Challenges in Mathematics Learning at the University: An Activity to Motivate Students and Promote Self-awareness

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Abstract. Math anxiety is always just around the corner. At the university, it makes students continuously postpone the Calculus exam, leading them into a vicious circle of low confidence and poor performance. To get out of this situation, students need to be motivated and involved. They also need to master metacognitive strategies that can support their learning process. In this paper, we present a digital activity entitled *Advent Calendar*, focused on storytelling and proposed through the logic of spacing. The aim is to increase students' motivation and self-awareness, but also to obtain learning analytics useful to monitor progress and solve any possible weaknesses with appropriate feedback. The activity was proposed using the tools offered by the Learning Management System (LMS) Moodle. It was tested in three university courses at two Italian universities, the University of Milan and the Polytechnic University of Turin, with students' active participation. This participation had a high impact on the results of the final examination. Feedback demonstrated positive feelings and good results in the motivation process, while the analytics showed a continuous approach to the study of mathematics.

Keywords: Teaching Mathematics, Learning Strategies, Educational Technology.

1 Introduction

The main purpose of general scientific degrees' Calculus courses is to introduce students to the scientific method of analysis, providing a suitable language and useful skills in order to effectively face other disciplinary courses. Unfortunately, this does not happen frequently. Many students see the Calculus exam as a stumbling block and try to postpone it as much as possible. This negative mindset can quickly turn into a cycle of low confidence, lower motivation, and poor performance [1, 2]. Math anxiety is always just around the corner [3]. The impact of the COVID-19 pandemic on education, with

the distinctive rise of e-learning and digital platforms, has added to this emotional stress and forced teachers to quickly develop new teaching strategies.

In this framework, designing preliminary actions aimed at fostering students' motivation and self-awareness is an essential challenge. Literature shows that students with higher learning motivation "achieve significantly higher test scores, enjoy learning more, have more positive self-concepts, make greater use of deep learning strategies and engage to a greater extent in autonomous self-regulated learning" [4; see also 5, 6].

Storytelling can support motivation. Its application to mathematics gives a concrete form and a familiar connotation to abstract concepts, involving emotions and imagination at the same time. "A story tends to have more depth than a simple example" [7]. Several authors have investigated the benefits of storytelling as a powerful medium for teaching and learning mathematics [8, 9, 10]. A narrative scenario downplays disciplinary topics and leads students to tackle challenging problems that they would seldom face in a more serious context. Stories and their characters can involve students and provide a structure for remembering concepts, and creating vivid mental images [7, 11]. Bruner underlined that "many scientific and mathematical hypotheses start their lives as little stories or metaphors" [12].

Metacognition can strengthen students' self-awareness. It is essential to focus on the ways in which students manage their study, teaching them appropriate strategies to monitor their own learning. One of the most effective metacognitive control strategies is *spacing*. Spacing consists of spreading out study time across a sufficiently long period in order to produce a more long-lasting learning [13]. It requires planning, however, and students are often unable to do that.

In a digital environment, learning analytics are essential. Tools provided by Learning Management Systems (LMS) — such as Canva, Moodle and Blackboard — make it easy to track students' progress. Teachers can provide global or individual feedback and act promptly on any potential weaknesses. In addition, they can monitor students' participation in the course activities, especially if it is delivered entirely online. In the university context, learning analytics are a powerful resource in view of the final exam.

This paper describes the *Advent Calendar*, a non-conventional digital learning activity designed to work on motivation and awareness (see *Activity Design & Implementation*). We present its features (see *Activity Features*), we provide some examples (see *Unwrapping the Gifts*), and we comment on the results of a pilot test (see *Results*).

2 Activity Design & Implementation

The *Advent Calendar* activity is inspired by the logic of the special calendar used to count the days until Christmas. It combines storytelling and spacing. Every day, students carry out a Christmas-themed mathematical exercise: Santa Claus, the elves and the reindeer are facing problems that can be solved by exploiting suitable mathematics tools introduced during a Calculus' course.

The activity can be carried out using Moodle assignments, along with a Google Drawings file. Google Drawings makes it possible to create an interactive image of the Advent Calendar, which can be embedded in a Moodle page (see Fig. 1). Each gift box

is linked to the page of a single exercise hosted inside a Moodle assignment[1]. The Moodle assignment's *restrict access* feature can be used in order to allow students to enter and submit their exercise only on the corresponding day[2]. The first student who submits the correct solution is awarded with his/her name on the corresponding calendar's gift box.

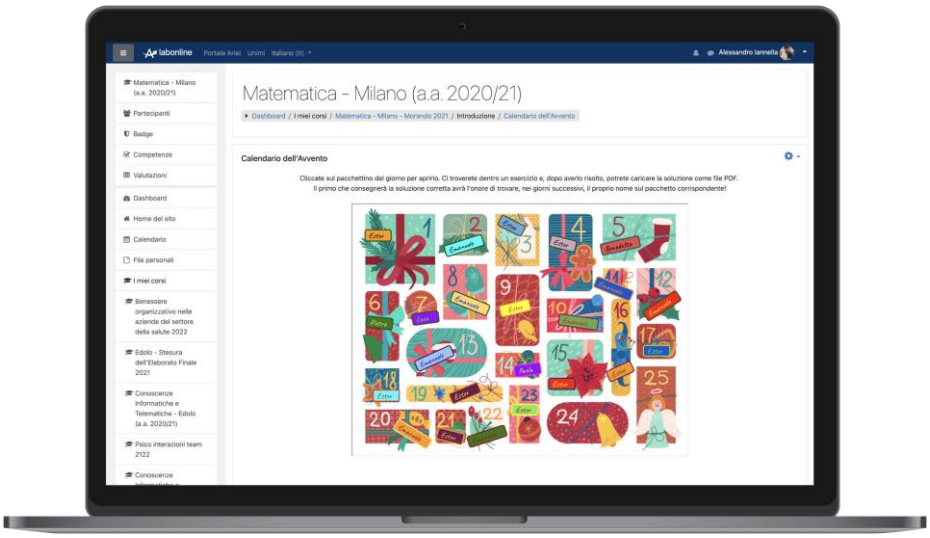

Fig. 1. The *Advent Calendar* activity inside a Moodle page. Each gift box is linked to an exercise.

In order to make the activity effective, it is crucial to give students personalised feedback on their answers or to publish a complete solution to compare with. Moodle assignments provide teachers with a wide range of combinable feedback, such as global or between-the-lines comments, evaluation rubrics, scoring scales or offline gradesheets[3]. In addition, students' submission offers ongoing assessment information that can help them make instructional decisions. For example, teachers can identify and resolve possible weaknesses or track the developing proficiency levels of the individual student in view of the final assessment.

The activity can also be gamified using a system of rewards with the aim of fostering participation and involvement. Students obtain a score for every correct exercise completed within the required time frame. The score distribution starts from 100 for the first exercise submitted and decreases by one for subsequent uploads. This decision aims to

[1] Teachers can also use H5P to create an interactive Advent Calendar. H5P is a free, content authoring plugin that can be integrated with Moodle (see https://h5p.org/content-types/advent-calendar, https://moodle.org/plugins/mod_hvp).

[2] Teachers can use the *restrict access* feature to edit the availability of any Moodle's activity or course section according to certain conditions such as dates, groups, grades or activities completion (see https://docs.moodle.org/311/en/Restrict_access).

[3] See https://docs.moodle.org/311/en/Assignment_settings#Feedback_types, https://docs.moodle.org/311/en/Assignment_settings#Grade.

stimulate spacing and to promote regular participation rather than rewarding speed of delivery.

At the end of the activity, a forum can be opened in order to invite students to send Christmas greetings to their colleagues using a mathematical language. It may be useful to stimulate students' participation and curiosity by allowing them to view their colleagues' greetings only after they have sent their own. The Moodle forum type called Q&A enables this dynamic: students need to post before viewing other student's postings[4] (see Fig. 2).

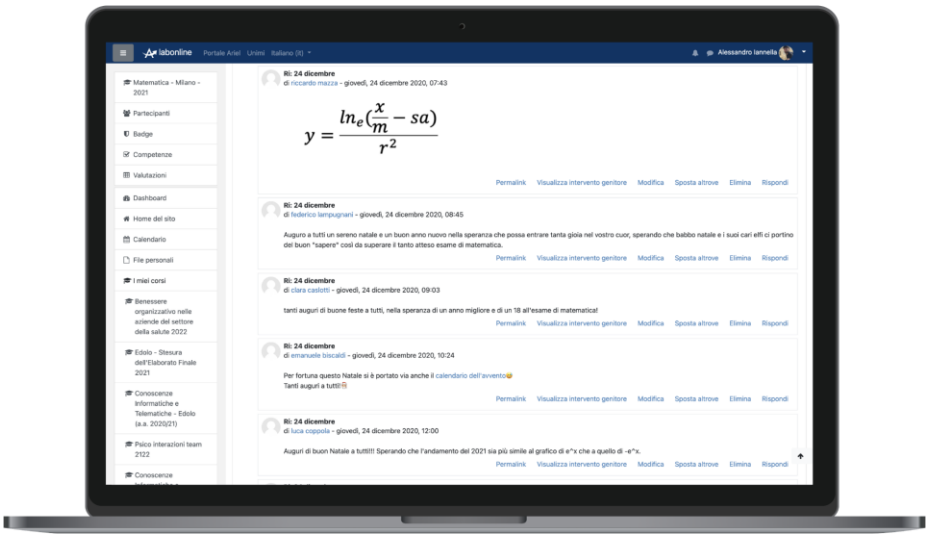

Fig. 2. The Q&A forum where students posted their wishes in mathematical language.

We proposed the *Advent Calendar* in the courses of *Mathematics* at the Polytechnic University of Turin (Degree in Architecture) and at the University of Milan (Degree in Agricultural Sciences and Technologies; Degree in Improvement and Protection of Mountain Environments) during the first semester of the academic year 2020-2021 (see *Results*). Due to the "easy-going" nature of the activity, the final score didn't contribute to the final exam's grade. However, we decided to reward students who sent at least one exercise by sending them a certificate of participation (see Fig. 3). In addition, following the gamification approach, the student with the highest score has been appointed *Mathematical Elf*, the second placed *Deputy Math Elf* and the third one *Second Deputy Math Elf*.

[4] See https://docs.moodle.org/311/en/Using_Forum#Which_forum_do_I_need.3F.

Fig. 3. Certificate given to the students who participated in the activity.

3 Activity Features

The teacher plays a key role in the proposed activity. S/he works on the emotional level, improving students' positive feelings, e.g. exploiting the Christmas spirit or stimulating competition. But s/he also works on the learning level, offering feedback and monitoring students.

From a methodological perspective, the *Advent Calendar* contains the two strategies mentioned above: storytelling in the case of the scenario, and spacing for the fruition modes. In terms of timing, the decision to open the submissions of each exercise at midnight turned out to be very effective. We noticed that many students waited until midnight to do the exercises immediately. The atmosphere was that of a real competition.

From a disciplinary perspective, there are some important aspects that allow us to reflect on the effectiveness of the experience. First of all, we think that the exercises should not be of increasing difficulty.

In fact, the surprise effect is very effective in encouraging students to open the gift box and have a look at the exercise every day, regardless of their ability to solve the previous day's exercise. Furthermore, to keep students engaged, the level of difficulty of each exercise must be chosen very carefully. Exercises that are too difficult or time-consuming risk discouraging students, while problems that are too simple are not challenging enough to stimulate reasoning and competition. A good solution taking both of these needs into account is to replace standard calculus exercises with authentic exercises, which require not only mathematical skills but also problem-solving abilities.

With regard to the review of the exercises, we believe that teachers can proceed in several ways, either by providing personalised feedback or by publishing the solved

exercise. We tried both methods, without finding any particular differences (see *Results*). In one course, we published solutions after the deadline of every activity, uploading the file on Moodle. Only students who had submitted the exercise were allowed to see the corresponding solution and most common mistakes were also reported and discussed. In the other two courses, we provided customised feedback, without publishing the solution.

The advantages of mixing didactic methodologies and digital tools through the *Advent Calendar* can be explored answering these research questions (see *Results*):

1. *Does the Advent Calendar promote positive emotions in Math learning?*
2. *Does the Advent Calendar improve each student's learning experience?*
3. *Is gamification effective in supporting spacing? Does it stimulate students to carry out the exercises day by day and accurately?*
4. *Does the participation in the proposed activities have a positive impact on the final exam?*

In order to answer these questions, we used the following parameters:

- students' feedback, measured via the comments posted in the forum and the e-mails sent to the teachers;
- students' reactions to the correction of an exercise, as evidenced by asking for further clarification and/or by repeating it;
- the analytics related to the participation to the activity, on an overall and student-centered level, with particular reference to persistence;
- the comparison between participation to the activity and the results of the final exam.

4 Unwrapping the Gifts

In order to better explain the activity, we provide some example exercises that appeared in the *Advent Calendar*.

1. *To accurately hang the light decorations, two pairs of elves place Santa's house in a reference system. If the first pair of elves hang the lights following a straight line from point A (5;0;4) to B (0;6;7) and the second pair from C (1;0;8) to D (3;5;0), will the wires touch each other? Use GeoGebra to check and demonstrate the solution* (see Fig. 4).

   The math purpose of this question is to revise the equation of straight lines in three-dimensional space and their intersections. The possibility of integrating a GeoGebra content in a Moodle assignment — through embedding or a specific plug-in[5] — allows students to check their computations (using CAS calculator) and provides a three dimensional representation of the problem improving their mathematical spatial visualization ability (using the 3D calculator).

[5] See https://moodle.org/plugins/assignsubmission_geogebra.

Fig. 4. Exercise 1 inside a Moodle assignment. Pictures by Nicola Spreafico.

2. *It's Christmas Eve and the elves Hildur, Hrólfur and Eyjólf are very busy wrapping the last few presents. Until now, they packed 24 gifts in all, but Hrólfur and Hildur together packed twice as many as Eyjólf did, and Hildur and Eyjólf together packed 2 more than Hrólfur did. How many presents did each elf prepare? Use Geogebra to provide a geometrical interpretation and representation of the problem.*

   The math content of this problem is very similar to the one of Exercise 1. In this case, instead of assigning a standard problem on the intersection of straight lines in 3D, we challenge the student to find the equations through an appropriate modeling of a real situation. Once the student has found the three equations, s/he has to use Geogebra to compare the algebraic and geometric representations of the problem.

3. *To surprise Santa Claus the elves are planning a ski slope. Following the suggestion of the Mathematical Elf they decided to build it following the graph of the function* $y=10xe^{-x/k}$ *when* $x\in[2,8]$*. Indeed, the Mathematical Elf had given some further indications on how to choose the value of* k*, but the Elves often get distracted when it comes to mathematics and they only remember that* k *had to be a positive number. Can you help them choose a value of* $k>0$ *so that the ski slope is all downhill? You can use GeoGebra to find or to check your solution.*

   This is a non-standard and quite challenging question, requiring an open mind and a problem-solving approach to be tackled. In fact, the problem does not have a unique solution and can be faced in at least two different ways. The first possibility is to solve the problem analytically, computing the derivative of the function and studying its sign on the interval [2,8] depending on the parameter k. In this case, after finding for which values of k the graphic of the function has a downhill slope for $x\in$ [2,8], students can check the correctness of their solutions by drawing with GeoGebra the

functions corresponding to the selected value of *k*. On the other hand, students with a more practical and less analytical inclination can approach the problem by drawing the family of functions depending on the parameter *k* using a slider in GeoGebra. In this way, visualizing the change of the slope of the function when *k* changes, they can easily find the solution experimentally, without performing any computation. In this case, we discussed pros and cons of the two approaches during the following lessons. In particular the analytic approach is more effective in finding the whole range of values of *k*, while the experimental one gives an immediate idea of how the shape of the function varies depending on *k*. We think that the use of dynamic softwares such as Geogebra can be very useful for carrying out a parametric problem such as this one (see Fig. 5).

Fig. 5. Solution to Exercise 3. Graph made using Geogebra.

4. *The graph shows the trend in the number of letters and emails received from Santa Claus in the period 2010-2019. Use a spreadsheet to calculate the mean and standard deviation of the number of letters and emails considering the whole period* (see Fig. 6).

   In this case students have to collect data reading the graph and insert them in a spreadsheet. Once again, students worked in two different ways: someone built the formulas for the mean and the standard deviation into the worksheet step by step, while others made direct use of Google Sheets' or Microsoft Excel's statistical functions.

Fig. 6. Exercise 4 inside a Moodle assignment. Pictures by Nicola Spreafico.

5 Results

Our pilot test of the *Advent Calendar* activity had 121 participants. This is about half of the students enrolled in the three university courses in which we proposed it. Despite the apparent "naive" scenario, the broad participation demonstrated that they got involved in the activity. We answer the research questions below (see *Activity Features*).

1. *Does the Advent Calendar promote positive emotions in Math learning?*

   Many students posted funny and positive messages in the Moodle forum opened at the end of the activity, showing how it stimulated their imagination and contributed to their reconciliation with Mathematics. We present some of these posts. Even if some of them are not mathematically correct, they highlight the good emotion that the activity sparkled in the participants:

   (a) "Merry Christmas to all! Hoping that the 2021 trend is more similar to the graph of e^x than to that of -e ^ x";

   (b) "For a Christmas in which nostalgia for normality tends to + ∞, I wish everyone an indeterminate form that can only be solved through so much joy and the hope of being able to return, very soon, to hug your loved ones again! Greetings to all!";

   (c) "I hope that the new year is represented for everyone by the function f (New Year) = hope + (joy) ^ 2 and that this can always be strictly increasing for [(January, 1, 2021) <= (month, day, 2021) <= (December, 31, 2021)]";

   (d) "Even if the holidays are over, my New Year's greetings for all of you are equal to infinity. I would like to understand if my intervention can be positive, calculating Lim x-> 0 of holidays for wishes (0 times infinity) =I. F. I use asymptotic estimates for e^x when x ->0 and I find that e^x is equivalent to 1 + x = 1, so 0

times 1=0 ...I don't like it!! So I calculate Lim x -> + infinity of holidays for wishes (+ infinity times + infinity) = + infinity-> yes, yes, I like it!!!".

Moreover, some students sent to the teachers emails sharing their positive feelings about the *Advent Calendar* experience:

(e) "I am finding this initiative very funny and above all very useful for reviewing some concepts of past topics. Finally, I congratulate her on the imagination with which she manages to create these exercises, they are truly incredible!";

(f) "This *Advent Calendar* is a beautiful idea, it had never been proposed to me";

(g) "Thank you for your dedication, thank you for the way you invented these games, thank you for teaching with all the passion in the world".

The engagement of the students is also confirmed by the fact that they played along, providing the answers to the questions in the Christmas spirit (for example: the elves wrap 19 gifts; the Snurf bear wins because its limit is greater; Santa Claus will pass by Marta's house…).

2. *Does the Advent Calendar improve each student's learning experience?*

The activity improved the overall student's learning experience. In fact, a problem-solving approach encourages students to believe in their ability to think mathematically and gives them the tools to apply their mathematical knowledge to solve hypothetical and real-world problems. When solving problems students are exploring mathematics throughout an authentic task instead of an abstract one. This helps them to make sense of mathematical ideas. Moreover, students were invited to use softwares, such as GeoGebra and spreadsheets, in order to explore the problems, to better visualise 3D situations, to check guesses and to find original solution strategies. Another reason that made the *Advent Calendar* experience effective in improving student's learning experience was the presence of a day-by-day feedback on students' activities. As we mentioned above (see *Activity Features*), we provide feedback in two different ways. Both the strategies turned out to be quite effective, since they made students struggle to solve the exercises and test ideas. Furthermore, personalised feedback highlighted a variety of mistakes and made it possible to suggest different approaches to solve the problem. In this way, the activity follows the logic of the *flipped classroom* [14], encouraging students to ask questions during lessons and to actively interact amongst them and with the teacher. It is worth noting also that a certain number of students viewed the exercise without then submitting it. Sometimes they were not able to solve the exercise, while other times they solved it only after the deadline, asking for suggestions or explanations by e-mail or during the following lesson. The day-by-day feedback on the level of understanding of the different topics allows teachers to suggest review activities for specific students, using Microsoft Teams' breakout rooms[6] or Moodle groups[7].

3. *Is gamification effective in supporting spacing? Does it stimulate students to carry out the exercises day by day and accurately?*

[6] See https://support.microsoft.com/en-us/office/use-breakout-rooms-in-teams-meetings-7de1f48a-da07-466c-a5ab-4ebace28e461.

[7] See https://docs.moodle.org/311/en/Groups.

Students grasped the spirit of the challenge: 48% of them submitted at least half of the proposed exercises and 28% submitted at least 75% of them. We were surprised that several students submitted their solutions shortly after midnight, obviously waiting awake for the new exercise. Considering that the exercises were also solved during the weekends and taking into account students' access to the Moodle course, the activity demonstrated a good spread of the study across a sufficiently long period of time (*spacing*) for at least 60 students. The Christmas scenario and the logic of gamification certainly played a crucial role, but we also think that students understood the underlying seriousness of the activity. The feedback from the last lesson and the enthusiasm shown during the awarding of diplomas demonstrated that they appreciated its value in improving mathematical skills and in disseminating the importance of distributing learning and practice over time.

4. *Does the participation in the proposed activities have a positive impact on the final exam?*

   Students involved in the activity passed the exam during the first available session with a success rate 15% higher than the class rate. In the Architecture course, 80% of the participants passed the exams during the first session with respect to a mean of 65% success. In the two Agriculture courses, 65% of the participants passed the exams during the first session with respect to the mean of 49% success. In all the courses the medium mark of the participants was 27.5/30, while the medium mark of the students who didn't attend the activity was 23.7/30. These results may suggest that the activity had a positive impact on the final exam, although it is possible that only the most interested or skilled students participated. To confirm the effectiveness of the project, it would be necessary to compare the data with those of a control sample. It is also interesting to remark that most of the students which have been awarded with the maximum grade (30 e lode) participated in the activity — 10/12 in the Architecture course and 11/12 in the two Agriculture courses.

6 Conclusions

Calculus course students generally experience a decline in interest and participation, probably due both to disaffection and to the growing complexity of the new subjects. Since Calculus courses are cumulative and require an active learning approach, it is very important to find strategies to keep students involved so that they do not fall behind as the teacher moves on. In this framework, the *Advent Calendar* activity provided an effective review tool, encouraging students to spread out study time across a sufficiently long period of time and to develop their problem-solving strategies.

The success of the activity can be quite surprising in an academic Calculus course because students usually just struggle to pass the exam. However, in this case students turned out to be very open minded and confident in their teachers. In addition, the activity was very useful to play down the subject, to revise previous topics throughout the daily assignments, and to bring students to a satisfactory level of understanding.

This positive attitude suggests that this kind of alternative teaching strategies are welcomed by students and deserve to be proposed again in the coming years, regardless of the online, blended, or face-to-face learning.

The activity can be replicated for different subjects and at different moments of the year, allowing a very effective review experience before the final exam. There are several scenarios into which exercises can be deployed in order to defuse the topics, enhance active participation and provide an unexpected creative side even to apparently hard subjects.

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