Robust Consensus of Second-Order Heterogeneous Multi-Agent Systems via Dynamic Interaction

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***Abstract*— A consensus problem is proposed for second-order multi-agent systems with heterogeneous mass distribution. The motivation of this work is mainly related to spacecraft attitude coordinated control, in which gyroless configuration is considered, to avoid drift errors and design of estimation filters. The considered spacecraft includes flexible modes and coupling between the rigid and flexible dynamics. Dynamic interaction between the agents is considered. Moreover, the achievement of the consensus and robust stabilization are shown for coordinated heterogeneous multi-agent systems, for undirected and connected graph topology. Finally, the effectiveness of the proposed controller is shown for a precise pointing mission of the Crab Nebula.**

***Index Terms*— multi-agent system, robust consensus, dynamic displacement interaction, cooperative control, flexible modes.**

I. Introduction

In recent years, different researchers explored works based on multi-agent systems as emerging systems for distributed coordination and for achievement of consensus [1]. Heterogeneous (second-order) multi-agent systems problem has attracted attention in different applications [2], [3], since heterogeneity implies that different agents might have different dynamics. Unknown mass distribution and cooperation control are usually also included. The motivation behind this work is mainly related to space applications, in which with the increase in the demand for space vehicles and for future space missions, particular attention is paid for coordination of spacecraft in rendezvous or pointing operations [4], [5] and for systems with movables appendages [6], [7].

A survey on distributed attitude coordination control was proposed by [8], in which different works on consensus for multi-body rigid systems are presented, mainly addressed to coordination. As clearly explained, multi-agent systems have higher accuracy, higher energy efficiency and even lower costs. Moreover, the satellites are often characterized by complicated shapes, with large appendages such as deployable solar panels or antennas. For this reason, in the last decades, several studies have been carried out on the flexibility of the satellite structure, and a reasonable way to model it is to consider a rigid hub and flexible appendages.

The main objective of this paper is to establish a consensus theory for second-order heterogeneous multi-agent system, where velocity of each agent is not assumed to be measurable and an attitude ”coordinated control” of multi-agent systems with flexible modes is analyzed. Moreover, thanks to advances in networking and distributed computing, coordinated multi-vehicle systems is proposed as applications of proximity operations of spacecraft. Rigid body attitude dynamics of space systems was widely studied, as first-order single integrator kinematics, including leader-follower consensus [9], [10]. However, as said before, the purpose of this paper is to present a consensus theory for multi-agent systems with *dynamic displacement interaction* and second-order differential equations with flexible modes.

The key features of this paper are the following: (i) instead of static weights, we employ dynamic weights in agent interaction, (ii) velocity of each agent is not measurable, (iii) a robust consensus is achieved, showing a stability condition for an heterogeneous dynamical system, represented by second-order differential equations, and (iv) flexible modes are included in the dynamics. For spacecraft attitude applications, even if spacecraft are usually equipped with gyroscopes, due to drift errors and possible failures [11], [12], gyroless attitude estimation and control [13], [14], [15] are proposed in the last years for small satellites. Since space maneuvers should handle high pointing accuracy, several missions include ”formation flight” [9], [16] or ”coordinated control” [10], [6], [7]. Our idea is to measure the Line-of-Sight or the attitude angles with inertial sensors (i.e Star Trackers) [17], [18], for an attitude ”coordinated control” of multi-agent flexible spacecraft. The main result in this paper is the achievement of the consensus for heterogeneous second-order multi-agent systems, if the graph of the overall system is undirected and connected, even if a flexible model is considered. Since heterogeneous properties have no effects on consensus convergence for first-order integrator [2], a sufficient condition for the consensus of second-order multi-agent systems is also given with some local information exchange.

As briefly introduced before, in some existing results [19], [20], [2], velocity as well as displacement of each agent are assumed to be available, though it is indeed a strong assumption for a certain practical case. For this reason, some researchers studied the problem of second-order multi-agent systems in which only position measurements are available, as in [21], [22]. Moreover, most of the researchers are focusing on directed graph topology, even if the robust consensus cannot be easily demonstrated, without assumptions related to graph topology itself and on system parameters. The mainly difference with these works are: (a) heterogeneity is included in our research, as well as uncertainties, (b) undirected graphs are considered, i.e. information on the

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system parameters are not requires, and (c) a sufficient condition for demonstrate the robust consensus tracking is shown.

The work proposed in this paper represents the extension of [23] and [24]. In [23] dynamic displacement feedback is achieved by robust stabilization. In [24] dynamic weights and homogeneous multi-agent systems are included. In this paper we extend the proof of robust stabilization for heterogeneous multi-agent systems with flexible modes. The effectiveness of the proposed solution is shown for a space proximity operation maneuver [25], in which some agents aim to perform a pointing mission, i.e. assigned desired rotational displacement, by interacting with the neighbors agents.

The paper is organized as follows. The main result related to the reached consensus and robust stabilization with dynamic weights and flexible modes is in Section II. A numerical example is included in Section III. Finally, conclusions are drawn in Section IV.

II. Main Result

The main result of this paper is described in this section, where we investigate a robust consensus for heterogeneous multi-agent systems with flexible appendages. A mathematical model of spacecrafts having flexible appendages is given in [26], [27], [28], and we employ this model. A motivation behind this work is the coordinated control of multi-agent spacecraft with heterogeneous mass distribution, where movable systems are included.

Let us consider N heterogeneous agents each of which consists of a rigid main body and some flexible appendages. That is, we can write each agent as

$$\begin{aligned} J_i\ddot{\theta}_i(t) &= u_i(t) - \delta_i^T\ddot{\eta}_i(t), \\ \ddot{\eta}_i(t) + C_i\dot{\eta}_i(t) + K_i\eta_i(t) &= -\delta_i\ddot{\theta}_i(t) \end{aligned} \tag{1}$$

in general, while we write it as

$$J_i\ddot{\theta}_i(t) = u_i(t) \tag{2}$$

if it does not have any flexible appendage. Here the index $i \in \{1, 2, \dots, N\}$ is used for identifying each agent, $u_i(t) \in \mathbb{R}^\ell$ denotes the consensus control input of agent i, and $\theta_i(t) \in \mathbb{R}^\ell$ denotes the linear/angular displacement vector of agent i. Thus $\dot{\theta}_i(t) \in \mathbb{R}^\ell$ and $\ddot{\theta}_i(t) \in \mathbb{R}^\ell$ represent the linear/angular velocity and acceleration of agents i, respectively, and J_i is the rigid body mass/inertia matrix of agent i. We assume that all of $\theta_i(t)$ $(i = 1, 2, \dots, N)$ are in the same coordinate system since we consider an attitude consensus in this paper. On the otter hand, we allow the modal coordinate vectors $\eta_i(t) \in \mathbb{R}^{m_i}$ $(i = 1, 2, \dots, N)$ to be different, where m_i denotes the number of flexible modes of agent i. The matrices C_i and K_i denote damping and stiffness, while δ_i represents coupling between rigid dynamics and flexible dynamics.

It should be noted that the coefficient matrices J_i, C_i, K_i, and δ_i in (1) and (2) generally satisfy

$$J_i = J_i^T > 0, \quad C_i = C_i^T > 0, \quad K_i = K_i^T > 0 \tag{3}$$

and

$$\begin{bmatrix} J_i & \delta_i^T \\ \delta_i & I_{m_i} \end{bmatrix} > 0 \tag{4}$$

if the matrices of the agent are derived under physical and mechanical constraints, where $I_q \in \mathbb{R}^{q\times q}$ denotes the identity matrix of dimension q. We assume these conditions (3) and (4) for all $i \in \{1, 2, \dots, N\}$ throughout this paper.

To these agents (1) and (2), we apply the following type of interaction with dynamic weights

$$\begin{aligned} \dot{z}_i(t) &= -\gamma z_i(t) - \gamma \sum_{j=1}^N (\gamma p_{ij} - q_{ij}) R(\theta_i(t) - \theta_j(t)), \\ u_i(t) &= -z_i(t) - \gamma \sum_{j=1}^N p_{ij} R(\theta_i(t) - \theta_j(t)), \end{aligned} \tag{5}$$

where $z_i(t) \in \mathbb{R}^\ell$ is the state of the dynamic weight. The gain $R \in \mathbb{R}^{\ell\times\ell}$ is chosen so that it satisfies

$$R = R^T > 0, \tag{6}$$

and the interaction parameters $p_{ij} \geq 0$ and $q_{ij} \geq 0$ $(i = 1, 2, \dots, N,\ j = 1, 2, \dots, N)$ are selected so that the corresponding graphs are undirected, i.e.,

$$p_{ij} = p_{ji}, \qquad q_{ij} = q_{ji}$$

for all i and j. The parameter $\gamma > 0$ will be determined later.

Notice here that the interaction (5) can be represented as

$$u_i(s) = -\sum_{j=1}^N w_{ij}(s) R(\theta_i(s) - \theta_j(s))$$

in the frequency domain, where the transfer function $w_{ij}(s)$ is

$$w_{ij}(s) = \gamma p_{ij} - \frac{\gamma(\gamma p_{ij} - q_{ij})}{s + \gamma} = \frac{p_{ij}s + q_{ij}}{(1/\gamma)s + 1}.$$

That is, if γ tends to infinity, $w_{ij}(s)$ tends to $p_{ij}s + q_{ij}$, which implies that (5) tends to

$$u_i(t) = -\sum_{j=1}^N \{p_{ij}R(\dot{\theta}_i(t) - \dot{\theta}_j(t)) + q_{ij}R(\theta_i(t) - \theta_j(t))\}.$$

In this sense, this dynamic interaction can be regarded as a PD type control law for achieving the consensus.

To derive a compact form of the overall multi-agent system composed of (1), (2), and (5), we first prepare the vectors

$$\begin{aligned} \theta(t) &= \begin{bmatrix} \theta_1^T(t) & \theta_2^T(t) & \cdots & \theta_N^T(t) \end{bmatrix}^T \in \mathbb{R}^{N\ell}, \\ \eta(t) &= \begin{bmatrix} \eta_1^T(t) & \eta_2^T(t) & \cdots & \eta_N^T(t) \end{bmatrix}^T \in \mathbb{R}^{M}, \\ u(t) &= \begin{bmatrix} u_1^T(t) & u_2^T(t) & \cdots & u_N^T(t) \end{bmatrix}^T \in \mathbb{R}^{N\ell}, \\ z(t) &= \begin{bmatrix} z_1^T(t) & z_2^T(t) & \cdots & z_N^T(t) \end{bmatrix}^T \in \mathbb{R}^{N\ell}, \\ M &= \sum_{i=1}^N m_i. \end{aligned}$$

Then we see that the agents (1) can be represented as

$$J\ddot{\theta}(t) = u(t) - \Delta^T\ddot{\eta}(t),$$

$$\ddot{\eta}(t) + C\dot{\eta}(t) + K\eta(t) = -\Delta\ddot{\theta}(t), \tag{7}$$

where

$$\begin{aligned} J &= \mathrm{block\,diag}\{J_1, J_2, \ldots, J_N\}, \\ C &= \mathrm{block\,diag}\{C_1, C_2, \ldots, C_N\}, \\ K &= \mathrm{block\,diag}\{K_1, K_2, \ldots, K_N\}, \\ \Delta &= \mathrm{block\,diag}\{\delta_1, \delta_2, \ldots, \delta_N\}. \end{aligned}$$

Notice here that these matrices satisfy

$$J = J^T > 0, \quad C = C^T > 0, \quad K = K^T > 0 \tag{8}$$

and

$$\begin{bmatrix} J & \Delta^T \\ \Delta & I_M \end{bmatrix} > 0, \tag{9}$$

which follows (3) and (4). Notice also that, even if some of the agents are fully rigid and described as (2), we can still write the agents as the compact form (7) with (8) and (9), where we redefine $\eta(t)$, C, and K appropriately with setting $m_i = 0$ for fully rigid agent i. That is, we can include both of (1) and (2) in (7) without going into details when we start from (7) with (8) and (9). On the other hand, the dynamic interaction (5) can be written as

$$\begin{aligned} \dot{z}(t) &= -\gamma z(t) - \gamma\left((\gamma L_v - L_d) \otimes R\right)\theta(t), \\ u(t) &= -z(t) - (\gamma L_v \otimes R)\theta(t), \end{aligned} \tag{10}$$

where $\otimes$ denotes the Kronecker product and the matrices L_v and L_d are defined as

$$L_v = [\ell^v_{ij}]_{N\times N}, \qquad L_d = [\ell^d_{ij}]_{N\times N},$$
$$\ell^v_{ij} = \begin{cases} \sum_{j=1}^N p_{ij} & (i = j) \\ -p_{ij} & (i \neq j) \end{cases}, \quad \ell^d_{ij} = \begin{cases} \sum_{j=1}^N q_{ij} & (i = j) \\ -q_{ij} & (i \neq j) \end{cases}.$$

It is well known [29] that, if the graphs corresponding to L_v and L_d are undirected and connected, L_v and L_d satisfy

$$L_v = L_v^T \geq 0, \quad L_v\mathbf{1}_N = 0, \quad \mathrm{rank}\, L_v = N - 1, \tag{11}$$
$$L_d = L_d^T \geq 0, \quad L_d\mathbf{1}_N = 0, \quad \mathrm{rank}\, L_d = N - 1, \tag{12}$$

where $\mathbf{1}_N \in \mathbb{R}^N$ is the vector whose elements are all one. Throughout the paper, we assume the above conditions as well.

Using (7) and (10), we can represent the overall multi-agent system as

$$\begin{aligned} J\ddot{\theta}(t) + \Delta^T\ddot{\eta}(t) + (\gamma L_v \otimes R)\theta(t) + z(t) &= 0, \\ \Delta\ddot{\theta}(t) + \ddot{\eta}(t) + C\dot{\eta}(t) + K\eta(t) &= 0, \\ \frac{1}{\gamma}\dot{z}(t) + \left((\gamma L_v - L_d) \otimes R\right)\theta(t) + z(t) &= 0. \end{aligned} \tag{13}$$

To investigate consensus condition of this system, let us define a matrix $S \in \mathbb{R}^{N\times(N-1)}$ which satisfies

$$\begin{bmatrix} \mathbf{1}_N^T/\sqrt{N} \\ S^T \end{bmatrix} \begin{bmatrix} \mathbf{1}_N/\sqrt{N} & S \end{bmatrix} = \begin{bmatrix} \mathbf{1}_N/\sqrt{N} & S \end{bmatrix} \begin{bmatrix} \mathbf{1}_N^T/\sqrt{N} \\ S^T \end{bmatrix} = I_N.$$

It should be noted that such an orthonormal complement S to $\mathbf{1}_N/\sqrt{N}$ always exists. With this S, we have the main result of this paper.

Theorem 1: Suppose that the agents (7) satisfy the conditions (8) and (9). Suppose also that the dynamic interaction (10) is designed so that R, L_v, and L_d satisfy (6), (11), and (12). Then the overall multi-agent system (13) composed of (7) and (10) achieves an attitude consensus

$$\begin{aligned} &\lim_{t\to\infty}\left(\dot{\theta}(t) - (\mathbf{1}_N \otimes a)\right) = 0, \\ &\lim_{t\to\infty}\left(\theta(t) - (\mathbf{1}_N \otimes (at + b))\right) = 0, \\ &\lim_{t\to\infty}\dot{\eta}(t) = 0, \quad \lim_{t\to\infty}\eta(t) = 0, \quad \lim_{t\to\infty} z(t) = 0 \end{aligned} \tag{14}$$

for some $a \in \mathbb{R}^\ell$ and $b \in \mathbb{R}^\ell$ if γ is selected so that it satisfies

$$S^T(\gamma L_v - L_d)S > 0. \tag{15}$$

An important implication of this theorem is that the dynamic interaction (10) in fact achieves *robust* consensus for the *heterogeneous* agents (7). That is, we do *not* need J, C, K, and Δ of (7) in order to choose R, L_v, L_d, and γ of (10), which means that we can design the consensus control law (10) *independently* of the agent dynamics (7). This is a significant feature of the dynamic interaction (10).

In this regard, although passivity plays a crucial role in standard multi-agent consensus [30], it cannot help us for the multi-agent consensus of (7) since the velocity $\dot{\theta}_i(t)$ is not measurable and thus the agent dynamics from $u_i(t)$ to $\theta_i(t)$ *cannot be passive*. Instead of passivity, we employ a stability theory for second-order differential equations [23].

We also remark that there always exists γ satisfying (15). In fact, since (11) holds, it turns out that $S^T L_v S > 0$, which ensures the existence of γ for (15).

In the rest of this section, we establish the proof of this theorem. To this end, we first introduce a variable transformation

$$z(t) = \left(\left(\gamma L_v - L_d + \frac{\mathbf{1}_N\mathbf{1}_N^T}{N}\right) \otimes R\right)\hat{z}(t), \tag{16}$$

where we see that $\gamma L_v - L_d + \mathbf{1}_N\mathbf{1}_N^T/N > 0$ from (15) and thus this is a nonsingular transformation. Then we have another representation of the overall system

$$\begin{aligned} &J\ddot{\theta}(t) + \Delta^T\ddot{\eta}(t) + (\gamma L_v \otimes R)\theta(t) \\ &\qquad + \left(\left(\gamma L_v - L_d + \frac{\mathbf{1}_N\mathbf{1}_N^T}{N}\right) \otimes R\right)\hat{z}(t) = 0, \\ &\qquad \Delta\ddot{\theta}(t) + \ddot{\eta}(t) + C\dot{\eta}(t) + K\eta(t) = 0, \\ &\frac{1}{\gamma}\left(\left(\gamma L_v - L_d + \frac{\mathbf{1}_N\mathbf{1}_N^T}{N}\right) \otimes R\right)\dot{\hat{z}}(t) \\ &\qquad + \left((\gamma L_v - L_d) \otimes R\right)\theta(t) \\ &\qquad + \left(\left(\gamma L_v - L_d + \frac{\mathbf{1}_N\mathbf{1}_N^T}{N}\right) \otimes R\right)\hat{z}(t) = 0. \end{aligned}$$

We further employ variable transformations for $\theta(t)$ and $\hat{z}(t)$

$$\begin{bmatrix} \bar{\theta}(t) \\ \tilde{\theta}(t) \end{bmatrix} = \begin{bmatrix} \left(\mathbf{1}_N^T/\sqrt{N}\right) \otimes I_\ell \\ S^T \otimes I_\ell \end{bmatrix} \theta(t), \quad (17)$$

$$\theta(t) = \begin{bmatrix} \left(\mathbf{1}_N/\sqrt{N}\right) \otimes I_\ell & S \otimes I_\ell \end{bmatrix} \begin{bmatrix} \bar{\theta}(t) \\ \tilde{\theta}(t) \end{bmatrix}, \quad (18)$$

$$\begin{bmatrix} \bar{z}(t) \\ \tilde{z}(t) \end{bmatrix} = \begin{bmatrix} \left(\mathbf{1}_N^T/\sqrt{N}\right) \otimes I_\ell \\ S^T \otimes I_\ell \end{bmatrix} \hat{z}(t), \quad (19)$$

$$\hat{z}(t) = \begin{bmatrix} \left(\mathbf{1}_N/\sqrt{N}\right) \otimes I_\ell & S \otimes I_\ell \end{bmatrix} \begin{bmatrix} \bar{z}(t) \\ \tilde{z}(t) \end{bmatrix}. \quad (20)$$

Then we can describe the overall system as

$$\bar{J}\ddot{\bar{\theta}}(t) + H^T\ddot{\tilde{\theta}}(t) + \bar{\Delta}^T\ddot{\eta}(t) + R\bar{z}(t) = 0, \quad (21)$$

$$H\ddot{\bar{\theta}}(t) + \tilde{J}\ddot{\tilde{\theta}}(t) + \tilde{\Delta}^T\ddot{\eta}(t) + \tilde{L}_v\tilde{\theta}(t) + \tilde{L}_{vd}\tilde{z}(t) = 0, \quad (22)$$

$$\bar{\Delta}\ddot{\bar{\theta}}(t) + \tilde{\Delta}\ddot{\tilde{\theta}}(t) + \ddot{\eta}(t) + C\dot{\eta}(t) + K\eta(t) = 0, \quad (23)$$

$$\frac{1}{\gamma}R\dot{\bar{z}}(t) + R\bar{z}(t) = 0, \quad (24)$$

$$\frac{1}{\gamma}\tilde{L}_{vd}\dot{\tilde{z}}(t) + \tilde{L}_{vd}\tilde{\theta}(t) + \tilde{L}_{vd}\tilde{z}(t) = 0, \quad (25)$$

where we use the following definitions:

$$\begin{bmatrix} \bar{J} & H^T \\ H & \tilde{J} \end{bmatrix} = \begin{bmatrix} \left(\mathbf{1}_N^T/\sqrt{N}\right) \otimes I_\ell \\ S^T \otimes I_\ell \end{bmatrix} J \begin{bmatrix} \left(\mathbf{1}_N/\sqrt{N}\right) \otimes I_\ell & S \otimes I_\ell \end{bmatrix},$$

$$\begin{bmatrix} \bar{\Delta} & \tilde{\Delta} \end{bmatrix} = \Delta \begin{bmatrix} \left(\mathbf{1}_N/\sqrt{N}\right) \otimes I_\ell & S \otimes I_\ell \end{bmatrix},$$

$$\tilde{L}_v = \left(\gamma S^T L_v S\right) \otimes R, \quad \tilde{L}_{vd} = \left(S^T\left(\gamma L_v - L_d\right)S\right) \otimes R.$$

Notice that the overall multi-agent system (21)-(25) has a cascade structure. To see this fact, we first rewrite (21) as

$$\ddot{\bar{\theta}}(t) = -\bar{J}^{-1}H^T\ddot{\tilde{\theta}}(t) - \bar{J}^{-1}\bar{\Delta}^T\ddot{\eta}(t) - \bar{J}^{-1}R\bar{z}(t), \quad (26)$$

where we regard the signals $\ddot{\tilde{\theta}}(t)$, $\ddot{\eta}(t)$, and $\bar{z}(t)$ as the input to this double integrator system (26). Next, substituting (26) into (22) and (23), we regard (22), (23), and (25) as a system

$$\begin{aligned} &(\tilde{J} - H\bar{J}^{-1}H^T)\ddot{\tilde{\theta}}(t) + (\tilde{\Delta}^T - H\bar{J}^{-1}\bar{\Delta}^T)\ddot{\eta}(t) \\ &\qquad + \tilde{L}_v\tilde{\theta}(t) + \tilde{L}_{vd}\tilde{z}(t) = H\bar{J}^{-1}R\bar{z}(t), \\ &(\tilde{\Delta} - \bar{\Delta}\bar{J}^{-1}H^T)\ddot{\tilde{\theta}}(t) + (I_M - \bar{\Delta}\bar{J}^{-1}\bar{\Delta}^T)\ddot{\eta}(t) \\ &\qquad + C\dot{\eta}(t) + K\eta(t) = \bar{\Delta}\bar{J}^{-1}R\bar{z}(t), \\ &\frac{1}{\gamma}\tilde{L}_{vd}\dot{\tilde{z}}(t) + \tilde{L}_{vd}\tilde{\theta}(t) + \tilde{L}_{vd}\tilde{z}(t) = 0 \end{aligned} \quad (27)$$

with the input $\bar{z}(t)$. Finally, we rewrite (24) as

$$\dot{\bar{z}}(t) = -\gamma\bar{z}(t). \quad (28)$$

We therefore see a cascade structure in the overall system. In fact, the system (26) is governed by the systems (27) and (28), and the system (27) is governed by the system (28).

Since γ is selected as a positive number in order to satisfy (15), apparently the system (28) is stable. That is,

$$\lim_{t\to\infty}\bar{z}(t) = 0,$$

which implies

$$\lim_{t\to\infty}\dot{\tilde{\theta}}(t) = 0, \quad \lim_{t\to\infty}\tilde{\theta}(t) = 0, \quad \lim_{t\to\infty}\dot{\eta}(t) = 0,$$
$$\lim_{t\to\infty}\eta(t) = 0, \quad \lim_{t\to\infty}\tilde{z}(t) = 0$$

if the system (27) is internally stable. In this case, the behavior of the system (26) asymptotically follows

$$\ddot{\bar{\theta}}(t) = 0,$$

which immediately implies

$$\lim_{t\to\infty}(\dot{\bar{\theta}}(t) - a) = 0, \quad \lim_{t\to\infty}(\bar{\theta}(t) - (at + b)) = 0$$

for some a and b. If these convergences are established, the transformations (16), (18), and (20) say that the attitude consensus (14) is actually achieved.

Thus the rest is to investigate internal stability of the system (27). The following lemma is useful for this purpose.

Lemma 1: [23] A dynamical system given by

$$\begin{aligned} M_1\ddot{x}_1(t) + D_1\dot{x}_1(t) + K_{11}x_1(t) + K_{12}x_2(t) &= 0, \\ D_2\dot{x}_2(t) + K_{21}x_1(t) + K_{22}x_2(t) &= 0 \end{aligned}$$

is stable if its coefficients satisfy

$$M_1 = M_1^T > 0, \quad D_1 + D_1^T \geq 0, \quad D_2 + D_2^T > 0,$$
$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^T > 0,$$
$$\begin{bmatrix} D_1 + D_1^T & K_{12} \end{bmatrix} \text{ is of full row rank.}$$

Let us define the matrices in this lemma as

$$M_1 = \begin{bmatrix} \tilde{J} & \tilde{\Delta}^T \\ \tilde{\Delta} & I_M \end{bmatrix} - \begin{bmatrix} H \\ \bar{\Delta} \end{bmatrix} \bar{J}^{-1} \begin{bmatrix} H^T & \bar{\Delta}^T \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}, \quad D_2 = \frac{1}{\gamma}\tilde{L}_{vd},$$
$$K_{11} = \begin{bmatrix} \tilde{L}_v & 0 \\ 0 & K \end{bmatrix}, \quad K_{12} = \begin{bmatrix} \tilde{L}_{vd} \\ 0 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} \tilde{L}_{vd} & 0 \end{bmatrix}, \quad K_{22} = \tilde{L}_{vd}.$$

Then we see that the system (27) has in fact the form of the system described in the lemma. We also see that the matrices defined above satisfy the stability condition of the lemma when they are chosen so that (6), (8), (9), (11), (12), and (15) hold true. This establishes Theorem 1.

III. Numerical Example

The proposed controller can be applied to a proximity space maneuver, in which a consensus between $N = 5$ spacecraft should be reached. A numerical example for the attitude control of spacecraft is proposed, in which a rigid hub and flexible appendages represent the dynamics of each agent. The agent can be represented as in Figure 1. The considered reference frame is a body reference frame and Euler angles [25] $\Theta = [\theta_x, \theta_y, \theta_z]^T$ (3-2-1 order of rotation) are considered for the attitude visualization. All the agents start with an initial attitude angle $\Theta_{0,i}$, which is different for each agents and it is randomly defined. J_i

TABLE I

FLEXIBLE MODE CHARACTERISTICS

Mode	Natural frequency $\omega_n [rad/s]$	Damping ζ [-]
Mode 1	0.7681	0.05607
Mode 2	1.1038	0.008620
Mode 3	1.8733	0.01283
Mode 4	2.5496	0.02516

is randomly changed over the agents. Moreover, $\delta_m \in \mathbb{R}^m$, $C_m = \text{diag}(2\zeta_m \omega_{n,m})$ and $K_m = \text{diag}(\omega_{n,m}^2)$, with $\omega_{n,m}$ natural frequency and ζ_m damping of m flexible modes. The δ matrix is the one defined in [26], with $m = 4$, instead the flexible mode properties are in Table I. The generalized coordinates represent the modal coordinates. For each agent i, $C_i = \text{diag}(C_{m,i})$ and $K_i = \text{diag}(K_{m,i})$. The moments of inertia of each agent are randomly defined from the nominal value of [26], in which $J_{x,0} = 350$ kgm^2, $J_{y,0} = 280$ kgm^2, $J_{z,0} = 190$ kgm^2, $J_{xz,0} = 4$ kgm^2, $J_{xy,0} = 3$ kgm^2, $J_{yz,0} = 10$ kgm^2. So, $J_{k,i} = J_{k,0} + J_{k,0}\text{rand}(1)$, with $k = [x, y, z, xy, xz, yz]$. $J_i - \delta_i^T \delta_i > 0$ is positive-definite, for all the agents, even if a deviation of the moment of inertia is included (as previously explained).

The consensus to be reached is zero angular velocities $\omega_f = [0, 0, 0]^T$ and the desired Euler angles are $\theta_f = [0.7684, 0, 0.224]^T$ rad, i.e. the scientific observation of the Crab Nebula. This final point defines the equilibrium constant value to be reached by all the agents.

The flexible spacecraft is composed of a rigid body and some flexible appendages. Its attitude can be described by two sets of equations: the kinematic equations and the dynamics equations. The overall dynamics (1) combines rigid and flexible dynamics, in which $\delta_m \in \mathbb{R}^m$, $C_m = \text{diag}(2\zeta_m \omega_{n,m})$ and $K_m = \text{diag}(\omega_{n,m}^2)$, with $\omega_{n,m}$ natural frequency and ζ_m damping of m flexible modes. The δ matrix is the one defined in [26], with $m = 4$, instead the flexible mode properties are in Table I. The generalized coordinates represent the modal coordinates. For each agent i, $C_i = \text{diag}(C_{m,i})$ and $K_i = \text{diag}(K_{m,i})$. Moreover, $J_i - \delta_i^T \delta_i > 0$ is positive-definite, for all the agents, even if a deviation of the moment of inertia is included (as previously explained). As in [27], 4 flexible modes are included, modeled as in Eq. (1). The coupling between the rigid and flexible models are included. The consensus is reached, even if the control input is defined as in Eq. (5), in which the flexible dynamics is not considered, for both the

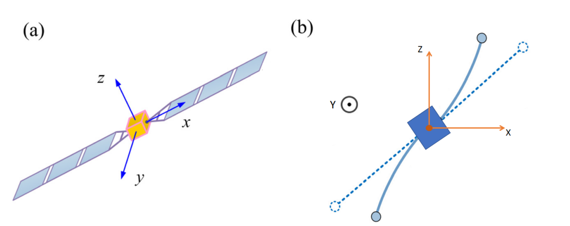

Fig. 1. A single agent dynamics, including flexible appendages: (a) representation of a flexible spacecraft, (b) representation of the bending mode and rigid hub

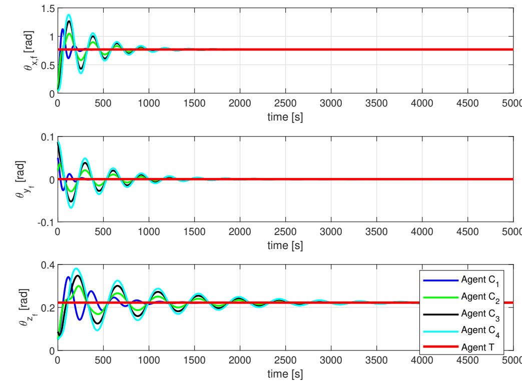

Fig. 2. Rotational displacement along X, Y and Z axes with flexible modes (nominal weights)

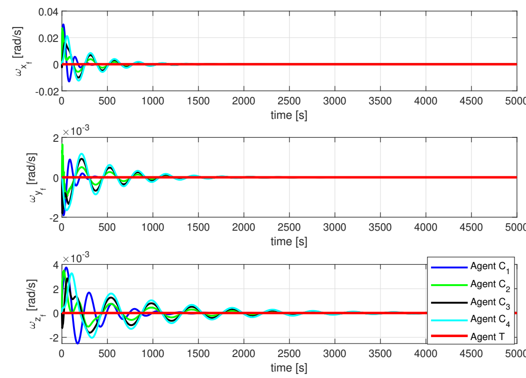

Fig. 3. Rotational angular velocity along X, Y and Z axes with flexible modes (nominal case)

rotational displacement and velocity, as in Figures 2-3. The assigned control input is the same for both cases.

At the time step $t_0 = 0$ seconds, the error in the rotational displacement is about $err_\theta = [40, 10, 3]^T$ deg, for all the agents. Due to the high error, a high control inputs is required. If the initial conditions are selected in order to reduce the error at the beginning of simulations, even with $\gamma = 100$, a reduced control input is requested. The settling time is not affected by the choice of the initial conditions.

Moreover, different weights $p_{i,j}$ and $q_{i,j}$ are tested, to evaluate the performance of the distributed and coordinated control. The list of selected weights are in Table II. The nominal one is $p_{i,j} = 10$ and $q_{i,j} = 1$, for all the agents. If γ and $p_{i,j}$ are increasing, the settling time to consensus is reduced, even if the required control input is increased. As an example, for the same agent, the rotational displacement along X axis for all the cases is shown (Figure 4). The consensus is reached, even if the settling time is higher when flexible modes are included with nominal weights.

IV. CONCLUDING REMARKS

In this paper, a second-order multi-agent consensus via a dynamic displacement interaction and flexible modes is stud-

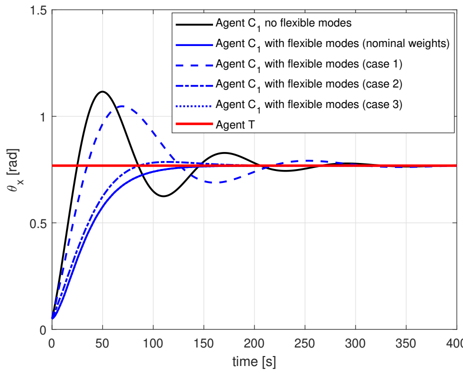

Fig. 4. Rotational displacement along X axis for all cases

TABLE II

DYNAMICS WEIGHT TRADE-OFF

CASE	γ	$p_{i,j}$	$p_{i,j}$
nominal	100	10	1
1	100	10	0.5
2	100	20	0.5
3	1000	10	0.5

ied, assuming that velocity of each agent is not measurable. We have established a sufficient condition for the consensus, where the condition is represented by using graph Laplacians. We have seen that such a dynamic interaction achieves the consensus always exists if the graph of the overall system is undirected and connected, where the dynamics of the weights of the interaction should be selected adequately. The proposed methodology is applied to an attitude control for space pointing maneuvers.

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