Summary

This thesis discusses a statistical physics approach to linear estimation problems using expectation propagation (EP) based methods. EP dates back to about twenty years ago, when it was independently discovered in the context of statistical physics by Opper and later in the context of machine learning by Minka. The ideas underlying the method draw upon the Thouless-Anderson-Palmer approach introduced to study the physics of disordered systems and are well suited to be applied to probabilistic modeling in general. In this thesis, we have applied EP based schemes to a broad class of problems that, in their simplest form, can be cast as finding solutions of underconstrained systems of linear equations of the kind $\mathbf{F}\boldsymbol{x} = \boldsymbol{y}$, where $\boldsymbol{x} \in \mathbb{R}^N$, $\boldsymbol{y} \in \mathbb{R}^M$ and $M < N$, to be solved under additional constraints concerning the structure of the unknown vector $\boldsymbol{x}$, given the knowledge of the linear transformation $\mathbf{F}$ (often called the measurement matrix) and of the set $\boldsymbol{y}$ of partial observations. Generalizations of the problem just described often consist in sampling the vector $\boldsymbol{y}$ from a non linear or stochastic componentwise function of $\mathbf{F}\boldsymbol{x}$. The resulting set of problems is very general and widely arises in physics, information processing and engineering, especially in the context of applied optics, medical imaging and bioinformatics. Some important applications, for example, include tomography, magnetic resonance imaging and photon limited imaging. In this thesis, linear estimation problems are addressed in a Bayesian framework, where the additional constraints on the hypothesized structure of the sought solution are encoded in suitable prior distributions. Thus, the resulting problem to be solved approximately reduces to computing the marginals of some Boltzmann distribution, whose Hamiltonian is the sum of an interaction-like contribution penalizing solutions that do not fulfil the linear constraints and of an external field-like contribution related to the prior distributions associated with the unknowns.