backup ratio) has been widely explored in the literature, while the effect of the web thickness has been investigated only by Lewicki [1]. Moreover, the correlation between these two parameters has not been so emphasized.

Object of this work is to investigate the correlation between rim and web thickness on the crack propagation path in thin rim gears and the corresponding interaction in gear bending failures.

To this aim, numerical simulations have been performed, based on the extended finite elements method (XFEM) [12].

Results related to gear models with different web and rim thickness have been interpreted in ISO Standard environment [13], [14], relating the crack path to the so called gear blank factor C_R (additional correction factor with respect to the classical ones, referring to the tooth stiffness determination), useful in cases of mating gears consisting of rims and webs. For that reason, the results of this paper are presented referred to the module and not to the height of the tooth, so the reference value of backup ratio equal to 1 becomes 2.16 times the module, because the actual root fillet is considered.

2. Gear Models

Gear models have been chosen in order to emphasize the effect of both rim and web geometry. To do that, rim and web thickness has been normalized as regards to the module, so obtaining two parameters, respectively the backup ratio M_R and the web ratio M_W, expressed as:

$$M_R = \frac{t_R}{m_n} \tag{1}$$

$$M_W = \frac{t_W}{b} \tag{2}$$

where t_R is the rim thickness, t_W the web thickness, b is the face width and m_n is the normal gear module (see Fig. 1).

These two parameters are similar to those available in literature ([1], [3], [8]), where rim and web dimensions are referred to the tooth height.

General geometric parameters of the gear model are reported in Table 1.

On the basis of this gear geometry, twelve different configurations have been selected for simulations, involving different values of backup and wed ratios, and resumed in Table 2.

Here Fig. 1: gear geometric parameters.

Here Table 1: gear parameters.

In particular, the first column refers to the simulation number (test 1, 2, 3, ⋯), second and third columns report respectively backup and web ratio values as indicated in the present work (equations (1) and (2)), the last column shows the corresponding backup ratio (m_b) values referred to the tooth height, as indicated in literature ([1], [3], [8]).

Here Table 2: gear simulation parameters.

2.1 Numerical models

The crack propagation study presented in this work has been carried on by means of 3D extended finite elements (XFEM) models; in these models the crack initiation point has to be previously defined. In others words, the XFEM code requires an hypothesis allowing the crack creation and then its propagation.

In this paper the initial crack has been placed at the point in the tooth root fillet showing the maximum equivalent stress (Von Mises). This choice is due to the verification the authors made in [2]: the crack path is ruled by its initial position that is related to gear geometry. So the initial point in this research is set in the locus where maximum equivalent stress is reached. This point has been identified by a dedicated 2D FEM analysis; 2D FEM models have been realized by means of MSC Patran/Nastran software. Only a portion of the gear has been modeled in order to reduce the calculation time (Fig. 2). The global mesh has been realized with shell quad elements (four nodes) and the mesh near the tooth root has been further refined to have average element size of 0.1mm. To reproduce the different thicknesses of the wheel (rim, web, hub), the shell elements thickness has been varied according to the gear geometry (see Fig. 2).The load has been applied as a force on the highest point of single tooth contact (HPSTC) and boundary conditions have been imposed on the two sides of the gear slice in order to simulate the presence of the whole gear (see Fig. 2).

Here Fig. 2: 2D FEM model.

2.1.1 XFEM Method

Extended finite element method (XFEM) is a relatively new numerical technique that well fits problems related to the fracture mechanics. XFEM technique, developed by Moes et al. for 2D problems [15], consists in adding local displacement functions to the standard finite element model, to consider the jump in the displacements across the crack and the near-tip asymptotic.

XFEM technique allows the crack growth without re-meshing necessity but, in order to get accurate results, the mesh should be fine enough near the crack and around the crack tips [15].

XFEM has been also extended to study 3D problems. In particular the software Samcef [16] used in this work implements the method developed by Wyart et al. [16], [17] where the mesh is decomposed in FEM and XFEM domains. Concerning the generation of the XFEM problem in the associated domains, the computation of the stress intensity factors, the representation of the 3D cracks and its propagation, they are carried out according to the work of M. Duflot [16], fully described in [18].

2.1.2 XFEM Models

In this work 3D XFEM models have been used to investigate crack propagation; in particular 3D models have been used in place of 2D models because they allow to represent a more realistic failure cases (in 2D model the crack involve the whole tooth width, while in 3D models it is possible to place a crack on one side on the tooth width, as it happens in real cases). In addiction this paper is the first step of a deep investigation concerning the crack propagation in gear. The next step is considering also the helical gears, where the application point of the force varies during engagement emphasizing the tridimensional behavior of the crack propagation. As previously described, XFEM models have been created starting from a standard 3D finite element model where the crack propagation zone has to be defined and refined. In the refined zone, XFEM elements replace the traditional ones (Fig. 3).

In particular, XFEM domain has been created by defining three zones near the tooth root where the crack is hypothesized to propagate. These three zones are concentric and the elements in each zone have been refined with decreasing sizes: the external zone has 1.6mm average dimension, the middle zone 0.6mm and the internal one (where the initial crack is placed) 0.3mm average dimension. The initial crack has been chosen having an elliptic shape (0.1mm in the x direction and 0.25mm in the y direction, see Fig. 4) and placed at the point where the maximum equivalent stress is reached; in particular the maximum equivalent Von Mises stress has been previously calculated as explained in section 2.1, providing the corresponding point in which the initial elliptic crack has been located.

The initial crack is only numerically present, no defect (notches) are present inside the mesh.

Here Fig. 3: XFEM model with enriched zone.

The initial crack orientation is perpendicular to the tangent line to the tooth root fillet at the maximum equivalent Von Mises stress point (see Fig. 4).This positioning should be more representative of the real case and in authors experience the initial direction of the crack is not relevant for propagation [2].

The Paris’ law has been used as propagation law. Concerning crack propagation parameters, each simulation consists in 15 propagation steps, with an increment of crack length equal to 0.3mm each. These parameters have been selected in order to avoid the crack to propagate outside the refinement region.

All numerical models have been developed by assuming that the material is homogeneous and isotropic, without imperfections or damages.

The loading condition has been created by applying a force distributed on a line located at the highest point of the single tooth contact at HPSTC and the hub inner diameter has been clamped (Fig. 4).

Here Fig. 4: conceptual model: loads, boundary conditions and initial crack position.

3. Results

Results obtained in the present paper are resumed in the following, focusing on the use of XFEM technique and on the effect of geometrical parameters interaction on the crack propagation path. An attempt to explain the results in ISO Standard environment has also been drawn. An example of 3D XFEM result is shown in Fig. 5; in particular, in this Figure the propagated crack corresponding to Test 10 is emphasized (see Table 2).

Here Fig. 5: XFEM model with propagated crack.

Fig. 6 shows others examples of crack propagation referring test cases 1, 6, 7 and 12 (see Table 2).

Here Fig. 6: crack propagation for test cases 1 (a), 6 (b), 7 (c) and 12 (d).

For sake of clarity, in order to make easier the comprehension of propagation phenomena, all numerical results are resumed in Fig. 7.

Being the focus point of this work the watershed between safe fail and catastrophic failure, Fig. 7 at once emphasises this difference for as concerns the propagation path.

Here Fig. 7: effect of web and backup ratio on crack propagation path.

From the analysis of this Figure, it is possible to observe that, referring to the effect of the rim thickness, the results are aligned with those found in the literature: for backup ratio values lower than 1.08, the crack propagates through the rim and, for backup ratio values close to or bigger than 2.16, the crack goes through the tooth [1] (as stated before, in this work the module has been used as base parameter, so the correlation between actual backup ratio values and the corresponding available in literature is shown in Table 2). Concerning the web thickness, it is possible to observe that its effect is more evident for backup ratio values lower than 1 and it practically consists in shifting the crack path towards the rim. In particular, if the web ratio decreases (i. e. the web thickness decreases), the crack path tends more rapidly to propagate through the rim. This phenomenon may be overall appreciated by comparing the results obtained in gears with lowest backup ratio values (0.87 and 0.65), but with different web ratios (see Fig. 7).

A correlation between rim and web thickness may be noted by observing the obtained results.

In order to better highlight the influence of this interaction between geometrical parameters on crack propagation, the ISO gear blank factor C_R has been taken into account [13-14]. This parameter, related to the flexibility of gear rim and web, is defined as [13]:

$$C_R = 1 + \frac{\ln(M_w)}{5e^{(M_R/5)}} \tag{3}$$

where M_W is the web ratio (equation (1)) and M_R is the backup ratio (equation (2)).

Fig. 8 shows the curves obtained from equation (5), according to ISO 6336, representing the gear blank factor C_R as a function of the web ratio; the curve parameter is the backup ratio value.

It is possible to observe that the domain of these curves is limited in the lower region up to $M_W = 0.2$; when $M_W = 1$, corresponding to the solid disc gears.

Here Fig. 8: gear blank factor C_R according to ISO6336 [13].

Fig. 9, similar to Fig. 8, emphasizes the C_R curves involving both validity of equation (5) (ISO Standard zone) and curves related to gears analyzed in the present paper (M_R = 2.16, 1.08, 0.87 and 0.65). In the same plot, the points representing C_R values related to simulations (Tests 1 to 12, see Table 2) are represented, with a tag indicating the corresponding failure mode: [R] stands for rim failure and [T] stands for tooth failure.

Here Fig. 9: gear blank factor C_R obtained for the gears considered in this work ([R] = rim failure, [T] = tooth failure).

From the analysis of the obtained results, it is possible to highlight that safe failures involve gears with backup ratio M_R= 2.16 and, for the other cases, failures are catastrophic, involving the rim zone.

This phenomenon is in accordance with the literature [1], confirming that, for M_R values higher than 2.16, the crack propagates through the tooth (see Table 2).

Moreover, it is possible to observe (see Fig. 9) that the points representing gears with web ratio lower than 0.2 do not belong to the corresponding C_R curve (Tests from 5 to 12). This is reasonable because the validity of the ISO blank factor is defined only for web ratios higher than 0.2.

So, it may be possible to define a "safe region" in the C_R plot, where the crack propagates through the tooth. This region may be indicated as the zone over the C_R curve, calculated with M_R = 2.16.

A wider definition of the safe region may be done by considering the ISO C_R equation up to M_W = 0.2. For M_W values lower than 0.2, it may be considered the curve interpolating the C_R values related to tests 1, 5 and 9; and the value of C_R at M_W= 0.2 for the same M_R as tests 1, 5 and 9.

4. Conclusions

In the present paper the effect of the interaction between geometrical parameters of gears on the crack propagation path has been analyzed. This study has been performed by means of 3D XFEM technique, with principles of linear elastic fracture mechanics,

Crack propagation paths due to gear tooth bending fatigue have been predicted for a variety of backup and web ratio values, to establish design guidelines to prevent catastrophic failure modes.

Obtained results allowed us to draw the following conclusions.

Firstly, 3D XFEM technique was able to predict bending crack propagation paths for different gear geometries, related to the initial crack position along the face width. In this work, the initial crack has been placed at the point in the tooth root fillet showing the maximum equivalent stress (Von Mises), as indicated by practical cases or by experimental tests. Results available in literature have been confirmed by the present research activity.

The interaction between rim and web geometry has also been emphasized by means of the ISO blank factor C_R, making more clear the corresponding interpretation relating to safe or catastrophic failure modes.

Finally, rim and tooth failure zones have been determined also outside the ISO blank factor graph validity, providing a realistic design map for thin rimmed gears and for aerospace gears with particular web geometries.

Future work may be focused on better define the safe region limits also considering different gear configurations.

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