IAC-21-C1,3,3,x63869

Guidance and Control in Autonomous Debris Removal Space Missions via Adaptive Nonlinear Model Predictive Control

Marco Scaffidi Lallaro[a]*, Carlo Novara[b], Simone Vidano[c]

[a] *Mechatronic Engineering Student, Polytechnic of Turin, 24 Corso Duca degli Abbruzzi, Turin, Italy*, s253810@studenti.polito.it
[b] *Department of Electronics and Telecommunications, Polytechnic of Turin, 24 Corso Duca degli Abbruzzi, Turin, Italy*, carlo.novara@polito.it
[c] *Department of Electronics and Telecommunications, Polytechnic of Turin, 24 Corso Duca degli Abbruzzi, Turin, Italy*, simone.vidano@polito.it
* Corresponding Author

Abstract

Space debris orbiting around the Earth are becoming a major problem that could impair the future of space exploration. Among the different approaches to this problem that have been proposed in recent years, this work focuses on a possible innovative solution, consisting in an autonomous spacecraft that performs a rendezvous maneuver, collects a debris of unknown mass and then moves to a parking orbit. When the spacecraft collects a debris of unknown mass, the dynamics of the system may change substantially, and this may affect the control stability and performance of the spacecraft. In this paper, a control system is designed, capable of handling situations with time-varying and uncertain parameters, as it occurs in space debris removal missions. A control strategy based on an Adaptive Nonlinear Model Predictive Control (ANMPC) is considered. The unknown mass of the debris is treated as an uncertain parameter and is estimated by means of two different methods (Recursive Average and Extended Kalman Filter (EKF)). Then, the estimated mass is used to update the internal model of the ANMPC, which later solves an on-line optimization problem, providing an optimal trajectory and control action for reaching the debris and then the parking orbit. The simulations carried out show that the proposed control system is able to effectively accomplish the requested task.

Keywords: Guidance Navigation and Control (GNC), Adaptive Nonlinear Model Predictive Control (ANMPC), Extended Kalman Filter (EKF), Active Debris Removal (ADR).

Nomenclature

Coordinates of the Chaser in a Target-Centered Reference	$z_{1:3}$
Command Input (GE frame)	$u_{1:3}$
Command Input (LVLH frame)	$u_{rel\ 1:3}$
Input Initial Conditions (GE frame)	$u_{0\ 1:3}$
Chaser Mass	m_1
Angular Velocity	ω
Gravitational Parameter	μ
Position Vector (GE frame)	$r, x_{1:3}$
Cube of *r* Euclidean Norm	r^3
Position Vector (LVLH frame)	$r_{rel}, x_{rel\ 1:3}$
Velocity Vector (GE frame)	$v, x_{4:6}$
Velocity Vector (LVLH frame)	$v_{rel}, x_{rel\ 4:6}$
Eccentricity	e
Inclination Angle	i
Semi-Major Axis	a
Earth Radius	r_e
Drag Force	F_d
Local Atmospheric Density	ρ
Reference Density	ρ_0
Drag Coefficient	C_D
S/C Area projected along the direction of motion	S

Altitude from the Earth Surface	r_0
Scale Height Coefficient	H
Disturbances	d
S/C Mass (no fuel)	m_b, m_1
S/C Total Mass	m, x_7
Exhaust Velocity	v_e
S/C State Vector (GE frame)	$x_{1:7}$
S/C Initial Conditions (GE frame)	$x_{0\ 1:7}$
S/C State Vector (LVLH frame)	$x_{rel\ 1:7}$
Debris State Vector (GE frame)	$x_{d\ 1:7}$
Debris Initial Conditions (GE frame)	$x_{d0\ 1:7}$
Discrete State Vector (GE frame)	$x_{p\ 1:8}$
Output Equation	h, y
Estimated Debris Mass	$m_{2\ est}, x_8$
Constant Reference Signal	ref
Tolerance	tol
Debris Mass	m_2
Threshold	u_t
Semi-Major Axis Reference	a_r
Eccentricity Vector Reference	$e_{r\ 1:3}$
Inclination Angle Reference	i_r
Partial Simulation Time	t_{step}
Simulation Time	t_{total}

Sampling Time	T_s
Prediction Time	T_p
Fuel Mass	m_{fuel}
NMPC Weighted Matrices	(Q, P, R)
System Order	n
Predicted Tracking Error	$\tilde{y}_p$

Acronyms/Abbreviations
MPC Model Predictive Control
NMPC Nonlinear MPC
ANMPC Adaptive NMPC
MIMO Multiple Input - Multiple-Output
FBL Feedback Linearization
SMC Sliding Mode Control
GS Gain Scheduling
w.r.t. with respect to
RH Receding Horizon
LTI Linear Time Invariant
S/C Spacecraft
GNC Guidance, Navigation and Control
HCW Hill-Clohessy-Wiltshire
LEO Low-Earth Orbit
CoM Center of Mass
2B Two-Bodies
FR2B Free Two-Bodies
GE Geocentric Equatorial
LVLH Local-Vertical Local-Horizontal
DCM Direction Cosine Matrix
RLS Recursive Least Squares
EKF Extended Kalman Filter
ORE Orbit Equation
ADR Active Debris Removal
AOCS Attitude and Orbit Control Systems
LQR Linear Quadratic Regulator

1. Introduction

Space debris orbiting around the Earth are becoming a major problem that could impair the future of operational space missions and space. More than 15000 objects have been catalogued in orbit around the Earth at which only the 6% are active satellites [1].

The debris problem was formulated for the first time in the 60s by Ley [25]. Then, in 1978 Kessler formulated a possible dystopian scenario that takes the name of Kessler syndrome. In this dystopian future, the Space pollution could be great enough to make no more possible Space exploration [26].

In addressing this problem, the United Nations stated some guidelines for Space Debris Mitigation, but mitigation alone is not sufficient to solve the problem. For this purpose, several Active Debris Removal (ADR) methods were proposed in the last decade, while the Space agencies started their own space programs for these kinds of missions. Different methods to approach the debris problem can be found in the literature, see, e.g., [1] [2].

Depending on how a debris can be removed, the following classification can be outlined:

- Collective Methods
- Laser-based Methods
- Ion-beam Shepherd-based Methods
- Tether-based Methods
- Sail-based Methods
- Dynamical Systems-based Methods

The most effective ways to deal with large debris are the ones in which a system enters in contact with them. For example, in the collective ones a Spacecraft (S/C) is equipped with a mechanical system (such as a robotic arm or tentacles). After the Docking maneuver a debris can be collected and removed from its critical orbit.

This work focuses on a possible solution consisting in an autonomous S/C that performs a rendezvous maneuver, collects a debris of unknown mass and then moves it to a parking orbit.

In [4], a feedback linearization method is used to accomplish an autonomous non-cooperative rendezvous maneuver. The equations used to describe the relative motion are a modified version of the Hill-Clohessy-Wiltshire (HCW) equations. The limitations of the feedback linearization method do not make possible to consider constraints and the dynamics equations must be written in affine form. As it will be seen later, constraints must be considered, and the equations, that describe the dynamics of the problem, are not in affine form.

In [5], two CubeSats are provided with a Linear Quadratic Regulator (LQR). The command input is calculated from an optimization problem and applied in open-loop. Since the optimal sequence is applied in open-loop, it is not possible to dealing with varying constraints and/or to counteract unexpected behaviours.

In [6], the GNC problem is analyzed for an ADR mission that exploits a robotic arm as capture system. In this article two different controllers are proposed for Guidance and Control modules. A LQR is used for Guidance purposes, while both a LQR and a PID controller are used for Control purposes. An Extended Kalman Filter (EKF) and an Unscented Kalman filter are used in the Navigation module. Also in this case, the drawbacks coming from the LQR are not taken into account.

The aim of the present work is to develop a control algorithm that is able to accomplish the Guidance and Control tasks for a wide range of ADR missions. In order to consider time-varying constraints and to deal with the system's equations that are not in affine form, an adaptive version of NMPC is developed. This control method allows us also to obtain optimal solutions for the Guidance and Control problem.

The critical region for debris concentration is at an orbital altitude between 600 to 800 [km]. For this reason, the analysis is concentrated only on the control system that has to be installed on a hypothetical Spacecraft (S/C) that works in a LEO scenario (that includes the critical region seen before). In this way, the control algorithm can be extended to a wide range of ADR missions that could use different collective methods: robotic arm, tentacle system, harpoon, and nets.

One important issue for the automation of an ADR system is that when the space debris is collected by the S/C, its unknown mass affects the dynamics of the whole system. In this case the information on the S/C dynamics alone is not sufficient to successfully accomplish the different tasks of a space mission. For this purpose, the ANMPC control strategy is developed. This approach is implemented ad-hoc in the debris removal space mission, where the unknown debris mass, treated as a parameter in the control problem, is estimated via two different methods (Recursive Average and EKF) in order to obtain an optimal trajectory needed for the debris successfully removal.

A key feature of NMPC is its capability to optimally menage the constraints coming from the different maneuvers and also to counteract the differences between the used models.

The mission considered in this paper is divided in five different maneuvers. In each one the controller sets its parameter in a gain scheduling fashion. The scheduled parameters, as the references and the sample time, are selected thanks to some conditions. In this way, the system understands when a specific maneuver must start. It is important noting that the last Orbit-Change maneuver is considered to bring debris with large size in an empty orbit by increasing the apogee distance.

2. Mission Scenario

The considered debris removal mission is composed by several orbital maneuvers:

1. Rendezvous maneuver: the S/C approaches the debris;
2. Docking maneuver: the S/C gets in contact with the debris;
3. Estimation maneuver: the unknown mass of the debris is estimated;
4. Second Rendezvous maneuver: the system (composed by S/C and debris) goes near to a specific starting point for the other maneuver. This maneuver is not necessary, but may be convenient to have a known starting point, or also to start the orbit-change maneuver from an empty orbit;
5. Orbit-Change maneuver: the S/C brings the debris in another orbit to avoid collisions problem.

In this paper, we focus on S/C orbital/trajectory control, without considering attitude control. This last task would clearly require to estimate also the inertia matrix of the full body composed by the S/C and the debris.

2.1 Rendezvous Maneuver

In this maneuver, two objects approach to a very close distance. The S/C, that has to move in order to enter in contact with the second object, is called the Chaser. While the object that is moving in its own orbit is called the Target.

The Chaser moves by considering a specific final point near to the Target. The side on which the two bodies have to be, at the final, is very important. In order to satisfy this requirement and to avoid collision problems between them, specific constraints on the state (or on the output) must be accounted for. Another important constraint is represented by input saturation.

The equations used to describe the relative motion between the two objects are the HCW equations. They describe the dynamics of the Chaser in a neighborhood of the Target [10].

An important observation is that if the variation of the Chaser mass (due to the fuel consumption) is not considered, these equations become LTI [10].

2.2 Docking Maneuver

Docking, similarly to Rendezvous, is a precision maneuver. It consists in two bodies that establish a rigid and stable contact. After this maneuver the two bodies can be treated as an unique object.

This maneuver typically starts after the Rendezvous, but it has more stringent constraints. While the Chaser approaches the Target in the docking point, the maneuver space is reducing. This constraint is implemented by means of a conic constraint on the state (or as output constraint). This input saturation constraints have to be more restrictive than the ones considered for the Rendezvous. Indeed, the relative velocity between the two bodies must converge to zero in order to avoid collisions and damages [10].

The equations that describe the motion, also in this case, are the HCW equations.

2.3 Estimation Maneuver

This maneuver is executed in this specific mission in order to estimate the mass of the debris. In [21],[22], and [23] it can be seen that an higher estimate precision is obtained by moving the system randomly in a given region.

By means of some estimation algorithm the mass is estimated and then the maneuver is completed.

2.4 Orbit-Change Maneuver

This maneuver consists in changing the orbit of the debris. Considering this specific mission, an orbit-change maneuver is accomplished by increasing the semi-major axis. The S/C brings the debris faraway, in another orbit that is empty, in order to avoid collision problems with other satellites.

3. Models

3.1 S/C Model

The S/C model that we adopt is based on the 2B equations. Only one of the most relevant perturbations is considered, that is the Drag Force due to residual of atmosphere in LEO. Indeed, the contribution of the Atmospheric Drag is significant for the S/C motion in a LEO. The 2B equations are the following:

$$\begin{aligned} &\dot{r} = v \quad , \qquad \dot{v} = -\mu \frac{r}{|r|^3} + \frac{1}{m}(F_d + d + u) \\ &F_d = -\frac{1}{2}\rho C_D S |v| v \\ &\rho(r) = \rho_0 e^{-\frac{r-r_0}{H}} \\ &u = 0 \;\; if \;\; m \le m_b \\ &\dot{m} = \begin{cases} 0 & if\ u = 0 \\ -\dfrac{|u|}{v_e} & if\ u \neq 0 \end{cases} \end{aligned} \tag{1}$$

The equations that describe the motion of the debris are the FR2B equation:

$$\dot{r} = v \quad , \qquad \dot{v} = -\mu \frac{r}{|r|^3} \tag{2}$$

In the debris orbit, it is not considered any perturbation because:

1. If it is considered the atmospheric drag, it can be seen that it is proportional to S. In the debris case this is assumed to be lower than the S/C case;
2. It is supposed that the first two maneuvers (Rendezvous and Docking) happen too quickly to appreciate the effect of small perturbations on the debris orbit.

Inside the plant there are both the equations that describe the motion of the S/C and of the debris. The controller uses the information given by the solutions of these equations in order to accomplish the mission tasks. However, it is worth noting that the system model is not the same used by the ANMPC.

3.2 Prediction Models

The ANMPC algorithm is based on an internal prediction model that in general may be different from the plant model.

The precision maneuvers (Rendezvous and Docking) require a model that considers the relative motion between the S/C and the debris. For this purpose, the HCW equations are used. The model based on the HCW equations is:

$$\begin{aligned} &\dot{x}_{rel\,1} = x_{rel\,4}, \;\; \dot{x}_{rel\,2} = x_{rel\,5}, \;\; \dot{x}_{rel\,3} = x_{rel\,6} \\ &\dot{x}_{rel4} = 3\omega^2 x_{rel\,1} + 2\omega^2 x_{rel\,5} + {u_{rel\,1}}/{x_{rel\,7}} \\ &\dot{x}_{rel\,5} = -2\omega^2 x_{rel\,4} + {u_{rel\,2}}/{x_{rel\,7}} \\ &\dot{x}_{rel\,6} = -\omega^2 x_{rel\,3} + {u_{rel\,3}}/{x_{rel\,7}} \\ &\dot{x}_{rel\,7} = 0 \\ &h = x_{rel\,1:7} \end{aligned} \tag{3}$$

In this version the total mass is supposed to remain constant in the prediction interval. After the estimation maneuver, for finding the debris mass, the estimated value will enter in this model.

While the prediction model used in the last maneuver, (once the mass is estimated) is based on the 2B equation:

$$\begin{aligned} &\dot{x}_1 = x_4, \quad \dot{x}_2 = x_5, \quad \dot{x}_3 = x_6 \\ &\dot{x}_{4:6} = -\mu \frac{x_{1:3}}{r^3} + \frac{u_{1:3}}{x_7 + m_{2\,est}} \\ &\dot{x}_7 = 0, \qquad h = x_{1:7} \end{aligned} \tag{4}$$

4. Adaptive Nonlinear Model Predictive Control

NMPC is a general and flexible approach to control of complex nonlinear systems NMPC provides optimal solutions (over a finite time interval), can deal with input/state/output constraints, and can manage systematically the trade-off between performance and command activity. Successful applications of NMPC can be found in many areas, such as automotive engineering, aerospace engineering, chemical process management, robotics, biomedicine, etc. Here, a concise but self-contained formulation of NMPC is provided.

4.1 Nonlinear Model Predictive Control

NMPC is based on two fundamental operations, that are computed at each sample time:

- Prediction over a finite time horizon.
- Optimization.

In the Prediction step, the Controller uses a model (prediction model seen in section 3.2) of the plant to predict the future behavior of the system. Based on the predicted behavior, the algorithm chooses the best input sequence to be provided to the system (on-line Optimization) [7, 11-15].

4.1.1 Prediction

At each sampling time, NMPC makes a prediction of the system behavior over a given time interval. This time is called the *prediction horizon*, and it must be always bigger (or at least equal) than the sample time.

The prediction model is given by the state equation and by the output equation. To make the prediction, the controller integrates the state equations, that describe the system, starting from the initial conditions given by the current state. Then, the predicted output is obtained by considering the output equation.

One important observation is that, in the time interval, the input is calculated in open-loop. The idea of Predictive Control is to search, thanks to an optimization algorithm, an input signal such that the output will have the desired behavior. Then, at each sample time, this controller recalculates the optimal input and applies it to the system (Receding Horizon Principle). In this way it is possible to obtain a closed-loop system that is sensitive to the environment changes.

4.1.2 Optimization

In order to formalize the concept of desired behavior for the output, an *Objective Function* (or *Cost Function*) is defined:

$$J(u(t:t+T_p)) = \int_t^{t+T_p} \left(\|\tilde{y}_p(\tau)\|_Q^2 + \|u(\tau)\|_R^2\right) d\tau + \|\tilde{y}_p(t+T_P)\|_P^2 \qquad (5)$$

The optimization problem is to minimize the cost function w.r.t. the command input. Minimize *J* means minimizing the tracking error, the norm of the input signal (so the command activity), and the tracking error at the end of the time interval. In this way, the desired behavior is obtained. In order to obtain this, the designer chooses (by trial and error procedures) the coefficients of the weight matrices *Q*, *R*, and *P*.

4.2 Adaptivity

In order to make adaptive a controller, the following operations are carried out:

1. Characterize the desired behavior of the closed-loop system;
2. Determine a suitable control law with adjustable parameters;
3. Finding a mechanism for adjusting the parameters;
4. Implement the control law;

The idea is to combine NMPC with an adaptive parameter estimation method.

The estimation problem refers to the empirical evaluation of an uncertain variable, like an unknown characteristic parameter or a remote signal, on the basis of observations and experimental measurements of the phenomenon under investigation.

An adaptive controller is a controller with adjustment parameters and a mechanism for adjusting the parameters.

4.2.1 Estimation Maneuver

Thanks to the on-line estimation of the debris mass, it is possible to complete optimally the GNC problem. The NMPC adapts its behavior based on this estimation.

There are two main methods to estimate some variable: recursive algorithms and filters (observers). In this work three algorithms are tested to accomplish this task. The first two are recursive algorithms, while the third one is an observer:

- Recursive Average Algorithm;
- Recursive Least Squares (RLS) Algorithm;
- Extended Kalman Filter (EKF);

These work on-line, hence, at each sampling time they update the estimated debris-mass variable.

Once the Docking maneuver is completed, there is a rigid connection between the S/C and the debris. For this reason, in the equations used to model the plant will enter the real value of the debris mass.

In this maneuver, also the prediction model used by the NMPC is written according to the 2B equations. This time the estimated value of the debris mass enter as a state variable. In this way, it is possible to obtain an adaptive NMPC.

The controller gives to the system a command input that it is calculated on the basis of the estimated value of the real debris mass. The system, by using this command input, reacts according to the equations that describe its dynamics. If *u* is too small, it is not possible to appreciate a divergent trajectory from the planned one. For this reason, it is needed a condition that rejects estimated values when *u* is below a certain threshold.

The average recursive algorithm is built by considering the equations that describe the system motion. Indeed, it is possible to rewrite them in function of the unknown mass. In this way, it is possible to have three equations (one for each direction) on which collecting data. Here, it is reported only the algorithm used to collect $m_{2\,(1)}$ (on direction 1). $m_{2\,(2)}$ and $m_{2\,(3)}$ have the same form.

Algorithm 1: Algorithm for collecting estimated values of the debris mass in one direction
Input: u, u_t, x
Output: $m_{2\,(1)}$
Initialization:
1: IF cycle (6)
2: **if** $|u_1| \geq u_t$ **then**
3: $m_{2\,(1)} = \frac{u_1}{\dot{x}_4 + \frac{\mu x_1}{\|x_{1:3}\|^3}} - x_7$
4: **else** $m_{2\,(1)} = 0$
5: **end if**
6: **return** $m_{2\,(1)}$

Where u_t is used to discard wrong estimated values (if $u \to 0 \Rightarrow m_{2i} \to 0$). Only the estimated values of the mass, corresponding to a given command input (greater than a given threshold), are considered.

The algorithm, on the basis of the collected values coming from the three directions, first computes the average of them at each sampling time. Then, it calculates recursively the average in time. The output, that correspond to the estimated debris mass, is applied to the controller as state variable:

Algorithm 2: Algorithm for calculating the estimated values of the debris mass
Input: $m_{2\,(1:3)}$
Output: $m_{2\,est}$
Initialization:
1: IF cycle
2: **if** $m_{2\,(1)} > 0$ **and** $m_{2\,(2)} > 0$ **and** $m_{2\,(3)} > 0$ **then** (7)
3: $m_{2\,partial} = \frac{1}{3}(m_{2\,(1)} + m_{2\,(2)} + m_{2\,(3)})$
4: **elseif** $(m_{2\,(1)} \neq m_{2\,(2)} \neq m_{2\,(3)})$ **and** $(m_{2\,(1)} = 0$ **or** $m_{2\,(2)} = 0$ **or** $m_{2\,(3)} = 0)$
5: $m_{2\,partial} = \frac{1}{2}(m_{2\,(1)} + m_{2\,(2)} + m_{2\,(3)})$

6: **else** $(m_{2\,(1)} = m_{2\,(2)} = 0$ **or** $m_{2\,(2)} = m_{2\,(3)} = 0$ **or** $m_{2\,(1)} = m_{2\,(3)} = 0)$
7 $m_{2\,partial} = (m_{2\,(1)} + m_{2\,(2)} + m_{2\,(3)})$
8: **end if**
9: FOR cycle
10: **for** $i = 1$ to the dimension of $m_{2\,partial}$ **do**
11: $m_{2\,est\,(i,:)} = \frac{1}{i}\sum m_{2\,partial\,(i,:)}$
12: **end for**
13: **return** $m_{2\,est}$

Where i is the *i-th* present time. It contains all the previous collected values.

Another approach based on a RLS method for the estimation problem is considered. But this method is not applicable since the estimation result depends on the fuel consumption. A solution is to consider constant the total S/C mass in a certain range and then apply the RLS algorithm in sufficiently small number of samples. By repeating this procedure, every *N*-samples (in which *x(7,:)=constant* is considered), it is possible to find different estimation values. Then, by considering a recursive average, it is possible to refine the estimated value of the mass. Another option is to reduce the input saturation constraint and obtain a small mass consumption. In this case, the small divergence can be seen as error on the measured output. A sufficiently small number of samples, also in this case, are needed in order to avoid strong variations. Moreover, as seen before, if *u* is too small there can be wrong estimated values.

The EKF works in discrete time and with the Jacobians of the system computed along the trajectory. The working principle is based on two fundamental steps:

1. *Prediction:* The algorithm computes a prediction of the state using a model of the system;
2. *Correction:* The measurements on the output are used to improve the prediction and obtain a better state estimate;

In this application the following mathematical model and settings are chosen:

- EKF model: It corresponds to a discretization of the model used by the controller. The discretization is made using Euler forward method:

$$x_{p\,1:3} = x_{4:6}\tau + x_{1:3}$$
$$x_{p\,4:6} = \left(-\frac{\mu x_{1:3}}{r^3} + \frac{u_{1:3}}{x_7 + x_8}\right)\tau + x_{4:6} \qquad (8)$$
$$x_{p\,7} = x_7\,, \quad x_{p\,8} = x_8, \quad y = x_{1:7}$$

- Covariance Matrices:

$$P_0 = diag(10^{-6}_{7\times1}, 5\cdot10^{6})$$
$$Q^d = diag(10^{-3}_{7\times1}, 5\cdot10^{-9}) \qquad (9)$$
$$R^d = diag(10^{2}_{7\times1})$$

5. Simulation Results

The solutions coming from the recursive average method and from the EKF are compared. The first method is simple, it doesn't require any additive computational effort, or to modify the system. The solutions show that convergence towards the real mass value is higher than the one given by the EKF, and the estimation result is sufficiently accurate. The EKF can be used in those applications that require an high precision on the measurements. Indeed, it is possible to use multiple sensors combined with the EKF in order to increase precision.

In the following, the adopted simulation parameter values are reported.

Table 1. Simulation Parameters

Drag Force	*C_D=1, S=12 [m^2]* *ρ_0=1.22 [kg/m3]* *H=8e3 [m]*
Other Perturbations and Disturbances	*std(d(t))=1 [N];*
Condition tolerances (only for the precision maneuvers)	*tol_1=0.5 [m], tol_2=0.05 [m/s] (Rendezvous),* *tol_2=0.005 [m/s] (Docking)*
Measurements Errors (only in the Estimation maneuver):	*std(w(t))=($1_{(3x1)}$[m], $10^{-1}_{(3x1)}$[m/s],10^{-3}[kg])* *std(dx(t))=($1_{(3x1)}$[m], $10^{-1}_{(3x1)}$[m/s],10^{-3}[kg])* *std($\dot{dx}$(t))=$10^{-4}_{(7x1)}$ ($1_{(3x1)}$[m/s],$1_{(3x1)}$[m/s^2], 1 [kg/s])* *std(du(t)) = $10^{-1}1_{(3x1)}$[N].*

In order to have a smooth docking, a stringent condition is considered on the relative velocity.

In the following table, the times and Delta-V values obtained in each maneuver are reported.

Table 2. Time and Delta-V

	Time [s]	**Delta-V [m/s]**
Rendezvous	1100	0.1203
Docking	1550	0.0080
Estimation	1600	52.5443
Rendezvous	2500	52.5663
Orbit-Change	5000	267.6127
Total Mission	5000	372.8515

5.1 General Settings

In order to accomplish the various mission maneuvers, a scheduling strategy is needed. The controller algorithm is fixed, but other elements change, such as the used setting parameters, the constraints (on the input and on the state), the final reference point, and the prediction model used by the controller.

A specific maneuver starts when the previous one is completed. By considering the reference signals, that give the relative final points in the first two maneuvers and in the third one, it is possible to run the algorithm until a certain precision is achieved. This precision has the meaning of a tolerance on the S/C final point and velocity. In other words, for each condition, the algorithm run a specific part until the specific condition remains satisfied. When these conditions are no more satisfied, the algorithm sets the parameters needed for the next maneuver.

In the simulations carried out, the parameter values shown in Table 1 have been considered.

Table 3.

Parameter	Value
μ	$0.3986e15\ [m^3/s^2]$
r_e	$6.38\ 10^6\ [m]$
t_{step}	$50\ [s]$
t_{total}	$8\ 10^3 [s]$
m_b	$10^3 [kg]$
m_{fuel}	$10^4 [kg]$
m_2	$550\ [kg]$
v_e	$4.4\ 10^3\ [m/s]$
r_{deb}	$r_e + 400\ 10^3 [m]$
ω	$\sqrt{\mu/r_{deb}^3}\ [rad/s]$
x_{d0}	$\left(r_{deb}, 0,0,0, \sqrt{\mu/r_{deb}}, 0, m_2\right)$
$x_{0\,(1)}$	$200 + r_{deb}$
x_0	$\left(x_{0\,(1)}, 0,0,0, \sqrt{\mu/x_{0\,(1)}}, 0, m_2\right)$

It is worth noting that both S/C and debris at initial time are at perigee and in an equatorial circular orbit.

5.2 Maneuver Settings

5.2.1 First Rendezvous Maneuver

In this maneuver, the NMPC algorithm uses the HCW equations as the internal prediction model. The axes of the LVLH frame are constructed as follows:

- The debris CoM is the origin (O′);
- The Local-Vertical axis is defined along the direction OO' on the orbit plane (where O is the Earth CoM);
- The Local-Horizontal axis is perpendicular to the local vertical, it is on the orbit plane, and the sign is concordant with the orbital velocity;
- The Orbit Pole axis is given by the vector product of the first two axes, and it is perpendicular to the orbit plane;

Based on these considerations, it is possible to write the unit vectors of the three axes and to find the relation between the two frames (by means of rotation matrix).

In this way, it is possible to write the S/C position components in the LVLH frame.

Considering a final position point for the S/C, near to the debris, a constant reference signal is need. It is specified on the LVLH frame: $ref_1 = (20,0,0,0,0,0)$

Where the first three components are the reference values for the relative position, while the last three are the references values for the relative velocity.

The algorithm conditions for this maneuver are:

$$\begin{aligned} &|(\|x_{rel\,1:3}^T\| - \|ref_{1\,1:3}\|)| - tol_1 \geq 0 \\ &|(\|x_{rel\,4:6}^T\| - \|ref_{1\,4:6}\|)| - tol_2 \geq 0 \end{aligned} \tag{10}$$

By means of simulation, the algorithm collects at each partial simulation time the values of the relative state (it is composed by N samples with $N=(t_{step}/t_{sampling})$). The last collected values are used to understand if the final point is reached with a given tolerance. If this point is not reached, the algorithm reruns the piece of code relative to this maneuver until the previous conditions are no more satisfied.

The NMPC algorithm uses an inequality, representing the state constraint. In this maneuver, a collision path with the debris must be avoid. For this reason, a spherical constraint debris-centered (with radius equal to 10) is considered. It is important to note that if it is seen from the inertial GE frame, it becomes a varying constraint. Instead, in the LVLH frame the same constraint is static:

$$F \leq 0\ ,\ F = 10 - \left\|x_{rel\,1:3,:}\right\| \tag{11}$$

The following NMPC settings are considered for this maneuver (they were found by means of trial and error procedure).

Table 4. NMPC Settings

Variable	Value
T_s	$1[s]$
T_p	$200[s]$
n	7
Q	$0_{6\times6}$
P	$10\ I_{6\times6}$
R	$0.1_{3\times1}$
Input Saturation Constraint	$\lvert u\rvert \leq 10^4_{3\times1}[N]$

5.2.2 Docking Maneuver

The Docking maneuver takes place when the Rendezvous is completed. The only things that change wrt the Rendezvous are the settings used, the reference signal, and the constraint function.

Since this maneuver is more precise than the first one, the controller must have more stringent constraints on the state and on the input. In order to give more importance to the final reached point, higher coefficients of the weight matrix P are chosen (this matrix is related to the final predicted error, if P coefficients increase the algorithm takes care to minimize more these variables than others).

The final point that the S/C has to reach is the following, given by the constant reference signal:

$ref_2 = 0_{6\times1}$

The algorithm conditions are in the same form of the ones seen in (10). The only thing that changes is the reference that in this case are tolerances and reference.

The constraint function in this case has the shape of a cone. Indeed, while the S/C approaches the debris the space in which it can move is reducing. The considered function is a cone with vertex coincident with the debris CoM, height equal to the first component of the final point given by the Rendezvous maneuver and equal to the base radius. In other words, at the initial point, the height of cone is coincident with the distances between the S/C and the debris, the radius of the base circumference is equal to that distance. While the S/C approaches to the debris, the distance is reducing (hence the height) as well as the base radius (that remains equal to the height in the considered point). The mathematical formulation is the following:

$$F \leq 0$$
$$F = \begin{cases} \|x_{rel\,2:3,:}\| - \|x_{rel\,1,:}\| & if\ 0 \leq x_{rel\,1,:} \leq ref_{1\,(1)} + tol \\ 0 & otherwise \end{cases} \quad (12)$$

As can be seen this constraint is implemented by considering an initial condition. If the distance between the S/C and the debris is inside the cone the constraint is given, otherwise there are not constraints.

Table 5. Changed NMPC Settings

Variable	Value
Q	$10_{6\times6}$
P	$10^3\ I_{6\times6}$
R	$0_{3\times1}$
Input Saturation Constraint	$\lvert u\rvert \leq 5\ 10^3_{3\times1}[N]$

5.2.3 Estimation Maneuver

The constant reference signal is specified in the GE frame. In this way, the problem has the meaning of set-point maneuver. It is worth noting that the system doesn't reach this reference since the estimation maneuver takes place only in a finite period equal to the Partial Simulation Time. For this reason, the algorithm conditions, to repeat this maneuver, are not needed. There are only flags to understand when the maneuver is made. The considered state constraint is related to the Earth. The planned trajectory cannot pass near to the Earth surface, or worst it cannot be planned as a collision trajectory.

Table 6. Changed NMPC Settings

Variable	Value
State Constraint	$1.05r_e - \|x_{1:3,:}\| \leq 0$
ref_3	$(10^{10}_{1:3}, 0_{4:7})$
T_s	$0.01[s]$
T_p	$100[s]$
n	8
Q	$0_{6\times6}$
P	$10\ I_{6\times6}$
R	$0.1_{3\times1}$
Input Saturation Constraint	$\lvert u\rvert \leq 10^4_{3\times1}[N]$

5.2.4 Second Rendezvous Maneuver

This manuver is needed to have a convenient initial starting point for the Orbit-Change maneuver. The mission task of this work is to bring the debris faraway in another orbit. It is convenient to start from a point that is on an empty orbit, or that minimizes the planned trajectory without considering constraints due to the presence of other satellites.

This type of maneuver is Rendezvous/Docking-like. A final point in a relative frame is considered as reference. This time, there are not objects to approach and for this reason the constraints can be considered less stringent than in the first two precision maneuvers.

The system plant remains the same used in the estimation maneuver. While, once the estimation process is completed, the final value of the debris mass is applied as a constant parameter into the controller model.

For the sake of simplicity, the final point of this maneuver is considered equal to the one that would have the CoM of the debris in its previous orbit. The constant reference signal and the conditions algorithm are equal to the ones used in the Docking maneuver.

The considered state constraint function takes into account that the planned trajectory can not pass near to the Earth's surface, but this time it is written in the LVLH frame:

$F \leq 0, F = 1.05r_e - \|[x_1 - r_e + r_{deb}, x_2, x_3]\|$

Where r_{deb} is the orbit radius that the debris would have if it left in its own orbit.

Table 7. Changed NMPC Settings

Variable	Value
T_s	$1[s]$
T_p	$100[s]$
n	7
Q	$0_{6\times6}$
P	$I_{6\times6}$
R	$0_{3\times1}$

Input Saturation Constraint	$\lvert u\rvert \le 10^4_{3\times1}[N]$

5.2.5 *Orbit-Change Maneuver*

The last maneuver, to accomplish the mission task, is the Orbit-Change maneuver. As seen before, it consists of increasing the major-semi axis of the debris orbit.

The model used by the controller is written according to the 2B equations:

$$\begin{aligned}
&\dot{x}_{1:3} = x_{4:6} \\
&\dot{x}_{4:6} = -\mu\frac{x_{1:3}}{r^3} + \frac{u_{1:3}}{x_7 + m_{2\,est}} \\
&\dot{x}_7 = 0, \qquad y = or2oe(x)
\end{aligned} \tag{13}$$

Where the output y is expressed in orbital-elements thanks to the *rv2oe* transformation function.

Since the output is expressed in orbital-elements, also the constant reference signal, that has to be tracked, must have the same form:

$ref_4 = (a_r, e_{r1}, e_{r2}, e_{r3}, \cos(i)_r) =$
$= (r_e + 900 \cdot 10^3, 0,0,0,1)$

The reference orbit was considered circular, equatorial (since $e_r=0$ and $(\cos i)_r=1$), and it is at distance a_r from the Earth's CoM.

The state constraint in this case is equal to the one seen in the estimation maneuver.

Table 8. Changed NMPC Settings

Variable	Value
T_s	$5[s]$
T_p	$100[s]$
n	7
Q	$0_{5\times5}$
P	$diag(1, 10^6_{2:4}, 10^7)$
R	$0_{3\times1}$
Input Saturation Constraint	$\lvert u\rvert \le 132\ 10^3_{3\times1}[N]$

5.3 *Command Input, Position, and Velocity*

This section shows some figures, referred to the whole mission, where command input, position, and velocity signals can be seen. They are expressed for both reference frames used: Absolute GE frame, and Relative LVLH frame.

The constraints on the input for the precision maneuvers are expressed in the LVLH frame. While for the Estimation and Orbit-Change they are expressed in the GE frame. As it can be seen they are satisfied.

In the following relative and absolute plots of position and velocity signals are given:

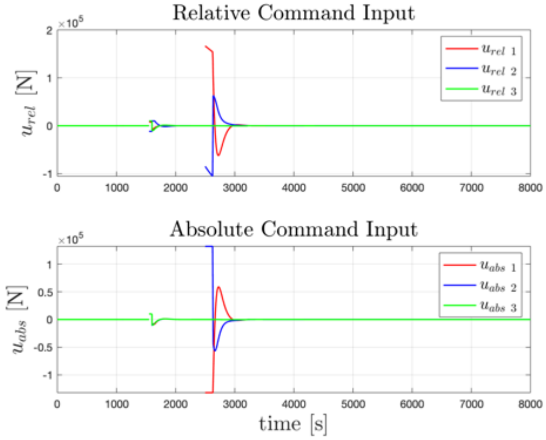

Fig. 1. Relative and Absolute Command Input Vs Time.

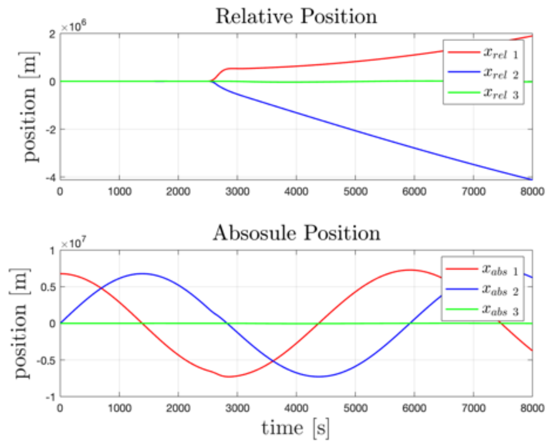

Fig. 2. Relative and Absolute Position Vs Time.

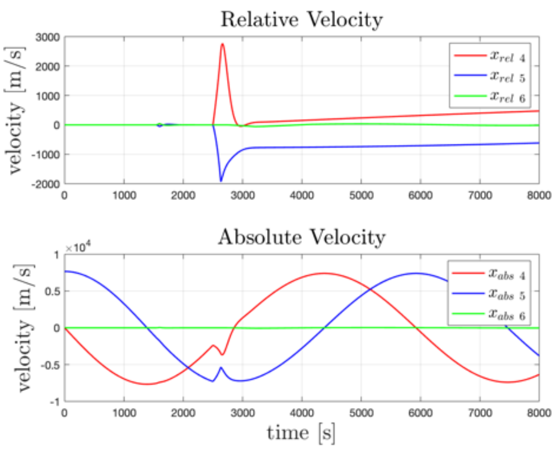

Fig. 3. Relative and Absolute Velocity Vs Time.

5.4 Trajectory

For the first two precision maneuvers a zoomed 2D plot is shown in the relative reference frame to appreciate the constraints for these maneuvers. In all the maneuvers the constraints on the state are satisfied.

The blue circumference (in 3D a sphere) indicates the constraint function used for the Rendezvous. While the blue triangle (in 3D a cone) is used for the Docking.

In the 3D plot, the initial position between S/C and debris is too small to appreciate a difference between the two trajectories. In figures 5 and 6 debris and S/C trajectories have red and blue colors respectively. In green the Earth constraint, and in yellow the Earth radius.

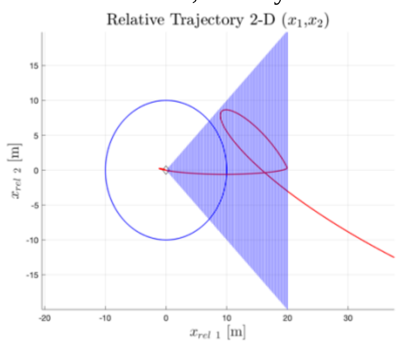

Fig. 4. Zoomed Relative trajectory after the first two maneuvers in a 2D plane.

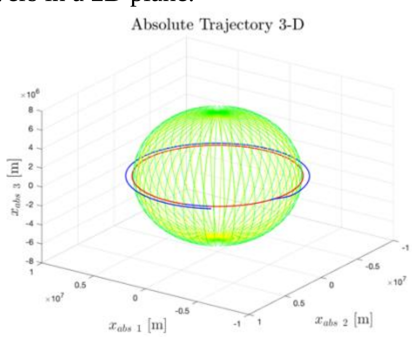

Fig. 5. Absolute trajectories in a 3D space.

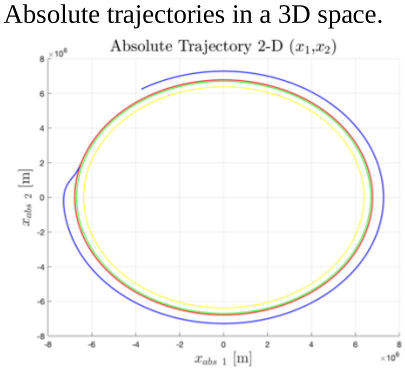

Fig. 6. Absolute trajectories in a 2D plane.

5.5 Fuel Consumption

A table containing fuel consumption mass in each maneuver, and a figure showing the decreasing total mass are now reported.

Tab. 9. Fuel Consumption in each maneuver

	m_{lost} [kg]
Rendezvous	2.96
Docking	0.8
Estimation	196.82
Rendezvous	344.43
Orbit-Change	8767
Total Mission	9312.10

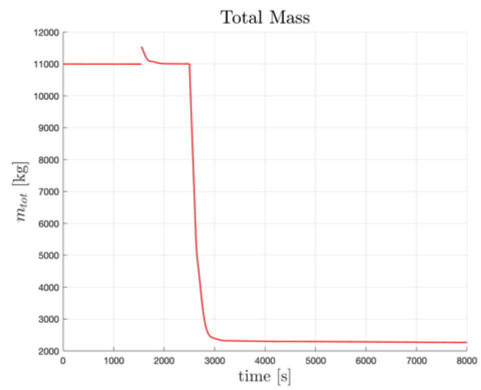

Fig. 7. Total Mass Vs Time.

5.6 Estimation Process

In the following, we show:

- The results given at each sampling time of the three equations used to estimate the debris mass (one for each direction);
- The result of the recursive average;
- The result of the EKF estimation;
- The Estimation errors;
- Final estimated values;
- Final Errors;

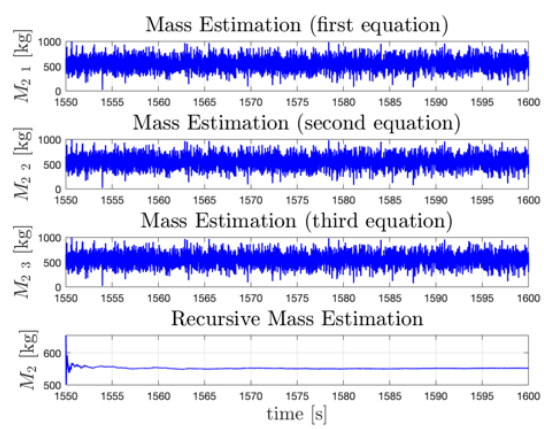

Fig. 8. Estimated Mass via Recursive Average Algorithm Vs Time.

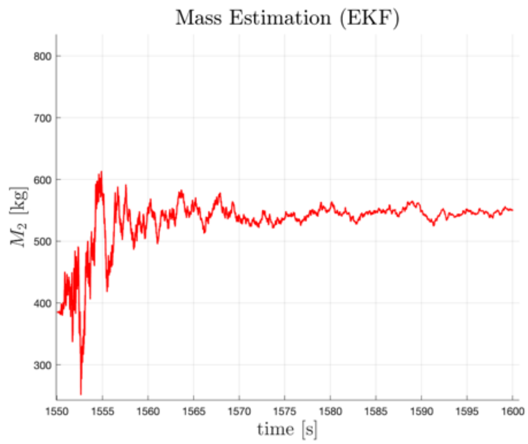

Fig. 9. Estimated Mass via EKF Vs Time.

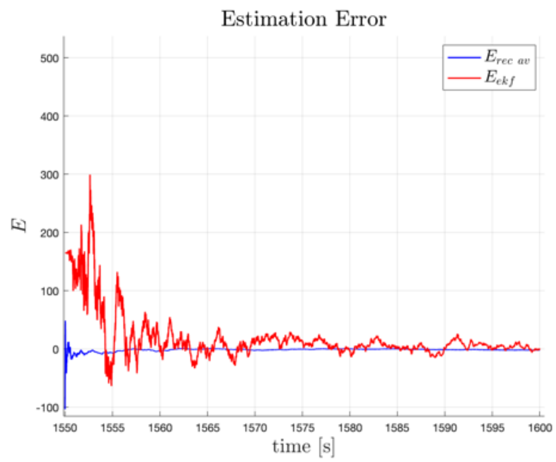

Fig. 10. Estimation Error Vs Time.

	AV	EKF
$\mathbf{m_{2\,est}}$ **[kg]**	552.03	549.07
$\mathbf{\lvert e_{final}\rvert}$ **[kg]**	2.03	0.92

It can be also seen that the errors slowly converge to zero.

5.7 Comparison without Estimating the Debris Mass

Thanks to its flexibility, the NMPC can find a solution (very near to the optimal one) also in the presence of unknown estimation of the debris mass. Indeed, the NMPC is very robust to system variations. The results taking into account are the ones related to the trajectories of the maneuver that takes place after the estimation process. The final point is reached at the same time: *t = 2500 [s]*.

The solution without the estimated value of the debris mass, inside the control model, doesn't diverge significantly from the one in which it is considered.

Thanks to this robustness feature, the controller can be applied also when it is not possible to have a precise mass estimation. This could be very important in that debris removal mission with small debris masses.

In any case, if an estimation algorithm is considered the solutions converge faster and there is a better management of the trade-off between performance and command activity. In all the cases where an high precision is needed, it is almost mandatory to use an estimation process in order to increase it and consequently to avoid possible collisions with other objects. It is also possible, thanks to these considerations, to save fuel and consequently money.

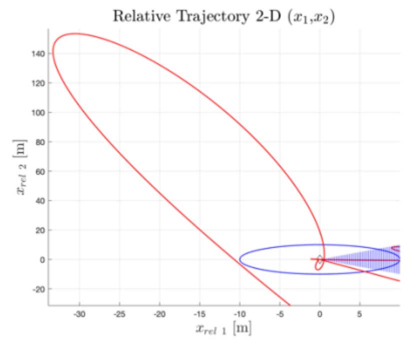

Fig. 11. Zoomed Absolute Trajectory with m_2 Estimation.

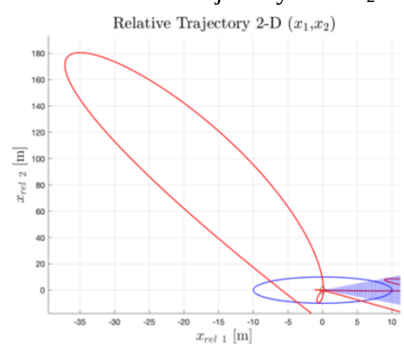

Fig. 12. Zoomed Absolute Trajectory without m_2 Estimation.

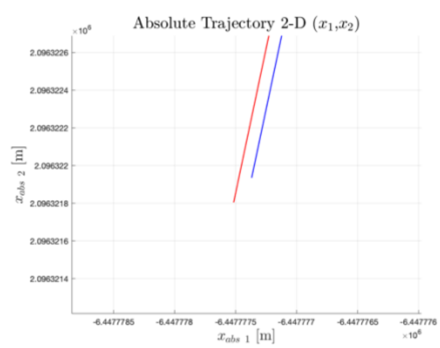

Fig. 13. Zoomed Relative Trajectory with m_2 Estimation.

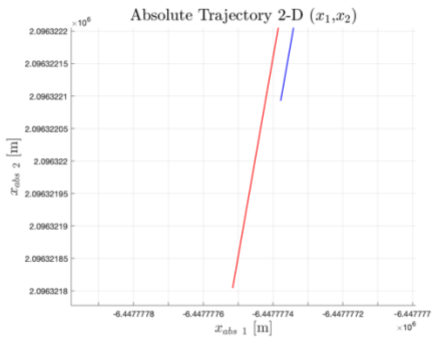

Fig. 14. Zoomed Relative Trajectory without m_2 Estimation

6. Conclusion

In this paper, a control strategy based on Nonlinear Model Predictive Control has been proposed, suitable for space debris removal missions. A key feature of this strategy is its flexibility. Indeed, it can be effectively used in different maneuvers such as rendezvous, docking, estimation maneuver, orbital change. A second relevant feature is that the underlying control algorithm of the strategy is the same for all the maneuvers. Only some parameters of the algorithm need to be tuned for each specific maneuver, implying a relatively simple and systematic design. Another important feature is that, thanks to an embedded estimation algorithm, the strategy is adaptive, allowing the spacecraft to correctly perform the required maneuvers even in the presence of significant changes of some spacecraft parameter. The effectiveness of the proposed strategy has been demonstrated by means of several simulations, where the strategy was able to correctly perform all the maneuvers of the considered debris removal mission.

References

[1] W. Pulliam, Catcher's Mitt Final Report, DARPA technical report, 2011;

[2] C. P. Mark, S. Kamath, Review of Active Space Debris Removal Methods, Space Policy 47, 194-206, 2019;

[3] J. L. Forshawa, et. al, The active space debris removal mission RemoveDebris, Acta Astronautica 168, 293-309, 2020;

[4] W. Wang, L. Chen, K. Li, Y. Lei, One active debris removal control system design and error analysis, Acta Astronautica 128, 499–512, 2016;

[5] C. Pirat, et. al, Mission design and GNC for In-Orbit Demonstration of Active Debris Removal technologies with CubeSats, Acta Astronautica 130, 114-127, 2017;

[6] M. Jankovic, J. Paul, F. Kirchner, GNC architecture for autonomous robotic capture of a non-cooperative target: Preliminary concept design, Advances in Space Research 57, 1715-1736, 2016;

[7] A. Bemporad, F. Borrelli, M. Morari, Predictive Control for Linear and Hybrid Systems, Cambridge University Press, 2017;

[8] UCS Satellite Database, https://www.ucsusa.org/resources/satellite- database;

[9] P. Mazzoldi, M. Nigro, C. Voci, Fisica 1, Edises, Moti Relativi, 72-90;

[10] E. Canuto, C. Novara, L. Massotti, C. Perez Montenegro, D. Carlucci, Spacecraft dynamics and control. The embedded model control approach, Butterworth-Heinemann, 2018;

[11] L. Gru ̈ne, J. Pannek, Nonlinear Model Predictive Control, Springer;

[12] M. Morari, J. H. Lee, Model predictive control. Past, present and future, Computers and Chemical Engineering 23, 667-682, 1999;

[13] Y. Wang, S. Boyd, Fast Model Predictive Control Using Online Optimization, IEEE Transactions on Control Systems Technology 18, 267- 278, 2010;

[14] H. Fukushima, T. Kim, T. Sugie, Adaptive model predictive control for a class of constrained linear systems based on the comparison model, Automatica 43, 301–308, 2007;

[15] J. Rowlings, D. Mayne, Model Predictive Control Theory and Design, Nob Hill Pub, 2009;

[16] K. J. Astrom, B. Wittenmark, Adaptive Control, Second edition, Dover Publications, 2008;

[17] G. Chowdhary, M. Mu ̈hlegg, J. P. How, Florian Holzapfel, Concurrent Learning Adaptive Model Predictive Control, Advances in Aerospace Guidance, Navigation and Control Selected Papers of the Second CEAS Specialist Conference on Guidance, Navigation and Control, Springer, 29-47, 2013;

[18] J. P. Nelson, M. J. Balas, R. S. Erwin, Model Reference Adaptive Control of Mildly Non-Linear Systems with Time Varying Input Delays, Advances in Aerospace Guidance, Navigation and Control Selected Papers of the Second CEAS Specialist Conference on Guidance, Navigation and Control, Springer, 49-59, 2013;

[19] V. Adetola, D. DeHaan, M. Guay, Adaptive model predictive control for constrained nonlinear systems, Systems & Control Letters 58, 320-326, 2009;

[20] K. Yang, Y. Kang, S. Sukkarieh, Adaptive nonlinear model predictive path-following control for a fixed-wing unmanned aerial vehicle, International Journal of Control, Automation and Systems 11, 65–74, 2013;

[21] B. Dolega, R. Glebocki, D. Kordos, M. Zugaj, Advances in Aerospace Guidance, Navigation and Control - Selected Papers of the Fourth CEAS, Springer, 2018;

[22] J. Bordeneuve-Guibe, A. Drouin, C. Roos, Advances in Aerospace Guidance, Navigation and

Control - Selected Papers of the third CEAS, Springer, 2015;
[23] R. Tekin, et. al., Advances in Aerospace Guidance, Navigation and Control - Selected Papers of the Second CEAS, Springer, 2013;;
[24] H. Shen, T. Zhang, L. Casalino, D. Pastrone, Optimization of ADR Missions with Multiple Targhets, Journal of Spacecraft and Rockets 55, 1-9, 2017;
[25] W. Ley, How to Slay Dragons, Galaxy Science Fiction. 57–72, 1960;
[26] D. J. Kessler, et. al., Collision Frequency of Artificial Satellites: The Creation of a Debris Belt, Journal of Geophysical Research. 83 (A6), 2637–2646, 1978;