A Numerical Study of the Spanwise Turbulence Past a Cylinder Flow

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Andrea Ferrero, Francesco Larocca, Guglielmo Scovazzi and Massimo Germano

Abstract Many flows of industrial interest and many important benchmark turbulent flows are statistically stationary in time and are provided with a spanwise direction of homogeneity. The numerical simulation of such flows is conditioned by the discretization in space and time, and the statistical analysis of the data is biased by the finite extent of the produced dataset. In this work the flow around a circular cylinder at Reynolds 3900 is numerically investigated by an implicit Large Eddy Simulation. The computations are performed by a modal Discontinuous Galerkin finite element solver and the produced database is analysed in order to quantify the temporal and spanwise contribution to the estimation of the statistics. The goal of the work is to investigate a procedure which allows to quantify the statistical efficiency of the operators which are used to perform the average in time and in the spanwise direction. Finally, the hierarchical nature of the modal basis used in each element is exploited to perform a local element-wise filtering operation which allows to quantify the contributions given by the smallest resolved scales to the statistics.

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1 Introduction

Many flows of industrial interest and many important benchmark turbulent flows are statistically stationary in time and are provided with a spanwise direction of homogeneity. Typical examples are the flow in blade cascades and more simply the flow around a cylinder. The numerical simulation of such flows is conditioned by the discretization in space and time, and the statistical analysis of the data is biased by the finite extent of the produced dataset. In a previous paper the authors have examined in detail the statistical error in the time averages [1], but we remark that, due to the spanwise homogeneity, statistics can be also computed with a combined average in time and in the spanwise direction. As remarked in [2], an insufficient spanwise extent can produce a overprediction of the streamwise and normal turbulent intensities and corrispondingly artificial suppression of the spanwise turbulent intensity. These considerations can be used to define a minimum spanwise extension required to correctly describe the evolution of the largest structures. However, this provides only a lower bound to the spanwise extension: in a general flow it is not clear a-priori whether it is more convenient to collect the statistics by performing a simulation with a relatively large spanwise extension and a small time window or the contrary. In order to find a numerical criterion which allows to quantify the efficiency of computing statistics in time and in the spanwise direction, the potential of some indices of statistical resolution is investigated.
In particular a database of the turbulent flow past a circular cylinder at Reynolds 3900 is produce by means of an implicit Large Eddy Simulation (LES). The computations are carried out by means of a modal Discontinuous Galerkin (DG) finite element solver based on the use of a modal orthonormal basis implemented following the guidelines of [3]. The solver is compressible but in this study a low far field Mach number ($M_\infty = 0.2$) is assumed in order to make comparisons with available experimental results in almost incompressible conditions [4].
Finally, the hierarchical nature of the modal orthonormal basis used inside each element allows to perform a local element-wise filtering operation: in this way it is possible to quantify the contributions given by the smallest resolved scales.

2 Indices of statistical resolution

The turbulent flow past a cylinder is characterised by two homogeneities, the time t and the spanwise spatial direction z, and we can average along one or both of them in order to estimate average fields and Reynolds stresses. Given the turbulent velocity field $u_i(x, y, z, t)$ we will introduce the following space and time *statistical* filtering operators $\mathcal{T}$ and $\mathcal{Z}$

$$\mathcal{Z}\{u_i\} \equiv \langle u_i \rangle_z \equiv \frac{1}{Z}\int_0^Z u_i(x,y,z',t)dz'$$
$$\mathcal{T}\{u_i\} \equiv \langle u_i \rangle_t \equiv \frac{1}{T}\int_0^T u_i(x,y,z,t')dt' \tag{1}$$

and we define as $\mathcal{E}$ the product of the two

$$\mathcal{E}\{u_i\} \equiv \mathcal{T}\mathcal{Z} \equiv \mathcal{Z}\mathcal{T} \equiv \langle u_i \rangle_e \equiv \frac{1}{ZT}\int_0^Z \int_0^T u_i(x,y,z',t')dz'dt' \tag{2}$$

where Z and T are the extents of the domain in the spanwise direction z and in time t. The turbulent stresses can be defined by means of the operators $\mathcal{T}$ and $\mathcal{Z}$

$$\begin{aligned}
\tau_z(u_i,u_j) &\equiv \langle u_i u_j \rangle_z - \langle u_i \rangle_z \langle u_j \rangle_z \\
\tau_t(u_i,u_j) &\equiv \langle u_i u_j \rangle_t - \langle u_i \rangle_t \langle u_j \rangle_t \\
\tau_e(u_i,u_j) &\equiv \langle u_i u_j \rangle_{zt} - \langle u_i \rangle_{zt} \langle u_j \rangle_{zt} \\
\tau_z(\langle u_i \rangle_t, \langle u_j \rangle_t) &\equiv \langle \langle u_i \rangle_t \langle u_j \rangle_t \rangle_z - \langle u_i \rangle_{zt} \langle u_j \rangle_{zt} \\
\tau_t(\langle u_i \rangle_z, \langle u_j \rangle_z) &\equiv \langle \langle u_i \rangle_z \langle u_j \rangle_z \rangle_t - \langle u_i \rangle_{tz} \langle u_j \rangle_{tz}
\end{aligned} \tag{3}$$

We note that due to $\mathcal{E} = \mathcal{Z}\mathcal{T} = \mathcal{T}\mathcal{Z}$ we have the two identities

$$\begin{aligned}
\tau_e(u_i,u_j) &\equiv \tau_z(\langle u_i \rangle_t, \langle u_j \rangle_t) + \langle \tau_t(u_i,u_j) \rangle_z \\
&\equiv \tau_t(\langle u_i \rangle_z, \langle u_j \rangle_z) + \langle \tau_z(u_i,u_j) \rangle_t
\end{aligned} \tag{4}$$

and we can define two measures of turbulence resolution, the first related to the time average and the second to the spanwise average, given by

$$\begin{aligned}
M_t(x,y) &= \frac{\langle \tau_t(u_i,u_i) \rangle_z}{R_{ii}} = 1 - \frac{\tau_z(\langle u_i \rangle_t, \langle u_i \rangle_t)}{R_{ii}} \\
M_z(x,y) &= \frac{\langle \tau_z(u_i,u_i) \rangle_t}{R_{ii}} = 1 - \frac{\tau_t(\langle u_i \rangle_z, \langle u_i \rangle_z)}{R_{ii}}
\end{aligned} \tag{5}$$

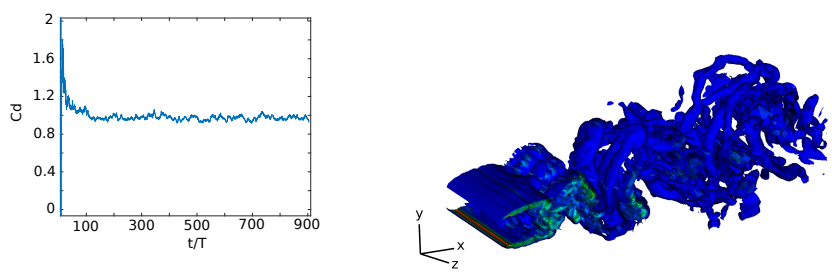
where $R_{ij} \equiv \tau_e(u_i,u_j)$. The indices $M_t(x,y)$ and $M_z(x,y)$ allow to quantify the relative contribution given by time and spanwise direction to the evaluation of the statistics. For example, where $M_t \to 0$ the statistics are mainly captured by sampling in the spanwise direction while where $M_t \to 1$ the statistics are mainly captured by sampling in time. The index M_z shows an opposite behaviour.

3 Numerical results

The simulations are performed on an unstructured mesh with approximately $3 \cdot 10^5$ elements by a third order accurate DG scheme ($p = 2$): $3 \cdot 10^6$ degrees of freedom are employed for each equation. Time integration is performed by means of an explicit

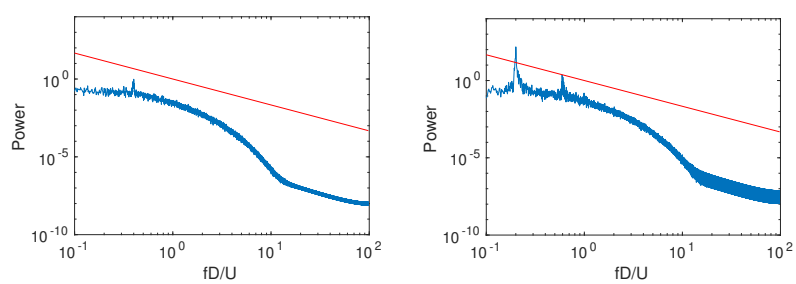
RK3 method. The spanwise extension of the domain along the direction z is set equal to three diameters D. The simulation is carried out for several hundreds of convective times CT, as shown in Figure 1a which reports the time evolution of the drag coefficient C_d. A snapshot of the vorticity magnitude field is reported in Figure 1b which puts in evidence the development of turbulence structures in the wake.
The spectrum of the streamwise and normal velocity in a station located at $x/D = 4$ and $y/D = 0$ is reported in Figure 2. The spectrum is computed from a time window with $400CT$ and it is averaged along the spanwise direction z. The plots show a peak at $fD/U = 0.4$ for the streamwise velocity and two peaks at $fD/U = 0.2$ and $fD/U = 0.6$ for the normal velocity: this is in line with the results reported by [4].
The R_{11} component of the Reynolds stress tensor is evaluated in the station at $x/D = 2.02$ according to Eq. 4. The results are reported in Figure 3a for two different choices of the average windows T and Z and they are compared with the experimental results from [4]. The numerical simulation is performed for $T = 400CT$ with a spanwise extension $Z = 3D$ and it is used to generate a global database. Two subsets are extracted from this database by selecting different extensions in time and spanwise direction: $(T = 100CT, Z = 3D/20)$ and $(T = 5CT, Z = 3D)$. In this problem the computational cost is directly proportional to both the spanwise extension and the duration of the simulated time window. This means that the same computational cost could be associated to the two subsets. However, the results reported in Figure 3a show that the estimation of the Reynolds stresses performed from the dataset with $(T = 100CT, Z = 3D/20)$ is significantly closer to the experimental results. In contrast, the results obtained from the dataset with $(T = 5CT, Z = 3D)$ appear to be far from statistical convergence since they are characterised by a strongly asymmetric distribution. This first test suggests that in the considered test case time averaging is more efficient with respect to averaging in the spanwise direction. This can be explained by the fact that in this problem the flow field is dominated by the streamwise velocity and so the transport of turbulent structures in the streamwise direction determines strong temporal fluctuations for a fixed control station. Furthermore, the streawise fluctuations are significantly stronger with respect to the spanwise fluctuations, as confirmed by the distribution reported in Figure 3b.
In order to verify the possibility to link the statistical efficiency of time and spanwise directions to a measurable quantity the indices M_t and M_z are evaluated and reported in Figure 4a: the plot shows clearly that the index M_t is systematically higher than the index M_z for all the values of y. This result suggests a general strategy which can be applied for statistically steady flows with a direction of spatial homogeneity. First of all, a preliminary simulation can be done by choosing the minimum spanwise extension which is necessary to allow the correct evolution of the largest structures and running the simulation for a few convective times. Then the results of this preliminary simulation can be used to estimate the indices M_t and M_z which provide insight in the statistical efficiency of time and spanwise direction: finally, a full simulation can be set up by increasing the runtime or the spanwise extension, according to the indications provided by M_t and M_z.
Finally, the plot in Figure 4b shows a comparison between the experimental values of R_{11} and the numerical predictions evaluated according to Eq. 4 for the full order

results ($p = 2$) and for a filtered solution ($p = 0$) obtained by truncating the results of the $p = 2$ simulation to the first term in each element. This makes it possible to quantify the contributions given by the turbulent structures whose size is comparable to the size of the element. The results show that this contribution is small in the present simulation and so it is possible to assume that the contribution of the subgrid scales is even smaller. This explains why the implicit LES approach provides reasonable results. However, for problems characterised by higher values of Reynolds number, the contributions associated to the subgrid scales can be more important and so an explicit LES with a subgrid model would be more suitable. In that case, the modal nature of the DG solution can be exploited to develop dynamic approaches for the subgrid scale model, following for example the guidelines provided by [5]. As a final remark, in this work the extensions of the spanwise and temporal windows are normalised with respect to the diameter and the convective time: it would be possible to get more physical insight by normalising with respect to the spatial and temporal turbulence scales. However, the use of diameter and convective time simplifies the investigation since these values are known a-priori.



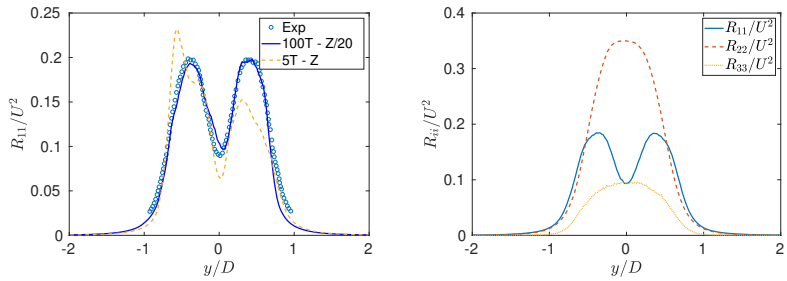
(a) (b)

Fig. 1 Drag coefficient history (a) and instantaneous vorticity magnitude field (b).



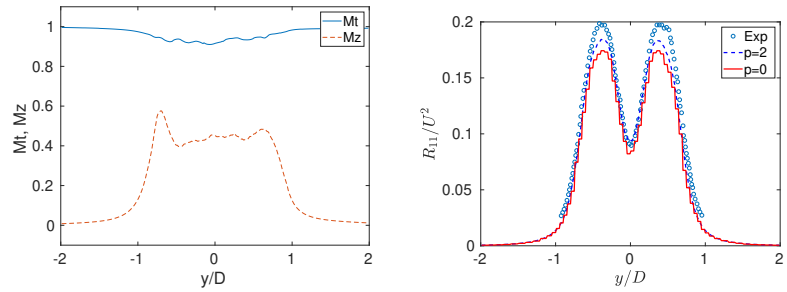
(a) (b)

Fig. 2 Spectrum for the streamwise (a) and normal (b) velocity at $x/D = 4$ and $y/D = 0$.



(a) (b)

Fig. 3 Effects of the choice on the spanwise extension and time window size on R_{11} (a) and comparison of the Reynolds stresses in the three directions (b) for the control station at $x/L = 2.02$.



(a) (b)

Fig. 4 Indices of statistical resolution (a) and element-wise modal filtering (b) for the control station at $x/L = 2.02$.

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References

1. A Ferrero, F Larocca, M Germano and G Scovazzi (2020) A study on the statistical convergence of turbulence simulations around a cylinder, AIP Conference Proceedings, 2293(1):200002
2. MR Mankbadi and NJ Georgiadis (2015) Examination of parameters affecting large-eddy simulations of flow past a square cylinder, AIAA J 53:1706–1712
3. F Bassi, L Botti, A Colombo, DA Di Pietro and P Tesini (2012) On the flexibility of agglomeration based physical space discontinuous Galerkin discretizations, J Comput Phys 231(1):45–65
4. P Parnaudeau, J Carlier, D Heitz and E Lamballais (2008) Experimental and numerical studies of the flow over a circular cylinder at Reynolds number 3900, Phys Fluids 20:085101
5. A Abbà, L Bonaventura, M Nini and M Restelli (2015) Dynamic models for Large Eddy Simulation of compressible flows with a high order DG method, Comput Fluids 122:209–222