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# **Steady-State Analysis of Pollutant Transport To Assess Landfill Liner Performance**

by

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## **ABSTRACT**

New analytical solutions are presented for the assessment of the impact of contaminant transport through landfill liners on groundwater quality under steady-state conditions. These solutions can be applied to evaluate the equivalency and the effectiveness of landfill liners, even those that include a geomembrane, taking into account the presence of a natural attenuation layer interposed between the engineered barrier and the underlying aquifer. The impact of a contaminant on groundwater quality is quantified through the determination of the contaminant concentration along the horizontal direction of the groundwater flow for the case of thin aquifers that are only a few meters thick. For the case of thick aquifers, both a closed-form analytical solution and a step-by-step numerical solution are provided for the calculation of the variations in the contaminant concentration in horizontal and vertical directions within the aquifer.

**Keywords chosen from ICE Publishing list:** Groundwater, Numerical methods, Waste containment & disposal system

## List of notation

$\alpha_T$	is the transverse dispersivity within the aquifer
$\gamma, \Gamma, \eta, \eta_D, \eta', \eta'_D, \chi$	are dimensionless parameters of the analytical solutions
$\Delta h$	is the hydraulic head difference across the landfill bottom barrier
$\Delta X$	is the interval of the horizontal distance for the numerical solution
$\Delta Y'$	is the interval of the vertical distance for the numerical solution
$\theta$	is the volumetric water content
$\Theta$	is the transmissivity of the interface between the geomembrane and the underlying soil
$D_g$	is the diffusion coefficient of the geomembrane
$D_h$	is the hydrodynamic dispersion/diffusion coefficient of the landfill bottom barrier
$D_{h,x}$	is the hydrodynamic dispersion/diffusion coefficient in the horizontal direction within the aquifer
$D_{h,y}$	is the hydrodynamic dispersion/diffusion coefficient in the vertical direction within the aquifer
$D_{hi}$	is the hydrodynamic dispersion/diffusion coefficient of the i-th layer of the landfill bottom barrier
$J_s$	is the pollutant vertical mass flux coming from the landfill
$K_g$	is the partition coefficient between the geomembrane and the solute
$L$	is the thickness of the landfill bottom barrier
$L_g$	is the geomembrane thickness
$L_i$	is the thickness of the i-th layer of the barrier
$L_w$	is the length of the geomembrane wrinkle
$N_l$	is the number of mineral layers in the landfill bottom barrier
$N_X$	is the number of nodes in the horizontal direction for the numerical solution
$N_{Y'}$	is the number of nodes in the vertical direction for the numerical solution
$P_L$	is the Peclet number of the landfill bottom barrier
$Q$	is the leakage rate through a single hole of the geomembrane
$RC$	is the dimensionless relative concentration of the pollutant
$X$	is the dimensionless horizontal distance below the landfill

$Y, Y'$	are dimensionless vertical distances from the top of the aquifer
$Y_{aq}$	is the dimensionless depth of the aquifer
$a, d, G$	are the dimensionless parameters of the numerical solution
$b$	is the half-width of the geomembrane wrinkle
$c$	is the pollutant concentration in a thick aquifer
$c_0$	is the pollutant concentration in the landfill leachate
$c_b$	is the pollutant concentration at the bottom of the landfill barrier
$c_x$	is the pollutant concentration in a thin aquifer
$c_{x0}$	is the pollutant concentration in the aquifer upstream from the landfill
$h$	is the thickness of the aquifer
$h_b$	is the height of the water level at the bottom of the landfill barrier
$h_p$	is the height of the ponded leachate above the landfill bottom barrier
$k_{eq}$	is the equivalent hydraulic conductivity of the landfill bottom barrier
$k_i$	is the hydraulic conductivity of the $i$ -th layer of the landfill bottom barrier
$\ell$	is the horizontal length of the landfill in the direction of the groundwater flow
$n_{aq}$	is the porosity of the aquifer
$n_h$	is the number of the geomembrane defects per unit area
$n_i$	is the porosity of the $i$ -th layer of the landfill bottom barrier
$q$	is the vertical infiltration flux coming from the landfill
$q_x$	is the groundwater horizontal flux
$q_{x0}$	is the groundwater horizontal flux upstream the landfill
$q_y$	is the groundwater vertical flux
$r$	is the number of the series terms of the analytical solution for aquifers with a finite thickness
$t$	is the time
$v_{x0}$	is the horizontal seepage velocity of groundwater upstream from the landfill
$x$	is the horizontal distance below the landfill
$y$	is the vertical distance from the top of the aquifer

## 1. INTRODUCTION

The design of landfill liners is aimed at minimizing both leachate flow and contaminant transport due to advection and molecular diffusion from the waste to the groundwater system (Rowe et al., 2004; Shackelford, 2014). Modern liners comprise compacted clay layers (CCLs) and/or geosynthetic components, such as geomembrane layers (GMLs) and geosynthetic clay layers (GCLs). GMLs can be coupled with CCLs and GCLs to form composite liners that provide optimal groundwater protection. Moreover, the aquifer located beneath the landfill is typically separated from the waste not only by these engineered barriers, but also by a natural foundation or attenuation layer (AL), which plays an important role in limiting contaminant migration (Rowe and Brachman, 2004).

The preliminary design of a lining system generally requires the assessment of both its *equivalency* with the lining system that is prescribed by the environmental regulations in force and its *effectiveness* with respect to specified performance criteria (Katsumi et al., 2001; Foose, 2010). One common performance criterion is that the design should ensure an acceptable risk for human health and the environment, which is typically quantified through a prescribed maximum value of the contaminant concentration in the groundwater. For this reason, the assessment of liner performance generally requires one to ascertain that the concentration of pollutants that are released by the waste remains below a prescribed threshold level at a specified compliance point, which is commonly represented by a monitoring well located in the underlying aquifer and downstream from the landfill (Figure 1).

Numerical solutions implemented in software products are currently available to evaluate the groundwater contaminant concentration below the landfill (Rowe and Booker, 1985a, 1985b; El-Zein and Rowe, 2008; El-Zein et al., 2012; Xie et al. 2016). However, such numerical solutions do not exclude the interest in implementing analytical solutions, which may be considered useful analysis tools, due to their simplicity and repeatability. In this paper, analytical solutions are developed under the restrictive assumptions of steady-state conditions and constant source concentration in the waste leachate. These conditions, which were also assumed by Guyonnet et al. (2001) and Foose (2010), exclude the possibility of modeling time-varying properties and time-dependent phenomena, and

27 typically result in conservative predictions of the groundwater contaminant concentration  
28 (Shackelford, 1990; Rabideau and Khandelwal, 1998; Rowe et al., 2004). As a result, such solutions  
29 should not be considered as long-term, realistic simulations of contaminant migration, but rather as  
30 conservative estimates of the risk related to a given contaminant concentration in the waste leachate,  
31 in a similar way to a Tier-2 analysis of the ASTM risk-based corrective action (RBCA) standard  
32 (ASTM, 2015) for a polluted site.

33 The scenarios considered in this paper include vertical pollutant transport through an engineered  
34 barrier, which can overlie a foundation or attenuation layer, and horizontal transport in the underlying  
35 aquifer (Figure 1). With respect to the already available steady-state analysis approaches (Guyonnet  
36 et al., 2001; Foose, 2010), the solutions that are derived in this paper allow the contaminant  
37 concentration distribution to be evaluated within the aquifer beneath the landfill. A first analytical  
38 solution is derived for the case of aquifers that are sufficiently *thin* in order to neglect the vertical  
39 distribution of the contaminant concentration in the groundwater. When the aquifers are *thick* and the  
40 vertical dispersion of the contaminants cannot be neglected, a two-dimensional mass balance in the  
41 aquifer has to be considered, and a set of analytical and step-by-step numerical solutions is developed.  
42 An example of the application of the proposed analytical solutions is presented to assess the migration  
43 of an organic pollutant (toluene,  $C_6H_5-CH_3$ ) for the case of composite liners, in which the mineral  
44 layers (CCL or GCL) are coupled with a GML.

## 2. POLLUTANT CONCENTRATION IN THIN AQUIFERS

If the thickness of the aquifer,  $h$ , is no more than a few meters, the vertical component of the groundwater volumetric flux can be neglected with respect to the horizontal one. The water balance inside an aquifer volume of infinitesimal length,  $dx$ , can be expressed as follows:

$$h \cdot \frac{dq_x}{dx} = q \quad (1)$$

where  $q_x$  is the horizontal groundwater flux,  $q$  is the vertical (infiltration) flux coming from the landfill and  $x$  is the horizontal distance below the landfill taken in the direction of the groundwater flow (Figure 2). After integration of Eq. 1, the groundwater flux  $q_x$  varies linearly beneath the landfill as follows:

$$q_x = q_{x0} + \frac{q}{h} \cdot x \quad (2)$$

where  $q_{x0}$  is the groundwater flux just upstream from the landfill, i.e. at location  $x = 0$ .

For thin aquifers, the pollutant concentration can be assumed to be invariant with the vertical position, and transport by advection is dominant relative to transport by longitudinal hydrodynamic dispersion/diffusion in the horizontal direction (Rowe and Booker, 1985a). Under steady-state conditions, the pollutant mass balance inside the aquifer can be obtained by combining the horizontal advective mass flux with the vertical mass flux derived from the landfill,  $J_s$ , as follows:

$$h \cdot \frac{d}{dx} (q_x \cdot c_x) = J_s \quad (3)$$

where  $c_x$  is the pollutant concentration in the aquifer beneath the landfill (Figure 2).



69 The vertical flux of a miscible pollutant through a multi-layer barrier is given by (Manassero et al.,  
70 2000; Guyonnet et al., 2001):

71

$$72 \quad J_s = q \frac{c_0 \cdot \exp(P_L) - c_b}{\exp(P_L) - 1} \quad (4)$$

73

74 where  $c_0$  is the pollutant concentration in the leachate on the top of the barrier,  $c_b$  is the pollutant  
75 concentration at the bottom of the barrier, which is supposed to coincide with the top of the aquifer  
76 located beneath the landfill, and  $P_L$  is the dimensionless Peclet number of the barrier.

77 The Peclet number, which represents the ratio of advective transport rate to diffusive-dispersive  
78 transport, can be expressed as follows:

79

$$80 \quad P_L = \frac{q}{\Lambda} \quad (5)$$

81

82 where  $\Lambda$  is the equivalent diffusivity of the multi-layer barrier. For a volatile organic compound  
83 (VOC), which can diffuse through GMLs,  $\Lambda$  is given by (Manassero et al., 2000; Guyonnet et al.,  
84 2001; Katsumi et al., 2001; Foose, 2010; Shackelford, 2014):

85

$$86 \quad \Lambda = \frac{1}{\frac{L_g}{K_g \cdot D_g} + \int_0^L \frac{dz}{\theta \cdot D_h}} \quad (6)$$

87

88 where  $L_g$  is the thickness of the geomembrane that is placed at the top of the barrier,  $K_g$  is the partition  
89 coefficient between the geomembrane and solute,  $D_g$  is the diffusion coefficient of the geomembrane,  
90  $\theta$  is the volumetric water content,  $D_h$  is the hydrodynamic dispersion/diffusion coefficient in the  
91 vertical direction and  $L$  is the total thickness of the barrier system. If pollutant transport occurs under

saturated conditions, the volumetric water content is equal to the porosity of the layers which constitute the barrier system, and the equivalent diffusivity can be expressed as follows:

$$\Lambda = \frac{1}{\frac{L_g}{K_g \cdot D_g} + \sum_{i=1}^{N_l} \frac{L_i}{n_i \cdot D_{hi}}} \quad (7)$$

where  $L_i$  is the thickness of the  $i$ -th layer,  $n_i$  is the porosity of the  $i$ -th layer,  $D_{hi}$  is the hydrodynamic dispersion/diffusion coefficient of the  $i$ -th layer and  $N_l$  is the number of mineral layers in the barrier system (Figure 3).

In the absence of the GML or in the case of degradation of the GML,  $\Lambda$  can be calculated by omitting the first term related to the GML at the denominator of Eq. 6 or 7. For inorganic solutes, whose diffusion through GMLs is extremely slow and can be neglected with respect to the transport through GML holes, the mass flux through composite barriers,  $J_{s(\text{Composite})}$ , can be calculated by multiplying the mass flux without the GML,  $J_{s(\text{Mineral})}$ , by the ratio between the leakage area and the total area of the barrier, expressed as the ratio between the leakage rate through the composite barrier,  $q_{(\text{Composite})}$ , and the water volumetric flux through the mineral barrier without the GML,  $q_{(\text{Mineral})}$ , i.e.  $J_{s(\text{Composite})} = J_{s(\text{Mineral})} \cdot q_{(\text{Composite})} / q_{(\text{Mineral})}$ , as proposed by Katsumi et al. (2001) and Foose (2010).

In a thin aquifer, the pollutant concentration at the top of the aquifer,  $c_b$ , can be assumed to coincide with the concentration in the aquifer,  $c_x$ , as the change in pollutant concentration along the vertical direction is negligible.

As a result, when Eq. 4 is inserted into Eq. 3, and the variation of  $q_x$  along the  $x$ -direction, which is given by Eq. 2, is taken into account, the pollutant mass balance yields the following first-order differential equation:

$$\frac{dc_x}{dx} + \chi \cdot \left( \frac{q}{q_{x0} \cdot h + q \cdot x} \right) \cdot c_x = \chi \cdot \left( \frac{q}{q_{x0} \cdot h + q \cdot x} \right) \cdot c_0 \quad (8)$$

116

117 where

118

$$\chi = \frac{\exp(P_L)}{\exp(P_L) - 1} = \frac{1}{1 - \exp(-P_L)} \quad (9)$$

120

121 The analytical solution of Eq. (8), associated to the boundary condition

122

$$c_x(x=0) = c_{x0} \quad (10)$$

124

125 where  $c_{x0}$  is the initial groundwater contaminant concentration that comes from upstream of the  
126 landfill, is given by:

127

$$c_x = c_{x0} + (c_0 - c_{x0}) - (c_0 - c_{x0}) \cdot \left( \frac{q_{x0} \cdot h}{q_{x0} \cdot h + q \cdot x} \right)^\chi \quad (11)$$

129

130 Defining the relative concentration, RC, as follows:

131

$$RC = \frac{c_x - c_{x0}}{c_0 - c_{x0}}, \quad (12)$$

133

134 the analytical solution can also be expressed in the following dimensionless form:

135

$$RC = 1 - \left( \frac{\eta}{\eta + X} \right)^\chi \quad (13)$$

137

138 where  $X = x/\ell$  is the dimensionless distance beneath the landfill, with reference to the landfill length  
139 in the direction of the groundwater flow,  $\ell$ , and

140

141 
$$\eta = \frac{q_{x0} \cdot h}{q \cdot \ell} . \quad (14)$$

142

143 The parameter  $\eta$  represents the ratio of the horizontal-to-vertical volumetric flow rates. The relative  
144 concentration at a given point in the aquifer beneath the landfill is a decreasing function of  $\eta$ ; i.e.,  
145 when  $\eta \rightarrow \infty$ , the relative concentration tends to zero, which corresponds to “perfect flushing”, as  
146 shown in Figure 4. The influence of the Peclet number of the barrier on the relative concentration is  
147 limited to values of  $P_L$  lower than 4.

148 When  $P_L > 4$ , the vertical solute mass flux is dominated by advection, and can be expressed as follows:

149

150 
$$J_s = q \cdot c_0 . \quad (15)$$

151

152 Under such conditions, the relative concentration no longer depends on  $P_L$ , and can be expressed as  
153 follows:

154

155 
$$RC = \frac{X}{\eta + X} . \quad (16)$$

156

157 When the vertical volumetric flux is nil (i.e.  $q = 0$ ), the Peclet number is also nil and the vertical solute  
158 mass flux is given by:

159

160 
$$J_s = \Lambda \cdot (c_0 - c_b) . \quad (17)$$

161

162 Under such conditions, the relative concentration that is obtained from the solute mass balance within  
163 the aquifer can be expressed as follows:

164

165 
$$RC = 1 - \exp\left(-\frac{X}{\eta_D}\right) \quad (18)$$

166

167 where

168

169 
$$\eta_D = \frac{q_{x0} \cdot h}{\Lambda \cdot \ell} . \quad (19)$$

170

### 171 3. POLLUTANT CONCENTRATION IN THICK AQUIFERS

172 When the thickness of the aquifer is not limited to a few meters, the variation in pollutant  
173 concentration in the vertical direction becomes significant, and a two-dimensional geometry needs to  
174 be taken into account. Therefore, the pollutant mass balance, within a non-deformable aquifer, can  
175 be expressed as follows:

$$177 \quad n_{aq} \frac{\partial c}{\partial t} = n_{aq} \frac{\partial}{\partial x} \left( D_{h,x} \frac{\partial c}{\partial x} \right) + n_{aq} \frac{\partial}{\partial y} \left( D_{h,y} \frac{\partial c}{\partial y} \right) - \frac{\partial}{\partial x} (q_x \cdot c) - \frac{\partial}{\partial y} (q_y \cdot c) \quad (20)$$

178  
179 where  $n_{aq}$  is the aquifer porosity,  $c = c(x,y)$  is the pollutant concentration within the aquifer,  $x$  is the  
180 horizontal distance beneath the landfill,  $y$  is the vertical distance from the top of the aquifer,  $D_{h,x}$  and  
181  $D_{h,y}$  are the horizontal and vertical hydrodynamic dispersion/diffusion coefficients in the aquifer,  
182 respectively, and  $q_x$  and  $q_y$  are the horizontal and vertical components of the groundwater volumetric  
183 flux in the aquifer, respectively. Under steady-state conditions, and assuming pure advection as the  
184 dominant transport mechanism in the horizontal direction, the mass balance is represented by a  
185 parabolic partial differential equation (Rubin and Buddemeier, 1996; Charbeneau, 2000):

$$187 \quad \frac{\partial}{\partial x} (q_x \cdot c) = n_{aq} \frac{\partial}{\partial y} \left( D_{h,y} \frac{\partial c}{\partial y} \right) - \frac{\partial}{\partial y} (q_y \cdot c). \quad (21)$$

188  
189 In an aquifer of thickness  $h$  with an impermeable layer at the bottom, the vertical volumetric flux  $q_y$   
190 can be assumed to vary linearly with the depth from the vertical infiltration value (i.e.  $q$ ) at  $y = 0$  to a  
191 null value at  $y = h$  (Charbeneau et al., 1995):

$$193 \quad q_y = q \cdot \left( 1 - \frac{y}{h} \right). \quad (22)$$

194

195 In order to preserve the volumetric balance within the aquifer

196

197 
$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0, \quad (23)$$

198

199 the horizontal volumetric flux must also vary linearly along the horizontal direction, as in Eq. 2:

200

201 
$$q_x = q_{x0} + \frac{q}{h} \cdot x. \quad (24)$$

202

203 If the horizontal volumetric flux in the aquifer is appreciably greater than the vertical volumetric flux,  
204 then the transverse mechanical dispersion can be assumed to be dominant relative to molecular  
205 diffusion and the longitudinal mechanical dispersion in the vertical direction (Rubin and Buddemeier,  
206 1996). As a result, the coefficient  $D_{h,y}$  can be calculated as follows:

207

208 
$$D_{h,y} = \alpha_T \cdot v_{x0} \quad (25)$$

209

210 where  $\alpha_T$  is the transverse dispersivity within the aquifer and  $v_{x0} = q_{x0}/n_{aq}$  is the horizontal seepage  
211 velocity of groundwater upstream from the landfill.

212 The pollutant mass balance can be expressed, using Eqs. 22, 24 and 25, as follows:

213

214 
$$q_x \frac{\partial c}{\partial x} = \alpha_T \cdot q_{x0} \cdot \frac{\partial^2 c}{\partial y^2} - q_y \frac{\partial c}{\partial y}. \quad (26)$$

215

216 The boundary condition associated with Eq. 26 at the top of the aquifer (i.e. at  $y = 0$ ) is obtained by  
 217 imposing continuity between the vertical solute flux coming from the landfill and the vertical solute  
 218 flux entering the aquifer:

$$220 \quad q_y \cdot c - \alpha_T \cdot q_{x0} \cdot \frac{\partial c}{\partial y} = q \cdot \frac{c_0 \cdot \exp(P_L) - c}{\exp(P_L) - 1} \quad \text{at } y = 0. \quad (27)$$

221  
 222 If the bottom of the aquifer is constituted by an impermeable layer, the boundary condition at  $y = h$   
 223 is given by

$$225 \quad \frac{\partial c}{\partial y} = 0 \quad \text{at } y = h. \quad (28)$$

226  
 227 The following initial condition is then sufficient to formulate the mathematical problem pertaining to  
 228 the mass balance (i.e. Eq. 26):

$$230 \quad c = c_{x0} \quad \text{at } x = 0. \quad (29)$$

231  
 232 A numerical solution to this problem can be obtained by adopting a step-by-step calculation  
 233 procedure, in which a discretization based on centered finite differences in direction  $y$  and the forward  
 234 Euler method are used to integrate with respect to variable  $x$  (see Appendix 1).

235 However, an analytical solution can be found for cases in which the aquifer thickness is very large,  
 236 by assuming a semi-infinite aquifer and neglecting vertical advection in comparison to vertical  
 237 transverse mechanical dispersion in the mass balance (but not in the boundary condition at  $y = 0$ ).

238 Since  $h \rightarrow \infty$  for a semi-infinite aquifer, the horizontal volumetric flux given by Eq. 24 can be  
 239 assumed constant (i.e.  $q_x = q_{x0}$ ) and the mass balance can be expressed as follows:



240

$$241 \quad \frac{\partial c}{\partial x} = \alpha_T \cdot \frac{\partial^2 c}{\partial y^2}. \quad (30)$$

242

243 An analytical solution to Eq. 30, associated with the boundary condition given by Eq. 27 and the  
 244 initial condition given by Eq. 29, can be derived from the set of solutions provided by Carslaw and  
 245 Jaeger (1959) and Crank (1975) for heat and diffusion problems as follows:

246

$$247 \quad c(x, y) = c_{x0} + (c_0 - c_{x0}) \cdot \left[ \operatorname{erfc} \left( \frac{y}{2 \cdot \sqrt{\alpha_T \cdot x}} \right) - \right. \\ \left. - \exp(\gamma \cdot y + \gamma^2 \cdot \alpha_T \cdot x) \cdot \operatorname{erfc} \left( \frac{y}{2 \cdot \sqrt{\alpha_T \cdot x}} + \gamma \cdot \sqrt{\alpha_T \cdot x} \right) \right] \quad (31)$$

248

249 where

250

$$251 \quad \gamma = \frac{q}{\alpha_T \cdot q_{x0}} \cdot \left[ \frac{\exp(P_L)}{\exp(P_L) - 1} \right]. \quad (32)$$

252

253 The same analytical solution can also be expressed in the following dimensionless form:

254

$$255 \quad RC = \operatorname{erfc} \left( \frac{Y}{2 \cdot \sqrt{X}} \right) - \exp(\Gamma \cdot Y + \Gamma^2 \cdot X) \cdot \operatorname{erfc} \left( \frac{Y}{2 \cdot \sqrt{X}} + \Gamma \cdot \sqrt{X} \right) \quad (33)$$

256

257 where

258

$$259 \quad RC = \frac{c - c_{x0}}{c_0 - c_{x0}} \quad (34a)$$

$$X = \frac{x}{\ell} \quad (34b)$$

$$Y = \frac{y}{\sqrt{\alpha_T \cdot \ell}} \quad (34c)$$

$$\Gamma = \gamma \cdot \sqrt{\alpha_T \cdot \ell} = \frac{\chi}{\eta'} \quad (34d)$$

$$\eta' = \frac{q_{x0} \cdot \sqrt{\alpha_T \cdot \ell}}{q \cdot \ell}. \quad (34e)$$

264

265 When the Peclet number is higher than 4 (i.e.  $P_L > 4$ ), the vertical transport is dominated by advection,

266  $\chi$  tends to unity and the parameter  $\Gamma$  reduces to:

267

$$\Gamma = \frac{q \cdot \ell}{q_{x0} \cdot \sqrt{\alpha_T \cdot \ell}} = \frac{1}{\eta'}. \quad (35)$$

269

270 If the vertical volumetric flux is nil (i.e.  $q = 0$ ), the vertical transport is purely diffusive, and the

271 parameter  $\Gamma$  is given by:

272

$$\Gamma = \frac{1}{\eta'_D}. \quad (36a)$$

274

275 where

276

$$\eta'_D = \frac{q_{x0} \cdot \sqrt{\alpha_T \cdot \ell}}{\Lambda \cdot \ell}. \quad (36b)$$

278

279 If a threshold value of the relative concentration,  $RC_{lim}$ , is assumed in the analysis of the pollutant  
280 migration from the landfill, the thickness of a pollutant plume in the groundwater can be derived  
281 through Eq. 33 by finding the depth at which  $RC = RC_{lim}$ . As an example, the thickness of the pollutant  
282 plume is plotted in Figure 5 for  $RC_{lim} = 0.001, 0.005, 0.01$  and  $\Gamma = 0.05$ . The use of the analytical  
283 solution given by Eq. 33 allows to avoid the introduction of a semi-empirical formula for the thickness  
284 of the pollutant plume, as described in Charbeneau et al. (1995), as well as the calculation of a  
285 boundary layer thickness through approximate solutions, as described in Rubin and Buddemeier  
286 (1996).

287 The relevance of neglecting the vertical advective transport in Eq. 30 can be appreciated by comparing  
288 the relative concentrations, calculated with Eq. 33, to the numerical solution described in Appendix  
289 1. The relative concentrations determined at  $X = 1$  with both the analytical and numerical solutions  
290 are shown in Figure 6 as a function of the dimensionless depth,  $Y' = y / h = Y \cdot \left( \sqrt{\alpha_T \cdot \ell} \right) / h$ , while  
291 varying the upstream horizontal groundwater flow,  $q_{x0}$ , from  $1 \cdot 10^{-8}$  to  $1 \cdot 10^{-6}$  m/s for the following  
292 set of parameters:  $q = 1 \cdot 10^{-10}$  m/s,  $\Lambda = 1 \cdot 10^{-10}$  m/s,  $P_L = 1$ ,  $\ell = 1000$  m,  $\alpha_T = 1$  m,  $h = 100$  m. The  
293 analytical solution appears to be in good agreement with the numerical results, at least as long as the  
294 ratio between the vertical volumetric flux,  $q$ , and the horizontal volumetric flux,  $q_{x0}$ , is less than 1%,  
295 i.e.  $q/q_{x0} < 0.01$ .

296 The approximation arising from the assumption of infinite thickness for the aquifer can be appreciated  
297 in Figure 7 by comparing the relative concentration that has been calculated at  $X = 1$  with Eq. 33 to  
298 the results provided by the numerical solution, varying the aquifer thickness from  $h = 20$  m to  $h = 100$   
299 m for the following set of parameters:  $q = 1 \cdot 10^{-10}$  m/s,  $\Lambda = 1 \cdot 10^{-10}$  m/s,  $P_L = 1$ ,  $\ell = 1000$  m,  $\alpha_T = 1$   
300 m,  $q_{x0} = 1 \cdot 10^{-6}$  m/s. The analytical solution results are sufficiently accurate for  $h = 100$  m when the  
301 ratio of the aquifer thickness,  $h$ , to the landfill length,  $\ell$ , is greater than 10% ( $h/\ell > 0.1$ ). If this  
302 condition is not fulfilled, an improvement to the analytical solution given by Eq. 33 can be obtained  
303 by reflecting the concentration curve at the bottom impermeable boundary (i.e. at  $y = h$ ) and

304 superimposing the reflected curve onto the original one. Repeating this procedure a number  $j = r$  of  
 305 times, the resulting solution can be expressed as follows:

306

$$\begin{aligned}
 \text{RC} = & \sum_{j=1}^r \text{erfc} \left( \frac{2 \cdot Y_{\text{aq}} \cdot (j-1) + Y}{2 \cdot \sqrt{X}} \right) - \exp \left[ \Gamma \cdot (2 \cdot Y_{\text{aq}} \cdot (j-1) + Y) + \Gamma^2 \cdot X \right] \cdot \\
 & \cdot \text{erfc} \left( \frac{2 \cdot Y_{\text{aq}} \cdot (j-1) + Y}{2 \cdot \sqrt{X}} + \Gamma \cdot \sqrt{X} \right) + \\
 307 & + \text{erfc} \left( \frac{2 \cdot Y_{\text{aq}} \cdot j - Y}{2 \cdot \sqrt{X}} \right) - \exp \left[ \Gamma \cdot (2 \cdot Y_{\text{aq}} \cdot j - Y) + \Gamma^2 \cdot X \right] \cdot \\
 & \cdot \text{erfc} \left( \frac{2 \cdot Y_{\text{aq}} \cdot j - Y}{2 \cdot \sqrt{X}} + \Gamma \cdot \sqrt{X} \right)
 \end{aligned} \tag{37}$$

308

309 where  $Y_{\text{aq}} = h / \sqrt{\alpha_T \times \ell}$  is the relative depth of the aquifer. When Eq. 37 is used, the analytical  
 310 solution is also in good agreement with the numerical results obtained for  $h = 20$  m, as shown in  
 311 Figure 8.

#### 312 4. APPLICATION EXAMPLES

313 The following examples are provided to illustrate how the previously derived steady-state solutions  
314 can be employed in order to assess the *equivalency* and *effectiveness* of different landfill barriers.

315 Because any realistic analysis should be based on specific data that have been measured by means of  
316 field and/or laboratory tests, the results of the following examples are only representative of the  
317 proposed analysis approach and should therefore not be generalized to analogous barriers that are  
318 characterized by different parameter values and/or are exposed to different boundary conditions.

319 Two barriers scenarios are considered herein. The first barrier scenario is a composite barrier  
320 comprising a 1.5-mm-thick *geomembrane liner* (GML) and a 1-m-thick *compacted clay liner* (CCL),  
321 which overlies a 3-m-thick attenuation layer (AL). The second barrier scenario is a composite barrier  
322 comprising a 1.5-mm-thick *geomembrane liner* (GML) and a 10-mm-thick *geosynthetic clay liner*  
323 (GCL), which overlies a 4-m-thick attenuation layer (AL) (Figure 9). The two barriers are therefore  
324 characterized by approximately the same total thickness (i.e.  $L \cong 4$  m).

325 The height of the ponded leachate in the leachate collection and removal system (LCRS),  $h_p$ , is  
326 assumed to be equal to 0.5 m, which is the minimum thickness of the LCRS that is required by  
327 European Directive 1999/31/EC, and the hydraulic head at the bottom of the barrier,  $h_b$ , is assumed  
328 to be equal to 1.5 m (Figure 9). As a result, the difference in the hydraulic head between the top of  
329 the mineral layers and the bottom of the attenuation layer,  $\Delta h$ , is equal to 3 m for the barrier with the  
330 CCL and 3.01 m for the barrier with the GCL. The physical, hydraulic and transport parameters that  
331 have been assigned to the geomembrane and the mineral layers are reported in Figure 9 and in Table  
332 1.

333 The CCL is hypothesized to be characterized by an average value of the hydraulic conductivity that  
334 corresponds to the maximum value that is permitted by the European and USA regulations, i.e.  $k =$   
335  $1 \cdot 10^{-9}$  m/s. The porosity,  $n$ , and tortuosity factor,  $\tau_a$ , values have been estimated from the data on the  
336 kaolinite specimens that were tested by Shackelford and Daniel (1991a,b).

337 The GCL hydraulic conductivity and porosity values are derived from the results of the laboratory  
338 test conducted by Puma et al. (2015) with an aggressive permeant solution of 0.25 M of CaCl<sub>2</sub> under  
339 an effective confining stress of 70 kPa. These selected values take into account the increase in  
340 hydraulic conductivity and the reduction in void ratio that are induced by a long-term permeation  
341 with an aqueous solution having a high salt concentration. The GCL tortuosity factor is derived from  
342 the data on the sodium bentonite specimen tested by Dominijanni et al. (2013), neglecting the solute  
343 restriction effect that is related to chemico-osmotic phenomena, which represents a conservative  
344 assumption. Typical parameter values of a silty soil have been selected for the AL (Manassero et al.,  
345 2000; Rowe and Brachman, 2004; Rowe et al., 2004). The analysis is developed for *toluene* (C<sub>6</sub>H<sub>5</sub>-  
346 CH<sub>3</sub>), which is a common component of municipal solid waste landfill leachates.

347 The leakage through composite barriers that include the GML has been calculated as the product of  
348 the number of holes per unit area in the GM,  $n_h$ , and the leakage rate through a single hole that  
349 coincides with a wrinkle, which, assuming no interaction between adjacent wrinkles, can be written  
350 as follows (Rowe, 1998):

351

$$352 \quad Q = 2 \cdot L_w \left[ k_{eq} \cdot b + (k \cdot L \cdot \Theta)^{0.5} \right] \frac{\Delta h}{L} \quad (38)$$

353

354 where  $L_w$  is the length of the wrinkle,  $k_{eq}$  is the hydraulic conductivity the underlying mineral barrier,  
355  $2b$  is the width of the wrinkle,  $L$  is the thickness of the underlying mineral layer (or the total thickness  
356 of the underlying mineral barrier),  $\Theta$  is the transmissivity of the interface between the GML and the  
357 underlying soil, and  $\Delta h$  is the hydraulic head loss across the barrier system.

358 The equivalent hydraulic conductivity,  $k_{eq}$ , in Eq. 38 is calculated as the harmonic mean of the  
359 hydraulic conductivities of individual layers:

360

$$k_{eq} = \frac{L}{\sum_{i=1}^{N_i} \frac{L_i}{k_i}} \quad (39)$$

362

363 where  $k_i$  is the hydraulic conductivity of the  $i$ -th layer. The calculation of  $k_{eq}$  also includes the  
 364 contribution of the attenuation layer that is placed between the engineered barrier system and the  
 365 underlying aquifer.

366 The following parameters have been assumed for the calculation of the leakage rate per unit area,  $q$ :  
 367  $n_h = 1$  hole in a wrinkle per hectare,  $L_w = 3$  m,  $2b = 0.2$  m,  $\Theta = 4 \cdot 10^{-8}$  m<sup>2</sup>/s for the contact between  
 368 GML and CCL and  $\Theta = 3.5 \cdot 10^{-11}$  m<sup>2</sup>/s for the contact between GML and GCL. The value of  
 369 transmissivity,  $\Theta$ , that has been assigned to the GML - CCL contact represents the average value of  
 370 the range estimated by Rowe (1998), which varies from  $1.6 \cdot 10^{-8}$  m<sup>2</sup>/s to  $1 \cdot 10^{-7}$  m<sup>2</sup>/s for this type of  
 371 composite barrier. Analogously, the value of transmissivity,  $\Theta$ , that has been assigned to the GML -  
 372 GCL contact represents the average value of the range provided by Harpur et al. (1993), which varies  
 373 between  $6 \cdot 10^{-12}$  m<sup>2</sup>/s and  $2 \cdot 10^{-10}$  m<sup>2</sup>/s (Rowe and Brachman, 2004).

374 The obtained results (Table 2) show that the leakage rate through the composite barrier with GCL ( $q$   
 375 = 3.4 lphd) is appreciably lower than the leakage rate through the composite barrier with CCL ( $q$  =  
 376 9.8 lphd), because of the better contact conditions between the GML and GCL.

377 The free-solution diffusion coefficient,  $D_0$ , is equal to  $9.7 \cdot 10^{-10}$  m<sup>2</sup>/s for toluene (Yaws, 1995). The  
 378 average values of the geomembrane partition coefficient and the diffusion coefficient have been  
 379 assumed equal to  $K_g = 96$  and  $D_g = 0.47 \cdot 10^{-12}$  m<sup>2</sup>/s, respectively (Rowe, 1998), on the basis of the  
 380 data by Park and Nibras (1993) pertaining to the migration of toluene in aqueous solutions.

381 The effective diffusion coefficient for the mineral layers has been calculated as the product of the  
 382 apparent tortuosity factor,  $\tau_a$ , and the free-solution diffusion coefficient (Shackelford and Daniel,  
 383 1991a). The longitudinal dispersivity of the mineral layers has been assumed equal to a tenth of the  
 384 thickness of the barrier layers to calculate the mechanical dispersion coefficients, which are defined

385 as the product of the longitudinal dispersivity and the seepage velocity (Shackelford and Rowe, 1998;  
 386 Guyonnet et al., 2001). The hydrodynamic dispersion/diffusion coefficients in the vertical direction,  
 387  $D_{hi}$ , have been calculated as the sum of the effective diffusion coefficients and the mechanical  
 388 dispersion coefficients (Shackelford, 1993). The calculated values of the equivalent diffusivity,  $\Lambda$ ,  
 389 and the Peclet number,  $P_L$ , are reported in Table 2. In the presence of the GML, the Peclet number is  
 390 lower than 1, thus indicating the dominance of diffusion over advection in the migration process of  
 391 this organic contaminant, which is able to diffuse through the geomembrane. Similar conclusions  
 392 were reached by Katsumi et al. (2001), Foose (2010) and Pu et al. (2015, 2016a, 2016b, 2016c, 2019).  
 393 When the performance of a composite barrier is assessed, the finite service life of the geomembrane  
 394 needs to be taken into account (Sangam and Rowe, 2002; Rowe, 2005). For example, Rowe (2006)  
 395 pointed out that the service life of geomembranes is of the order of 15-50 years at temperatures of  
 396 50-60 °C. As a result, the volumetric flux through the mineral layers,  $q$ , has been calculated  
 397 conservatively for both of the barriers without taking into account the presence of a geomembrane,  
 398 in order to assess the barrier performance after geomembrane degradation. Under the assumption of  
 399 saturated conditions in the liners and the attenuation layer,  $q$  has been determined as follows:

400

$$401 \quad q = k_{eq} \frac{h_p + L - h_b}{L}. \quad (40)$$

402

403 The CCL barrier is characterized by a value of  $q = 2.91 \cdot 10^{-9}$  m/s, which is significantly lower than  
 404 the value  $q = 4.39 \cdot 10^{-8}$  m/s that has been found for the GCL barrier. This high value of the volumetric  
 405 flux of the GCL barrier is related to the degradation of the hydraulic containment ability of GCL, due  
 406 to the permeation of aggressive aqueous solutions, which has been considered in the selection of the  
 407 value to assign to the GCL hydraulic conductivity. The Peclet number of the considered example  
 408 mineral barriers are larger than 10, thus showing that, in the absence of the geomembrane, advection  
 409 controls the contaminant migration processes.



410

#### 411 *Thin aquifer*

412 If the aquifer beneath the landfill with a length  $\ell$  of 1,000 m is sufficiently thin ( $h = 3$  m), the  
413 analytical solution (Eq. 13) can be used to assess the variation of the contaminant concentration in  
414 the aquifer along the direction of the groundwater flow.

415 The values of  $\eta$  and the relative concentration at  $x = \ell$ , which have been calculated assuming that  
416 the horizontal groundwater volumetric flux just upstream from the landfill,  $q_{x0}$ , is equal to  $1 \cdot 10^{-6}$  m/s  
417 ( $= 31.6$  m/yr), are reported in Table 3 for all the considered example barriers. The calculated relative  
418 concentrations of toluene in the aquifer are shown as a function of the horizontal distance below the  
419 landfill in Figure 10, for all the considered cases.

420 The composite barrier with the GCL is more effective than the composite barrier with the CCL in  
421 reducing the toluene concentrations in the aquifer, even though a conservative value of the GCL  
422 hydraulic conductivity, which can be reached after a long-term permeation with an aggressive  
423 aqueous solution, has been assumed. The concentration of toluene increases along the direction of the  
424 groundwater flow beneath the landfill. Therefore, the maximum value of contaminant concentration  
425 is reached at  $x = \ell$ , i.e. just downstream from the landfill.

426 After the degradation of the geomembrane, the effectiveness of the barriers is reduced significantly  
427 and, as a result, the relative concentration below the landfill increases by more than one order of  
428 magnitude. Under such conditions, the CCL is more efficient than the GCL, due to the better ability  
429 of the CCL to reduce the contaminant diffusive flux.

430

#### 431 *Thick aquifer*

432 If the aquifer thickness is not limited to a few meters, the vertical distribution of the contaminant  
433 needs to be taken into account by means of the analytical solutions presented by Eqs. 33 or 37, or by  
434 means of the numerical solution presented in the appendix. In this case, the contaminant concentration  
435 in the aquifer is dependent not only on the horizontal flushing resulting from the groundwater flow,

436 but also on the vertical dispersion. In this example, the aquifer thickness,  $h$ , has been assumed equal  
437 to 100 m and the transverse dispersivity within the aquifer,  $\alpha_T$ , has been assumed equal to 1 m, based  
438 on the indications of Rowe et al. (2004) in case of availability of high-quality experimental data.  
439 The landfill length,  $\ell$ , and the upstream horizontal groundwater volumetric flux,  $q_{x0}$ , have been  
440 assumed equal to 1,000 m and  $1 \cdot 10^{-6}$  m/s ( $= 31.6$  m/yr), respectively, in the same way as for the thin  
441 aquifer example.

442 The calculated relative concentrations of toluene are shown in Figures 11 as a function of the aquifer  
443 depth,  $y$ , at the distances  $x = 100$  m, 500 m and 1,000 m (i.e. at  $X = 0.1$ , 0.5 and 1) beneath the landfill.  
444 The contaminant concentration decreases with depth and increases over the horizontal distance.

445 The analytical solution for the barrier constituted by a GCL overlying an attenuation layer is not  
446 accurate, as the ratio  $q/q_{x0} = 4.4\%$  is greater than 1% (Figure 11d).

447 If a limiting value of the relative concentration is selected, a contaminant plume can be defined within  
448 the aquifer by means of the available analytical solution. For example, the toluene plumes  
449 corresponding to the limiting value of the relative concentration  $RC_{lim}$  of 0.01% are shown in Figure  
450 12 for the two barrier examples with the intact geomembrane.

451

## 452 5. CONCLUSIONS

453 The illustrative examples have shown the application of steady-state analytical solutions for the  
454 assessment of the contaminant concentration within an aquifer that is located below a landfill. On the  
455 basis of such an analysis, the *equivalence* between landfill barriers which consist of different liners  
456 can be established. Moreover, the *effectiveness* of the barrier in limiting contaminant migration and  
457 the related risk to groundwater quality can also be evaluated. With respect to the previous steady-  
458 state analysis approaches (Manassero et al., 2000; Guyonnet et. al., 2001; Foose, 2010), the proposed  
459 analytical solutions allow the contaminant concentration distribution to be evaluated in the horizontal  
460 direction of the groundwater flow, and eventually in the vertical direction for the case of thick  
461 aquifers. However, when the ratio between the vertical and horizontal water fluxes, i.e.  $q/q_{x0}$ , is higher  
462 than 1%, the proposed analytical solution for thick aquifers is not accurate and should be replaced by  
463 the numerical solution that is presented in Appendix 1.

464 The principal benefit of using such solutions is the possibility of conducting an analysis that involves  
465 a limited number of parameters and allows the influence of the liner properties (e.g. hydraulic  
466 conductivity, thickness, defects) and the field conditions (e.g. aquifer thickness, groundwater  
467 velocity) on the final result to be evaluated. Since the assumed boundary conditions are conservative  
468 with respect to the evaluation of the contaminant concentration within the aquifer, the proposed  
469 analysis can be compared to a Tier-2 risk assessment of the ASTM risk-based corrective action  
470 (RBCA) standard (ASTM, 2015) for a polluted site.

471 A possible interesting development of the proposed steady-state analysis method is the application  
472 within a probabilistic approach in which the boundary conditions and the model parameters (such as  
473 the leachate contaminant concentration, the hydraulic conductivity of the mineral layers and the  
474 number, size and location of the geomembrane defects) take on a random nature. In fact, the obtained  
475 analytical solutions may be implemented in a Monte Carlo algorithm to relate the pollutant  
476 concentration in the aquifer not only to the most representative values of the involved model  
477 parameters, but also to their variance.

478 **APPENDIX 1**

479 The pollutant mass balance given by Eq. 26 can be expressed in the following dimensionless form:

480

$$481 \quad \frac{\partial RC}{\partial X} = d \cdot \frac{\partial^2 RC}{\partial Y'^2} - a \frac{\partial RC}{\partial Y'} \quad (A1)$$

482

483 where

484

$$485 \quad Y' = y / h \quad (A2)$$

$$486 \quad d = \frac{\alpha_T \cdot q_{x0} \cdot \ell}{\left( q_{x0} + \frac{q \cdot \ell}{h} \cdot X \right) \cdot h^2}, \quad (A3)$$

$$487 \quad a = \frac{q \cdot (1 - Y') \cdot \ell}{\left( q_{x0} + \frac{q \cdot \ell}{h} \cdot X \right) \cdot h}. \quad (A4)$$

488

489 The  $Y'$  variable can be discretized into  $N_{Y'} + 1$  nodes, which are numbered from 0 to  $N_{Y'}$  and are

490 separated by equal intervals of  $\Delta Y' = 1/N_{Y'}$ . Similarly, the  $X$  variable can be discretized into  $N_X + 1$

491 nodes, which are numbered from 0 to  $N_X$  and are separated by equal intervals  $\Delta X = 1/N_X$ .

492 Using a centered finite difference to approximate the derivatives in variable  $Y'$  and the Euler forward

493 method to integrate with respect to the variable  $X$ , the following step-by-step calculation procedure

494 is found:

495

$$496 \quad RC_i^{j+1} = RC_i^j + \left( d^j \cdot \frac{RC_{i+1}^j - 2 \cdot RC_i^j + RC_{i-1}^j}{\Delta Y'^2} - a_i^j \cdot \frac{RC_{i+1}^j - RC_{i-1}^j}{2 \cdot \Delta Y'} \right) \cdot \Delta Y' \quad (A5)$$

497

498 where

499

$$RC_i^{j+1} = RC(X_{j+1}, Y'_i)$$

$$RC_i^j = RC(X_j, Y'_i)$$

$$RC_{i+1}^j = RC(X_j, Y'_{i+1})$$

$$RC_{i-1}^j = RC(X_j, Y'_{i-1})$$

$$d^j = d(X_j)$$

$$a_i^j = a(X_j, Y'_i).$$

In order to apply Eq. A5 to all the nodes of the  $Y'$  variable, two fictitious nodes, which are numbered as  $-1$  and  $N_{Y'} + 1$  and are located outside the grid extremes, have to be introduced. The relative concentrations at these fictitious nodes are determined from the boundary conditions. The boundary condition that is obtained at the top of the aquifer by imposing flux continuity is given by Eq. 27, which can be discretized as follows:

$$G \cdot RC_0^j - \frac{RC_1^j - RC_{-1}^j}{2 \cdot \Delta Y'} = G \quad (A6)$$

where

$$G = \frac{q}{q_{x0}} \cdot \frac{\exp(P_L)}{\exp(P_L) - 1} \cdot \frac{h}{\alpha_T}. \quad (A7)$$

Similarly, at the bottom of the aquifer, Eq. 28 can be discretized as follows:

$$\frac{RC_{N_{Y'}+1}^j - RC_{N_{Y'}-1}^j}{2 \cdot \Delta Y'} = 0. \quad (A8)$$

The resulting step-by-step calculation procedure is:

*Initial condition*

For i varying from 0 to  $N_Y$ ,

$$RC_i^0 = 0$$

end

*Integration in X*

For j varying from 0 to  $N_X - 1$

$$RC_{-1}^j = RC_1^j - G \cdot 2 \cdot \Delta Y' \cdot RC_0^j + G \cdot 2 \cdot \Delta Y'$$

$$RC_{N_Y+1}^j = RC_{N_Y-1}^j$$

For i varying from 0 to  $N_Y$ ,

$$RC_i^{j+1} = RC_i^j + \left( d_i^j \cdot \frac{RC_{i+1}^j - 2 \cdot RC_i^j + RC_{i-1}^j}{\Delta Y'^2} - a_i^j \cdot \frac{RC_{i+1}^j - RC_{i-1}^j}{2 \cdot \Delta Y'} \right) \cdot \Delta X$$

end

end

In order to obtain a stable solution, the following condition has to be respected in the choice of the interval of integration  $\Delta X$  (Sewell, 2005):

$$\Delta X \leq \frac{\Delta Y'^2}{2 \cdot d_{\max}} = \frac{\Delta Y'^2 \cdot h^2}{2 \cdot \alpha_T \cdot \ell}. \quad (A9)$$

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623    **LIST OF TABLES**

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Table No.	Table Caption
1	Physical, hydraulic and transport parameters of the geomembrane and the mineral layers of the example landfill barriers.
2	Calculated values of transport parameters for the example landfill barriers.
3	Values of the dimensionless parameter $\eta$ and the relative concentration of the contaminant at $x = \ell$ , for a thin aquifer ( $h = 3$ m, $q_{x0} = 1 \cdot 10^{-6}$ m/s, $\ell = 1000$ m).

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Figure No.	Figure Caption
1	Reference scheme of the considered scenario, in which the pollutant released by the waste migrates vertically through the engineered barrier system and the natural attenuation layer to the underlying aquifer, where pollutant transport then becomes horizontal.
2	Reference scheme for the water volumetric balance and the contaminant mass balance within a thin aquifer beneath the landfill.
3	Vertical profile of a barrier constituted by engineered and/or natural layers.
4	Relative concentration in a thin aquifer at $X = x/\ell = 1$ , as a function of the Peclet number of the barrier and the dimensionless parameter $\eta$ .
5	Thickness of contaminant plume, i.e. the depth at which $RC = RC_{lim}$ , for $RC_{lim} = 0.001, 0.005, 0.01$ and $\Gamma = 0.05$ .
6	Relative concentration of the contaminant as a function of the depth in the aquifer, at the distance $X = x/\ell = 1$ beneath the landfill, for different values of the horizontal groundwater velocity, $q_{x0}$ .
7	Comparison of the analytical solution for a semi-infinite aquifer and the corresponding numerical solution, in terms of relative concentration of the contaminant as a function of the depth in the aquifer, at the distance $X = x/\ell = 1$ beneath the landfill, for different values of the aquifer thickness, $h$ .
8	Comparison of the analytical solution for a finite aquifer and the corresponding numerical solution, in terms of relative concentration of the contaminant as a function of the depth in the aquifer, at the distance $X = x/\ell = 1$ beneath the landfill, for different values of the aquifer thickness, $h$ .

9	Scheme of the two barriers considered in the example analysis: (a) composite barrier constituted by a geomembrane liner (GML) and a compacted clay liner (CCL); (b) composite barrier constituted by a geomembrane Liner (GML) and a geosynthetic clay liner (GCL).
10	Relative concentration of toluene in a thin aquifer ( $h = 3$ m, $q_{x0} = 1 \cdot 10^{-6}$ m/s) beneath the landfill ( $\ell = 1000$ m).
11	Relative concentration of toluene as a function of the relative depth, $Y' = y/h$ , at $X = 0.1, 0.5$ and $1$ for the following barriers: (a) GML + CCL + AL; (b) GML + GCL + AL; (c) CCL + AL (degraded geomembrane); (d) GCL + AL (degraded geomembrane).
12	Toluene plume beneath the landfill for $RC_{lim} = 0.0001$ (0.01%).

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