Identification of nonlinear damping using nonlinear subspace method

Rui Zhu[1], S. Marchesiello[2], D. Anastasio[2], Dong Jiang[3], and Qingguo Fei[1]

[1] Southeast University, Nanjing, Jiangsu, People's Republic of China
[2] Politecnico di Torino, Torino, Italy
[3]Nanjing Forestry University Nanjing, Jiangsu, People's Republic of China

Abstract. In this paper, the identification problem is discussed for damping nonlinearity. In practical applications, nonlinear damping is widespread, which is inevitable in the vibration response. Within the wide range of nonlinear damping mechanisms, friction is surely one of the most common, and with a high impact on the dynamical behavior of structures. Two common kinds of friction are investigated: quadratic friction and Coulomb friction. Nonlinear damping parameters are identified by nonlinear subspace identification, where the damping nonlinearity of the system is considered as a feedback force applied to the underlying linear system and is identified utilizing the time domain data. Two simulation examples are conducted to verify the effectiveness of the method. Results confirm the effectiveness of the methodology in identifying damping nonlinearities.

Keywords: Nonlinear damping, Quadratic friction, Coulomb friction, Nonlinear subspace identification.

1 Introduction

In engineering, structures often exhibit nonlinear behaviour. Nonlinear damping [1] is a common nonlinear type, which may lead to difficulties in predicting the system response [2] . Therefore, it is essential to identify the nonlinear damping parameters from the measured vibration data [3].

The reader can refer to the extensive review of Noël et al. [4] about the developments in nonlinear system identification during the past ten years, emphasizing the progress realized over that period. As for nonlinear damping, an identification method based on the harmonic balance analysis was implemented in [5], considering softening and hardening behaviors. Amabili et al. [6] identified the nonlinear damping at each excitation level in the nonlinear regime from the experimental data of a rubber plate. Moreover, it should be highlighted that damping identification can be a tricky task also in the linear case, as studied by Naylor et al. [7], characterizing the nonproportional damping distribution of a multi-degrees-of freedom system using the resonant decay method.

Among the several publications about the nonlinear system identification of structures, Marchesiello et al. [8] adopted the perspective of nonlinearities as internal feedback forces and proposed the nonlinear subspace identification technique (NSI). Given the robustness and efficacy of the subspace method, these nonlinear subspace algorithms open up new horizons for the identification of nonlinear mechanical systems.

In this paper, NSI is extended to identify the nonlinear damping. Two numerical examples are used to verify the proposed method.

2 Nonlinear subspace method considering friction

The nonlinear damping is considering. The equation of the system can be written as:

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)+f_d=f(t) \tag{1}$$

where M is the mass matrix, C is the damping matrix and K is the linear stiffness matrix.

The system can be viewed as subjected to the external forces $f(t)$ and the internal feedback forces due to nonlinearities f_d as shown in.

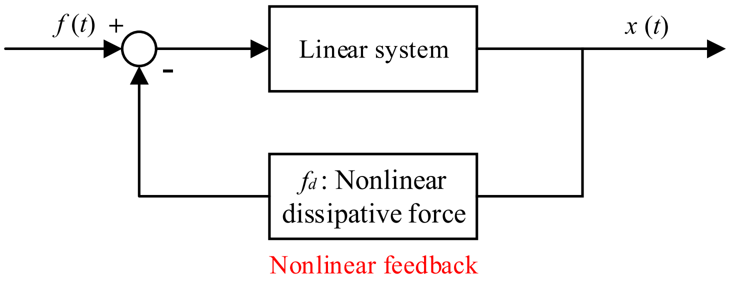

Fig. 1. Closed-loop representation with nonlinear damping

A one-degree-of-freedom mass-spring system with the Coulomb friction is used to elaborate the nonlinear subspace method.

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)+\alpha\,\mathrm{sgn}(\dot{x}(t))=f(t) \tag{2}$$

By moving the nonlinear term of Eq.(2) to the right-hand side

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=f(t)-\alpha\,\mathrm{sgn}(\dot{x}(t))=f(t)-f_d \tag{3}$$

A state vector is used by $z=[x \quad \dot{x}]^T$, the state-space formulation of the equation of motion can be expressed as

$$\begin{Bmatrix}\dot{x}\\ \ddot{x}\end{Bmatrix}=\underbrace{\begin{bmatrix}0 & 1\\ -\frac{\hat{k}_1}{m} & 0\end{bmatrix}}_{A_c}\begin{Bmatrix}x\\ \dot{x}\end{Bmatrix}+\underbrace{\begin{bmatrix}0 & 0\\ \frac{1}{m} & \frac{\alpha}{m}\end{bmatrix}}_{B_c}\begin{Bmatrix}f(t)\\ -Sgn(\dot{x})\end{Bmatrix} \tag{4}$$

$$y=\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{C}\begin{Bmatrix}x\\ \dot{x}\end{Bmatrix}+\underbrace{\begin{bmatrix}0 & 0\end{bmatrix}}_{D}\begin{Bmatrix}f(t)\\ -Sgn(\dot{x})\end{Bmatrix} \tag{5}$$

where A_c, B_c, C and D are the continuous state-space matrix.

The "extended" frequency response function can be derived based on authors' previous work about the nonlinear subspace identification.

$$H_E(\omega)=D+C(i\omega I-A_c)^{-1}B_c \tag{6}$$

where ω is the angular frequency and $i=\sqrt{-1}$.

Substituting Eq.(4) and Eq.(5) into Eq.(6), one can obtain

$$H_E(\omega)=[H \quad H\alpha] \tag{7}$$

where H is the underlying linear system receptance matrix. The nonlinear damping coefficients can be identified based on Eq.(7)

3 Simulation

3.1 Singe degree of freedom with cubic stiffness and quadratic friction

Consider the SDOF system with cubic stiffness and coulomb friction depicted in Fig.1, whose motion is described by the following equation:

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)+\alpha\dot{x}(t)\left|\dot{x}(t)\right|=f(t) \tag{8}$$

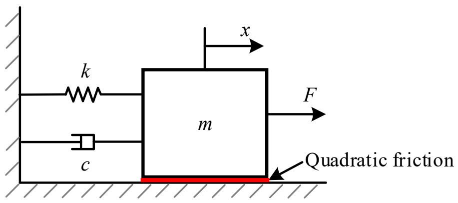

Fig. 2. A mass-spring system with quadratic friction

with system parameters summarized in. Assume that the type and the location of the nonlinearity are unknown.

Table 1. System parameters of Singe degree.

m (kg)	*k* (N/m)	*c* (Ns/m)	*α*(N)
2	1000	1	0.5

The SDOF system is excited by a zero-mean Gaussian random input, whose root-mean-square (r.m.s) value is 5. The response is calculated by Runge-Kutta. The effect of the measurement noise on the parameter estimation results is investigated by corrupting the previously generated output adding 5% Gaussian zero-mean noise. The displacement is shown in Fig.3.

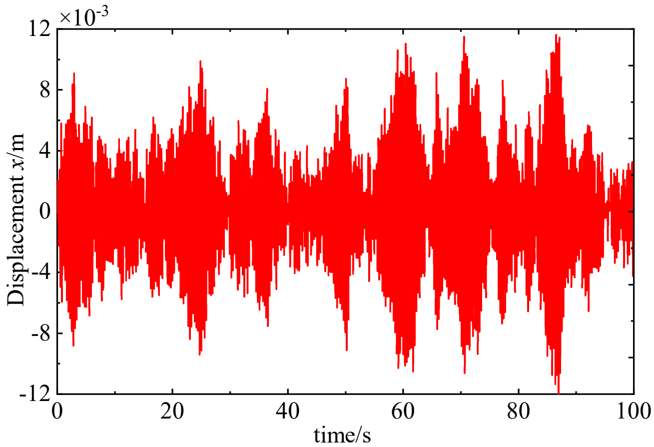

Fig. 3. The displacement of the system

The model order n=2 is determined by inspecting the singular value plot in Fig.4 (with i=20 block rows), where a jump of seven orders of magnitude between model order two and three is observed.

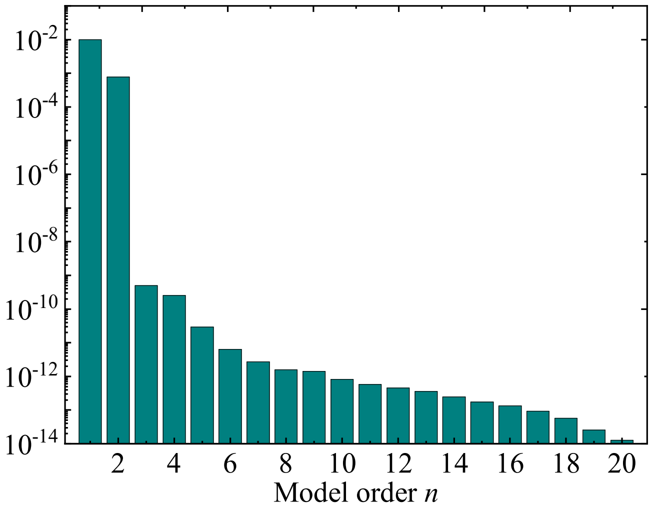

Fig. 4. Singular value plot with 5% measurement error and i=20

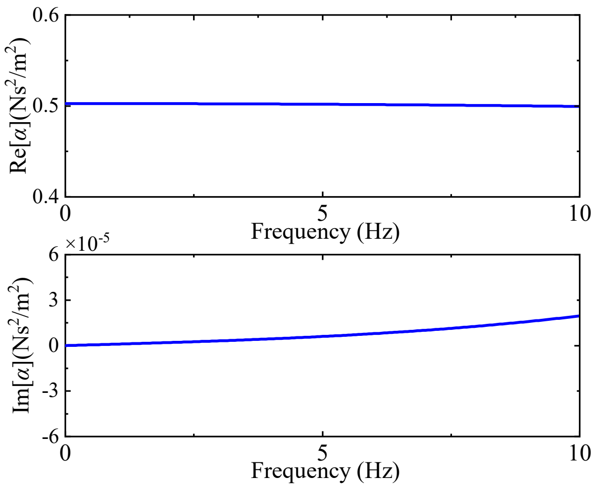

Fig. 5. Real and imaginary parts of the identified coefficients

The determined coefficients are shown in Fig.5. The error of the nonlinear damping coefficient α is only 1.2%. As shown in Fig.6, the underlying estimated FRF h can be obtained by the NSI. Results show that the underlying estimated FRF h is in quite good agreement with the true value.

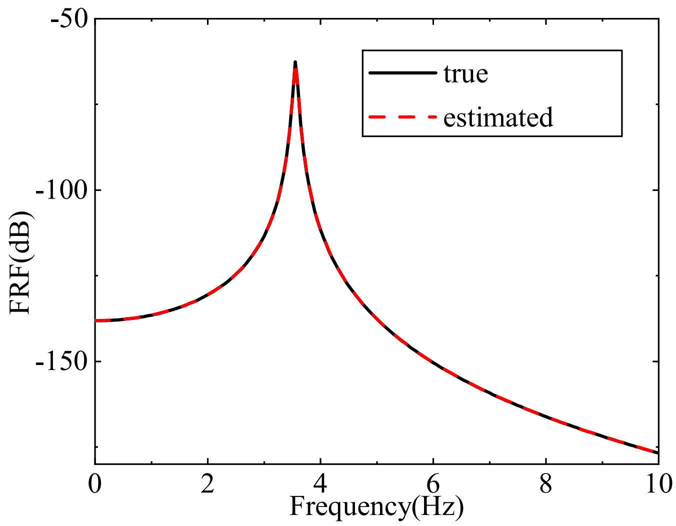

Fig. 6. Underlying linear FRF h_{11}

3.2 Three degrees of freedom with Coulomb and quadratic friction

The three-degree-of-freedom nonlinear system shown in Fig. 5 is excited by a zero-mean Gaussian random force at DOF 3 only. The system parameters are summarized in Table 2.

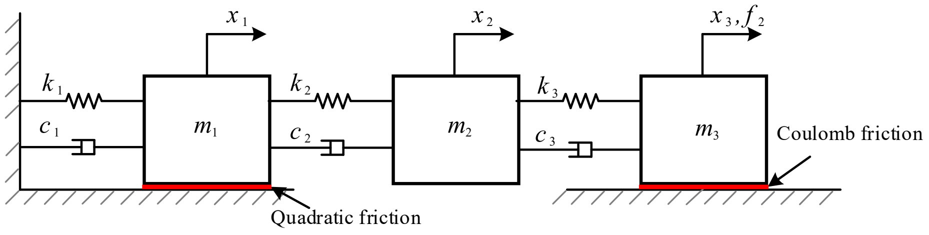

Fig. 7. Three-degree-of-freedom nonlinear system with a quadratic friction to ground at DOF 1 and the Coulomb friction to ground at DOF 3

Table 2. System parameters

Mass (kg)	Linear stiffness (N/m)	Damping (Ns/m)	Nonlinear damping
m_1=m_2=1	k_1= k_3=800	c_1= c_2=2	α_1=5
m_3=1.5	k_2=1000	c_3=1	α_2=0.2

The system is excited by a zero-mean Gaussian random input, whose root-mean-square (r.m.s) value is 5 at node 3. The response is calculated by 4th Runge-Kutta. The effect of the measurement noise on the parameter estimation results is investigated by corrupting the previously generated output adding 2% Gaussian zero-mean noise. The displacement is shown in Fig.8.

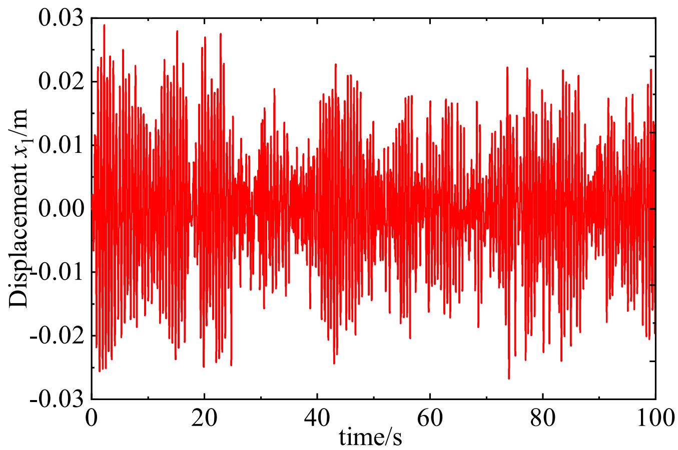

Fig. 8. The displacement of the system

The model order n=6 is determined by inspecting the singular value plot in Fig.4 (with i=40 block rows), where a jump of four orders of magnitude between model order six and seven is observed.

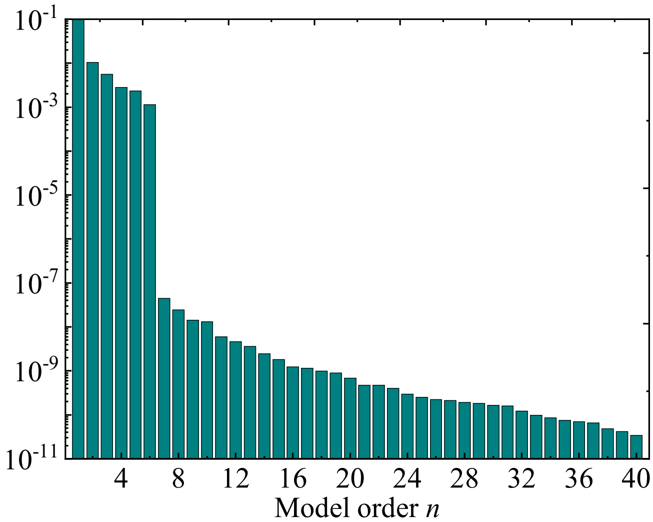

Fig. 9. Singular value plot with 2% measurement error and i=40

The determined coefficients are shown in Fig.5. It is worth highlighting that the imaginary part is always much lower than the absolute value of the real part in the selected frequency range, which assesses the goodness of the identification. The identified damping coefficients are reported in Table 3. The max error is only 0.18%.

Table 3. Identified results

Nonlinear damping coefficients	Exact value	Identified value	Error/%
α_1	5	5.010	0.18
α_2	0.2	0.199	0.11

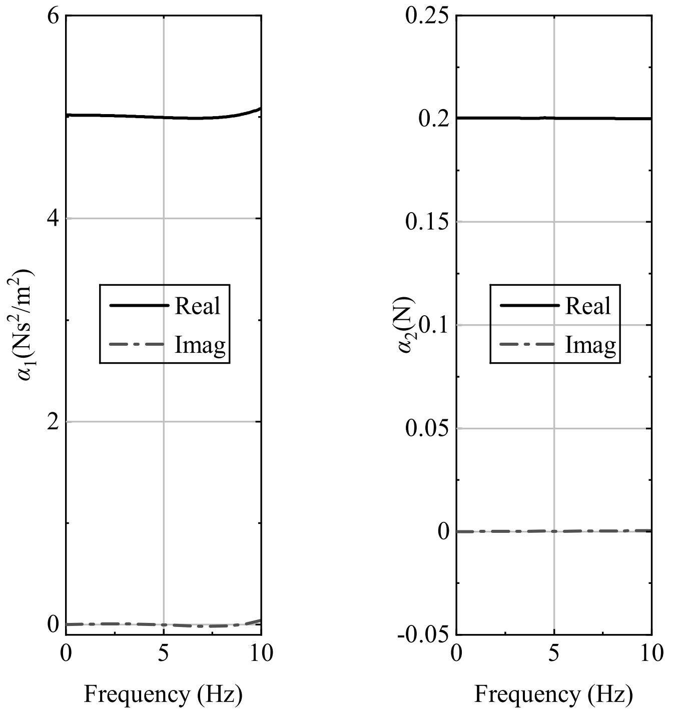

Fig. 10. Real and imaginary parts of the identified coefficients

As shown in Fig.10, the underlying estimated FRF h_{13} can be obtained by the NSI. Results show that the underlying estimated FRF h_{13} is in quite good agreement with the true value.

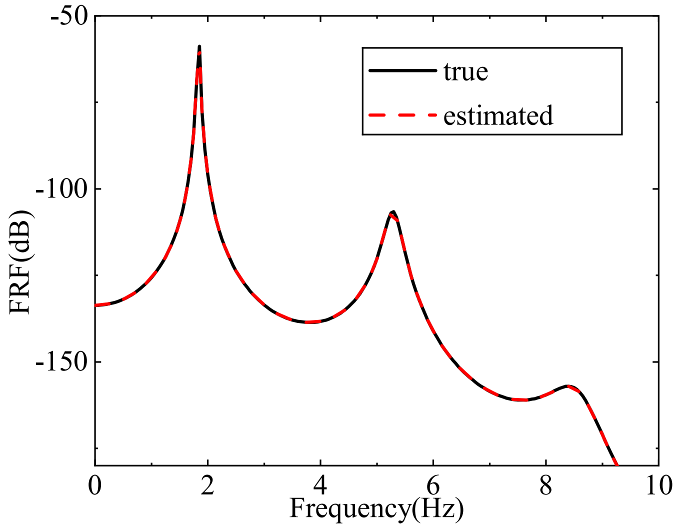

Fig. 11. Underlying linear FRF h_{13}

4 Conclusion

Two common kinds of nonlinear damping are successfully identified by nonlinear subspace method. The effect of the measurement noise on the parameter estimation results is investigated by corrupting the previously generated output adding different Gaussian zero-mean noise. Results show that the proposed method can fully characterize the nonlinearities in the structure and effectively identify the nonlinear damping parameters.

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