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Probing a r^{-n} modification of the Newtonian potential with Exoplanets

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The growing availability of increasingly accurate data on transiting exoplanets suggests the possibility of using these systems as possible testbeds for modified models of gravity. In particular, we suggest that the post-Keplerian (pK) dynamical effects from the perturbations of the Newtonian potential falling off as the square or the cube of the distance from the mass of the host star break the degeneracy of the anomalistic, draconitic and sidereal periods. The latter are characteristic temporal intervals in the motion of a binary system, and all coincide in the purely Keplerian case. We work out their analytical expressions in presence of the aforementioned perturbations to yield preliminary insights on the potential of the method proposed for constraining the modified models of gravity considered. A comparison with other results existing in the literature is made.

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I. INTRODUCTION

Almost 25 years after the first discovery of a planet orbiting a main sequence star [1], there are nowadays more than 4000 known planets in more than 3000 known planetary systems[1]. Different detection techniques such as radial velocity, transit photometry and timing, pulsar timing, microlensing, astrometry [2, 3], sensitive to specific physical properties of the planetary systems, have been developed and used so far. In general, it is fruitful to combine different and complementary approaches to obtain a knowledge that is as extensive and complete as possible of the key physical and orbital features of the planetary systems of interest. The large number of confirmed and potential exoplanets, besides giving us the possibility of improving our understanding of planets formation, provides a potentially viable opportunity of testing fundamental laws of physics, such as gravity models, outside our Solar System.

Actually, General Relativity (GR) is the best available model of the gravitational interaction: its predictions were verified with great accuracy during last century even though challenges to the Einsteinian paradigm come from cosmological observations [4, 5]. Moreover, besides the issues deriving from the observation of the Universe at very large scales, it is a well known fact that GR is not renormalizable and it cannot be reconciled with a quantum framework [6, 7]: this ultimately implies that our current model of gravitational interactions stands apart from the Standard Model of particle physics. As a consequence, there are strong motivations suggesting that GR is just a suitable limit of a more general theory of gravity we still do not know. A description of various modified gravity models can be found, e.g., in the review papers [8–10].

However, due to the reliability of Einstein's theory, it is manifest that any model of modified gravity should be in agreement with the known tests of GR: every extended theory of gravity is expected to reproduce GR in a suitable weak-field limit. As a consequence, modified gravity models must have correct Newtonian and post-Newtonian limits and, up to intermediate scales, the deviations from the GR predictions can be considered as perturbations described by specific model-dependent parameters, whose values can be constrained by observations. Here, we suggest that transiting exoplanets, thanks to the continuous improvement in the high precision measurements performed, could provide the opportunity of testing modified models of gravity outside the Solar System; for other works dealing with alternative theories of gravity and exoplanets, see, e.g., [11, 12]. This is quite similar to what happened to pulsars systems, which have become celestial laboratories for theories of gravity [13, 14]. Actually, a recent work by Blanchet et al. [15] suggests that the general relativistic orbital precession could be detected by observing the exoplanet HD 80606b.

In transiting exoplanets, the time span $T_{\rm exp}$ characterizing the orbital revolution, generically dubbed as 'orbital period', which is actually measured is the time interval $T_{\rm tra}$ between two consecutive passages at the positions in

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[1] See, e.g., `http://www.exoplanet.eu/catalog/` on the Internet.

the orbit, called transit centres, which minimize the sky-projected distance $r_{\rm sky}$ of the planet from the star [16]. From a theoretical point of view, it is unclear how $T_{\rm exp}$ can be mathematically modelled when post-Keplerian (pK) dynamical effects (star's oblateness $J_2^\star$, possible N-body disturbances due to other planets in the system, general relativity, modified models of gravity, etc.) are at work in the system at hand. Possible choices are as follows [17]. The draconitic period $T_{\rm dra}$ is defined for a perturbed path as the time span between two successive instants when the real position of the test particle coincides with the ascending node position on the corresponding osculating orbit. The anomalistic period $T_{\rm ano}$ can be defined as the time interval between two successive instants when the real position of the test particle coincides with the pericentre position on the corresponding orbit. The sidereal period $T_{\rm sid}$ is the time interval between two successive instants when the real position of the test particle lies on a given reference direction in the sky. The latter may be, e.g., the one from which the longitudes are reckoned, i.e. the x axis in the coordinate systems which are usually tied to the plane of the sky. In a purely Keplerian case, $T_{\rm dra}$, $T_{\rm ano}$, $T_{\rm sid}$ are degenerate, being identical to the unperturbed Keplerian period. If some pK disturbing accelerations are present, the degeneration is generally broken [17]. Here, using the approach described in [18], we consider pK disturbing accelerations starting from a general stationary and spherically symmetric spacetime which can be thought of a solution of the field equation of modified gravity model, describing the gravitational field around a point-like mass, which will be the model of an exoplanetary system. In particular, we will focus on the perturbations of the Newtonian potential falling off as the square or the cube of the distance from the central mass, and we will work out the expressions of $T_{\rm dra}$, $T_{\rm ano}$, $T_{\rm sid}$.

This paper is organised as follows: in Section II we obtain the perturbing acceleration, while in Section III we calculate the perturbations of the anomalistic, draconitic and sidereal periods. Conclusion are drawn in Section IV.

II. THE PERTURBING ACCELERATION

We assume that the gravitational field of a point mass in a generic modified model of gravity is described by the following static and spherically symmetric metric[2]

$$(ds)^2 = [1+\phi(r)]\,(dt)^2 - [1+\psi(r)]\left[(dx)^2+(dy)^2+(dz)^2\right], \tag{1}$$

where $r=\sqrt{x^2+y^2+z^2}$, and $\phi(r)$, $\psi(r)$ are functions depending on the mass M of the source, which is the host star in our case, and, possibly, on other parameters of the theory.

We suppose that the effects of the the modified model can be considered as perturbations of the known GR solution, i.e. the Schwarzschild spacetime. This amounts to saying that $\phi(r)$ and $\psi(r)$ must approach their GR values: in other words we suppose that in a suitable limit, the metric takes the form

$$\phi(r) = \phi_{\rm GR}(r)+\phi_{\rm mod}(r), \tag{2}$$

$$\psi(r) = \psi_{\rm GR}(r)+\psi_{\rm mod}(r), \tag{3}$$

where the GR values are given by $\phi_{\rm GR}(r)=-2M/r+2M^2/r^2$, $\psi_{\rm GR}(r)=2M/r$ and the perturbations $\phi_{\rm mod}(r), \psi_{\rm mod}(r)$ due to the modified gravity model are such that $\phi_{\rm mod}(r)\ll\phi_{\rm GR}(r)$, $\psi_{\rm mod}(r)\ll\psi_{\rm GR}(r)$. In order to obtain the perturbing pK acceleration, we proceed as follows. First, we assume that in the given theory, the matter is minimally and universally coupled, so that test particles follow geodesics of the metric (1). Then, we consider the (post-Newtonian) equation of motion of a test particle (see [19])

$$\ddot{x}^i = -\frac{1}{2}h_{00,i}-\frac{1}{2}h_{ik}h_{00,k}+h_{00,k}\dot{x}^k\dot{x}^i+\left(h_{ik,m}-\frac{1}{2}h_{km,i}\right)\dot{x}^k\dot{x}^m, \tag{4}$$

where "dot" stands for derivative with respect to the coordinate time. Since in our notation it is $h_{00}=\phi(r)$ and $h_{ij}=\psi(r)\delta_{ij}$, we can write the perturbing acceleration $\mathbf{A}$ in the form

$$\mathbf{A} = -\frac{1}{2}\left\{\Phi(r)\left[1+\psi_{\rm mod}(r)\right]+\Psi(r)v^2\right\}\hat{\mathbf{x}}+\left[\Phi(r)+\Psi(r)\right](\hat{\mathbf{x}}\cdot\mathbf{v})\,\mathbf{v}, \tag{5}$$

where we set $\mathbf{x}=\{x,y,z\}$, $\mathbf{v}=\{\dot{x},\dot{y},\dot{z}\}$, $\hat{\mathbf{x}}=\mathbf{x}/|\mathbf{x}|$, and[3]

$$\Phi(r)\doteq\frac{d\phi_{\rm mod}(r)}{dr}, \qquad \Psi(r)\doteq\frac{d\psi_{\rm mod}(r)}{dr}. \tag{6}$$

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[2] If not otherwise stated, as in (10) to (15) below, we use units such that the speed of light in vacuum c and the Newtonian gravitational constant G are set equal to 1; bold face letters like $\mathbf{x}$ refer to spatial vectors while Latin indices refer to spatial components.

[3] Notice that in Eq. (5) we neglected cross-terms deriving from the product of GR and modified gravity potentials.

Since we are interested in the lowest order effects on planetary motion of $\phi_{\rm mod}(r)$ and $\psi_{\rm mod}(r)$, we may also neglect the non linear terms (i.e. non linear perturbations with respect to flat spacetime). To this end, we start by noticing that to Newtonian order, it is $v^2 \simeq \phi_{\rm GR}(r) \simeq M/r$. As a consequence, in (5) we may neglect the terms proportional to v^2 and to $(\hat{\mathbf{x}}\cdot\mathbf{v})\,\mathbf{v}$ (which is also proportional to the orbital eccentricity) and also the term $\Phi(r)\psi_{\rm mod}(r)$. In summary, in the weak-field and slow-motion limit the perturbing pK acceleration that we are going to consider is purely radial and is given by

$$\mathbf{A} = -\frac{1}{2}\Phi(r)\hat{\mathbf{x}} \doteq A_r\hat{\mathbf{x}}. \tag{7}$$

We consider two kinds of quite generic perturbations, in the form of power laws. Given a constant $\alpha^{(N)}$, which is a parameter deriving from the modified gravity model, the perturbations that we focus on are in the form:

$$\phi_{\rm mod}(r) = \frac{\alpha^{(N)}}{r^N}, \quad N > 1, \tag{8}$$

According to our definition (7), we have to following pK perturbing acceleration:

$$A_r = \frac{1}{2}\alpha^{(N)} N \frac{1}{r^{N+1}}, \tag{9}$$

We notice that, for a logarithmic perturbation in the form $\phi_{\rm mod}(r) = \beta\log(r/r_0)$, it is $A_r = -\frac{1}{2}\frac{\beta}{r}$, which can be dealt with as in the case (9) above. In particular, we will consider below the cases $N = 2, 3$.

III. THE ANOMALISTIC, DRACONITIC AND SIDEREAL PERIODS

As we said before, in the purely Keplerian case the anomalistic, draconitic and sidereal periods $T_{\rm dra}$, $T_{\rm ano}$, $T_{\rm sid}$ are degenerate, since they coincide with unperturbed Keplerian period $T_{\rm K} = 2\pi\sqrt{a^3/\mu}$, where a is the semimajor axis and $\mu \doteq GM$ is the gravitational parameter of the host star of mass M. As shown in [17], when some pK perturbing accelerations are present, the degeneration is generally broken. Since, at present, it is unclear which out of the aforementioned three temporal intervals is actually measured in transiting exoplanets, we will analytically calculate the pK corrections to all of them induced by the radial accelerations A_r of (9) for $N = 2, 3$, assumed as small perturbations of the Newtonian monopole. In the following, we will adopt the approach used in [17] to deal with some standard classical and general relativistic pK accelerations. According to [17], it seems that the measured period $T_{\rm exp}$ should be $T_{\rm dra}$ for circular orbits, being possibly $T_{\rm ano}$ for nonzero values of the eccentricity e.

It turns out that, while it is possible to analytically work out $T_{\rm ano}$ without recurring to any simplifying assumptions in e, only approximate expressions to the lowest orders in e can be obtained for $T_{\rm dra}$, $T_{\rm sid}$. For the anomalistic period, the pK corrections $\Delta T_{\rm ano}$ to $T_{\rm K}$ due to (9) for $N = 2, 3$ are

$$\Delta T_{\rm ano} = \frac{3\pi\alpha^{(2)}\sqrt{a}\,(1+e\cos f_0)^2}{\mu^{3/2}\,(1-e^2)^2}, \; N = 2, \tag{10}$$

$$\Delta T_{\rm ano} = \frac{3\pi\alpha^{(3)}\,(1+e\cos f_0)^3}{\sqrt{a\,\mu^3}\,(1-e^2)^3}, \; N = 3, \tag{11}$$

while for the draconitic and sideral periods, the corrections $\Delta T_{\rm dra}$, $\Delta T_{\rm sid}$ to the Keplerian period are

$$\Delta T_{\rm dra} \simeq \frac{4\pi\alpha^{(2)}\sqrt{a}\,(1+3\,e\,\cos\omega\,\cos u_0)}{\mu^{3/2}} + \mathcal{O}\left(e^2\right), \; N = 2, \tag{12}$$

$$\Delta T_{\rm dra} \simeq \frac{6\pi\alpha^{(3)}\,(1+3\,e\,\cos\omega\,\cos u_0)}{\sqrt{a\,\mu^3}} + \mathcal{O}\left(e^2\right), \; N = 3, \tag{13}$$

and

$$\Delta T_{\rm sid} \simeq \frac{4\pi\alpha^{(2)}\sqrt{a}\,(1+3\,e\,\cos\varpi\,\cos l_0)}{\mu^{3/2}} + \mathcal{O}\left(e^2\right), \; N = 2, \tag{14}$$

$$\Delta T_{\rm sid} \simeq \frac{6\pi\alpha^{(3)}\,(1+3\,e\,\cos\varpi\,\cos l_0)}{\sqrt{a\,\mu^3}} + \mathcal{O}\left(e^2\right), \; N = 3, \tag{15}$$

respectively. In (10) to (15), f_0 is the initial value of the true anomaly f, ω is the argument of periastron, u_0 is the initial value of the argument of latitude $u \doteq f + \omega$, $\varpi \doteq \Omega + \omega$ is the longitude of periastron, Ω is the longitude of the ascending node, and l_0 is the initial value of the true longitude $l \doteq f + \varpi$. Note also that $\left[\alpha^{(2)}\right] = \mathrm{L}^4\,\mathrm{T}^{-2}$, while $\left[\alpha^{(3)}\right] = \mathrm{L}^5\,\mathrm{T}^{-2}$.

From Eqs. (10)-(15) we see that the three periods are degenerate also for circular orbits $e = 0$. In this case, if we set

$$T_{\rm ano} = T_{\rm K} + \Delta T_{\rm ano}, \tag{16}$$

$$T_{\rm dra} = T_{\rm K} + \Delta T_{\rm dra}, \tag{17}$$

$$T_{\rm sid} = T_{\rm K} + \Delta T_{\rm sid}, \tag{18}$$

we may estimate the relative impact of the perturbation of the Keplerian period:

$$\frac{T_{\rm ano} - T_{\rm K}}{T_{\rm K}} = \frac{3\alpha^{(2)}}{2\mu a},\ N = 2 \tag{19}$$

$$\frac{T_{\rm ano} - T_{\rm K}}{T_{\rm K}} = \frac{3\alpha^{(3)}}{2\mu a^2},\ N = 3 \tag{20}$$

$$\frac{T_{\rm dra} - T_{\rm K}}{T_{\rm K}} = \frac{2\alpha^{(2)}}{\mu a},\ N = 2 \tag{21}$$

$$\frac{T_{\rm dra} - T_{\rm K}}{T_{\rm K}} = \frac{2\alpha^{(3)}}{\mu a^2},\ N = 3 \tag{22}$$

$$\frac{T_{\rm sid} - T_{\rm K}}{T_{\rm K}} = \frac{2\alpha^{(2)}}{\mu a},\ N = 2 \tag{23}$$

$$\frac{T_{\rm sid} - T_{\rm K}}{T_{\rm K}} = \frac{2\alpha^{(3)}}{\mu a^2},\ N = 3 \tag{24}$$

From the above results, it is evident that the impact of the perturbation is more important for exoplanets orbiting in small orbits around a low-mass hosting star. Moreover, in this ideal case, it is possible to build the quantities

$$\left(\frac{T_{\rm exp} - T_{\rm K}}{T_{\rm K}}\right)\mu a \ \text{ for } \ N = 2 \tag{25}$$

$$\left(\frac{T_{\rm exp} - T_{\rm K}}{T_{\rm K}}\right)\mu a^2 \ \text{ for } \ N = 3 \tag{26}$$

which do not depend on the planetary system but on the modified gravity model only. For comparison, the lowest post-Newtonian corrections, as calculated in [17], lead to relative perturbations of the orbital period in the form

$$\frac{T_{\rm exp} - T_{\rm K}}{T_{\rm K}} \simeq \frac{R_S}{a} \tag{27}$$

where R_S is the Schwarzschild radius of the hosting star.

Using the exoplanet catalogue[4] we may evaluate the above quantities to obtain preliminary estimates for the model parameters $\alpha^{(2)}, \alpha^{(3)}$. Accordingly, if we consider the exoplanet Kepler-77b [20], $a = 0.04501$ AU, $M = 0.95\, M_{\rm Sun}$ and supposing that $T_{\rm exp} - T_{\rm K}$ is of the order of the orbital period error amounting to $\sigma_{T_{\rm exp}} = 2.3 \times 10^{-7}\,{\rm d} = 0.02\,{\rm s}$, we obtain as upper limit $|\alpha^{(2)}| \leq 10^{22}\,{\rm m}^4\,{\rm s}^{-2}$. Similarly, if we consider the exoplanet WASP-12b [21], characterized by $a = 0.0234$ AU, $M = 1.434\, M_{\rm Sun}$, $\sigma_{T_{\rm exp}} = 1.44 \times 10^{-7}\,{\rm d} = 0.01\,{\rm s}$, we obtain $|\alpha^{(3)}| \leq 10^{32}\,{\rm m}^5\,{\rm s}^{-2}$.

We may compare the above estimates to those obtained in Iorio and Ruggiero [22], where data records of the Earth's artificial satellites of the LAGEOS family were used. Taking into account the different parameterisation of the perturbing accelerations, the constraints for the K_2, K_3 parameters used in Iorio and Ruggiero [22], since $\left|\alpha^{(2)}\right| = 2\,|K_2|$ and $\left|\alpha^{(3)}\right| = 6\,|K_3|$, imply $\left|\alpha^{(2)}\right| \leq 4.2 \times 10^6\,{\rm m}^4\,{\rm s}^{-2}$ and $\left|\alpha^{(3)}\right| \leq 10^{13}\,{\rm m}^5\,{\rm s}^{-2}$. Hence, we see that the preliminary constraints obtained from exoplanets are worse. It is important to point out that, in general, a comparison is actually meaningful only for those cases where the theory parameter $\alpha^{(N)}$ does not depend on the characteristics of the planetary system (mass, charge of host star) but on the gravity model only. This is the case, for instance of a $f(T)$ gravity model, which has extensively been studied by Iorio and Saridakis [23], Xie and Deng [24], Ruggiero and Radicella [25] by looking at the supplementary advances of the perihelia of some planets of our Solar system: in this model a perturbation in the form (9) with $N = 2$ arises. In particular, if we consider our results, we may constrain the theory parameter α used in those works. Since it is $\left|\alpha^{(2)}\right| = 32c^2\,|\alpha|$, we obtain $|\alpha| \leq 9.5 \times 10^3\,{\rm m}^2$. It is interesting to notice that, if, on the one hand, this bound is worse than those obtained by Xie and Deng [24] ($|\alpha| \leq 1.2 \times 10^2\,{\rm m}^2$) and Ruggiero and Radicella [25] ($|\alpha| \leq 2.3 \times 10^1\,{\rm m}^2$) by about $\simeq 2 - 3$ orders of magnitude, on the other hand, it is better than, or at least comparable to, that obtained by Iorio and Saridakis [23] ($|\alpha| \leq 1.8 \times 10^4\,{\rm m}^2$).

IV. CONCLUSIONS

By analytically calculating the corrections to the anomalistic, draconitic and sideral periods due to the perturbations of the Newtonian potential falling off as the square or the cube of the distance from the central mass, we have preliminarily laid the groundwork for testing such modified models of gravity also in planetary scenarios other than our solar system with the aim of the increasing amount of data available from transiting exoplanets.

While in the purely Keplerian case the aforementioned periods are degenerate, pK accelerations break the degeneracy and give rise to different expressions that we have analytically obtained. We remark that, such perturbations, are quite general and may derive from different modified gravity models, as discussed in [22].

For those gravity models where the theory parameters are independent of the characteristics of the planetary system, such as the mass of the host star, our results show that the impact of these corrections is more important for planets orbiting in small orbits around low-mass stars.

However, it is important to emphasize that our results are just intended to yield preliminary insights on the potential of the method proposed in constraining the modified models of gravity considered if and when actual dynamical modeling will reach the experimental accuracy level, and to design sensitivity analyses. They should not be regarded as actual tests since, in this case, the data of the exoplanets of interest should be reprocessed with ad-hoc modified dynamical models including also the standard pK features of motion and the alternative model(s) one is interested in.

Indeed, the values of the currently available orbital and physical parameters which should be inserted in (10) to (15) come from purely Keplerian data analyses. As such, they are unavoidably a priori "imprinted" by all the unmodelled pK dynamics, standard (such as, for instance, terms in the form (27)) and exotic, thus making the resulting values of the corrections ΔT of (10) to (15) purely indicative of the potentiality of the method. As pointed out in [17], it would be important to find some exoplanetary systems for which it is possible to measure independently measured two different orbital periods identifiable with some of the ones examined here. Indeed, their difference δT would cancel out the common Keplerian component $T_{\rm K}$, which is a major source of systematic error because of the lingering uncertainty on the stellar mass, leaving just a pK correction. On the other hand, even in this case, some competing classical and general relativistic pK terms may still enter δT, depending on the system's orbital geometry [17]. A possible solution would be, at least in principle, a generalization of the method proposed in a different context [26] by setting up suitable linear combinations of as many different periods as possible for the same planet and/or of more planets belonging to the same extrasolar system in order to disentangle, by construction, $\alpha^{(N)}$ from the other

[4] See, again `http://www.exoplanet.eu/catalog/` on the Internet.

standard classical and relativistic pK corrections.

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