POLITECNICO DI TORINO
Repository ISTITUZIONALE

Magnetic Flux Sensing Exploiting Two SQUIPTs Connected by means of a Floating Island

(Article begins on next page)

25 April 2024

Magnetic Flux Sensing Exploiting Two SQUIPTs Connected by means of a Floating Island

L. Fasolo, D. Borchia and E. Enrico

Abstract—**Magnetometers exploiting interference effect of the superconducting wavefunction are known since the realization of the first SQUID, with several improvements in performance in the following years. In this field, the Superconducting Quantum Interference Proximity Transistor (SQUIPT) offers an interesting alternative to the conventional SQUID thanks to its lower power dissipation that makes it ideal for nanoscale and ultra-low temperature ($T < 1$ mK) applications, where very low dissipation is required. Here, a device composed of two facing fully superconducting SQUIPT is proposed. Numerical simulations demonstrate how the interplay between the behaviours of a couple of tunnel junctions in such a design accounts for promising performances as both a flux-to-current and a flux-to-voltage transducer.**

Index Terms - **Magnetometers, Nanoelectronics, Single electron transistors, Superconducting devices.**

I. INTRODUCTION

The Superconducting Quantum Interference Proximity Transistor [1] is a two terminal nanostructured electronic device composed of a superconducting loop interrupted by a normal metal or a superconducting nanowire. In the former case, due to the *proximity effect*, the contact between the superconducting material and the normal material leads to the appearance of a minigap in the electronic density of state (DoS) of the wire, that results to be modulated by the magnetic flux piercing the loop [2], [3]. It has also been demonstrated that a flux-induced quasiparticles DoS modulation can occur in a superconducting wire, with a dependency similar to the normal metal one when the weak link is short compared to the diffusive coherence length, $L < \sqrt{hD/2\pi\Delta}$ (where h is the Plank constant, D is the diffusion coefficient of the nanowire, and Δ is the superconducting energy gap of the ring) [4]. Theoretical proposals and experimental evidences [4], [5] showed that fully superconducting SQUIPTs exhibit better performances in terms of flux sensitivity with respect to the normal metal wire-based counterparts. More recently, the exploitation of a couple of facing SQUIPTs has been proposed and tested, demonstrating enhanced functionalities due to

(*Corresponding author: Emanuele Enrico*)
L. Fasolo is with INRiM, Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy and also with Politecnico di Torino, Corso Castelfidardo 39, 10129 Torino, Italy (email: luca_fasolo@polito.it).
D. Borchia is with Università degli Studi di Torino, Via Pietro Giuria 1, 10125 Torino, Italy (email: davide.borchia@edu.unito.it).
E. Enrico is with INRiM, Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy (email:e.enrico@inrim.it).

single-electron sensitivity [6], [7]. In the SQUIPT layout, the flux-dependent DoS is probed by the so-called probe (P) electrode through a negligible-size tunnel junction coupled with the center of the nanowire (NW). In a voltage-bias (V_b) configuration, a magnetic flux-dependent quasiparticles current $I(\Phi)$ can flow across this junction (Fig.1a). If the tunnel resistance (R_T) is sufficiently large, the Cooper pair current across the junction can be neglected with respect to the quasiparticles current. The net quasiparticle current flowing through the junction, as a function of the bias condition, can be expressed as

$$I(V_b) = \frac{1}{eR_T} \int \left[n_{NW}(E)\, n_P(E - eV_b) \cdot \right. \tag{1}$$
$$\left. \cdot (f_{NW}(E) - f_P(E - eV_b)) \right] \mathrm{d}E$$

where E is the quasiparticle energy with respect to the chemical potential, $f_{NW(P)}(E)$ is the Fermi-Dirac distribution for the nanowire (probe), and exploiting $n_{NW(P)}$ as the normalized superconducting quasiparticle DoS:

$$n_{NW(P)}(E) = \left| Re \left[\frac{E + i\,\gamma_{NW(P)}\,\Delta_{NW(P)}}{\sqrt{\left(E + i\,\gamma_{NW(P)}\,\Delta_{NW(P)}\right)^2 - \tilde{\Delta}_{NW(P)}^2}} \right] \right| \tag{2}$$

where $\Delta_{NW(P)}$ is the superconducting BCS gap and $\gamma_{NW(P)}$ is the reduced Dynes parameter modeling the finite lifetime of Cooper pairs in a superconductor-based tunnel junction connected with a noisy low-ohmic environment [8], [9]. Moreover, while for the probe holds $\tilde{\Delta}_P = \Delta_P$, to take into account the flux-modulated gap in the nanowire, one can introduce $\tilde{\Delta}_{NW} = \Delta_{NW}(s \cdot \cos(\pi\Phi/\Phi_0) + t)$, where $\Phi_0 = h/2e$ is the magnetic flux quantum and e is the elementary charge. The cosinusoidal dependency derives from the analytical solution of the quasi-classical Usadel equations for diffusive systems, under the assumption of a short limit junction (i.e., for a nanowire length much smaller than the diffusive coherence length) and for a perfectly centered probe [4]. The parameters s and t here introduced, phenomenologically model an incomplete suppression of the gap that can occur when the kinetic inductance of the loop isn't negligible with respect the kinetic inductance of the nanowire [5], [10].

In the present work we compare the performance, as a magnetic flux transducer, of a fully superconducting single SQUIPT with a device composed by two of them, electrically coupled by means of their probes. (Fig.1b). The two probes
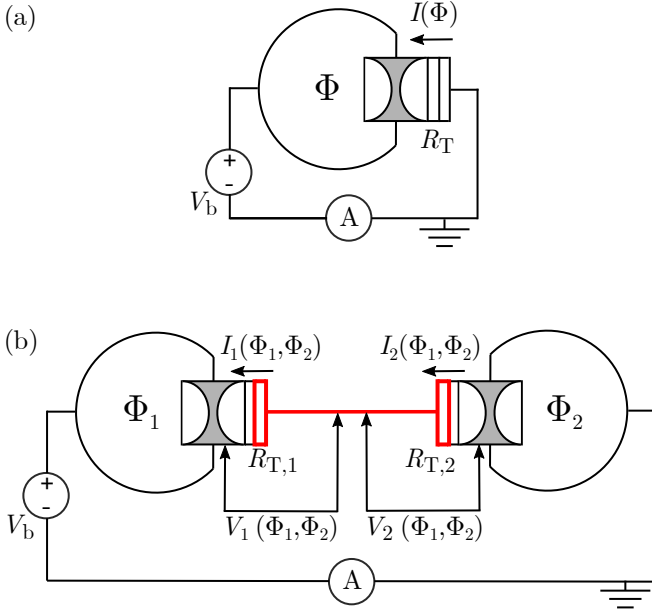
(a)

Fig. 1. a) Schematic representation of a voltage biased SQUIPT. In this figure, an equivalent circuital symbol, derived from the one typically used for a tunnel junction has been introduced, representing the tunnel junction between the nanowire and the probe. b) Schematic representation of the two facing SQUIPTs configuration, connected through their probes that constitute a superconducting island (thick red lines) with a floating potential.
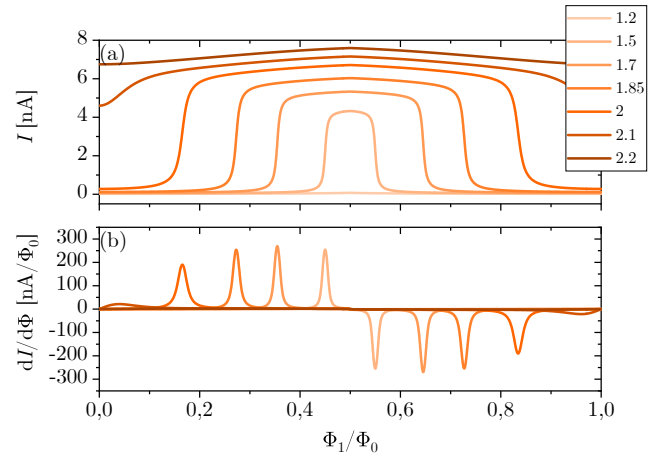

Fig. 2. a) Modulation of the net quasiparticle current flowing across the junction in a SQUIPT as a function of the concatenated reduced magnetic flux Φ/Φ_0 for different bias voltages eV_b/Δ_P. b) Flux-to-current transfer functions $\partial I/\partial \Phi$.

constitute a floating superconducting island with negligible charging energy. A similar layout can be found in [7]. Besides, we suppose the entire device to be thermally anchored to the substrate at a temperature much lower then its superconducting critical temperature.

For the sake of simplicity, the two junctions in the proposed configuration are assumed to have identical resistances ($R_{T,1} = R_{T,2} = R_T$). Furthermore, two magnetic fluxes (Φ_1 and Φ_2) are concatenated to the loops, having a constant difference $\Phi_1 - \Phi_2 = \Delta\Phi$, condition easily reached experimentally by exploiting local magnetic fluxes generated with on-chip coils [11].

With a general approach, when the device is voltage-biased (V_b), the two junctions experience different voltage drops (V_1, V_2) balanced on the base of the ratio of their conductances, such that $V_b = V_1 + V_2$.

To evaluate this latter ratio, we model the net quasiparticle current I_m flowing through the m-th junction as a generalization of (1):

$$I_m(V_m) = \frac{1}{eR_T} \int \Big[n_{m,L}(E)\, n_{m,R}(E - eV_m) \cdot \tag{3}$$
$$\cdot \big(f_{m,L}(E) - f_{m,R}(E - eV_m) \big) \Big] dE$$

where m $= \{1, 2\}$, and the index m,L(R) identifies the left (right) electrode of the m-th junction, that can be one of the nanowires or one of the ends of the central island. From (2) and (3) it turns out that the tunnel junctions conductances depend on their electrodes DoSes and consequently on Φ_1 and Φ_2. In addiction, the current conservation $I_1(V_1) = I_2(V_2) =$

I imply a dependence of the voltages on both the fluxes, hence $V_{1(2)}(\Phi_1, \Phi_2)$.

II. SIMULATIONS AND RESULTS

Numerical simulations have been performed by selecting the device parameters on the base of the measurements performed on a similar layout relying on aluminum technology [11]. Here, the tunneling resistance of all junctions is $R_T = 55$ kΩ, while the island (probe) superconducting gap value is $\Delta_{I(P)} = 215$ μeV. We suppose the two nanowires to be equal, such that $\tilde{\Delta}_{NW,1}$ and $\tilde{\Delta}_{NW,2}$ both sweep from a maximum value of 235 μeV to a minimum of 85 μeV (corresponding to the phenomenological parameters $t = 85/235 \approx 0.36$ and $s = 1 - t$). It's worth mentioning that in the reference experiment, the maximum gap of the nanowire results to be higher than the gap of the island, being the former a thinner nanostructure [12]. Furthermore, the value of the reduced Dynes parameter of each component was considered equal and set at $\gamma = 5.0 \cdot 10^{-3}$. Eventually, the difference between the two fluxes was fixed to the value $\Delta\Phi/\Phi_0 = 0.5$ (corresponding to the opposite phase condition in the modulation of the two nanowire gaps).

The curves in Fig. 2a represent the current flowing through the junction of a SQUIPT as a function of the concatenated reduced magnetic flux Φ/Φ_0 when the device is biased with different voltages eV_b/Δ_P. In the high bias voltage regime the junction is dominated by the normal metal character of its electrodes and behaves like a normal resistance, with a current weakly dependent on the magnetic flux piercing the loop. For intermediate bias voltages the current response acquires a non-linear flux dependency, highlighted by the presence of sharp peaks in the flux-to-current transfer function $\partial I/\partial \Phi$ reported in Fig. 2b. These peaks are due to the alignment between the maximum (peak) of the empty branch of the superconducting DoS of one side of the junction, and the maximum (peak)
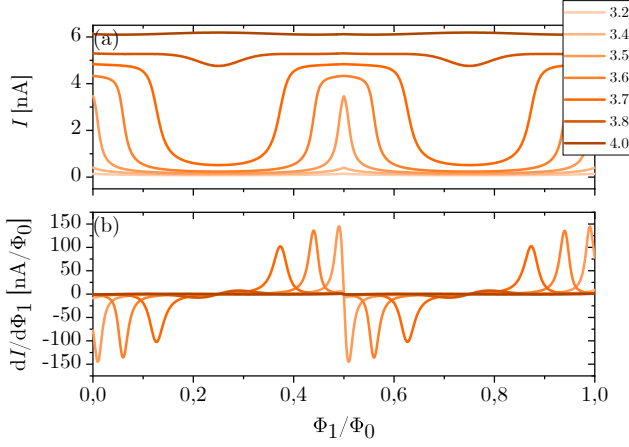
Fig. 3. a) Modulation in the two facing SQUIPT configuration of the net quasiparticle current flowing across the two junction as a function of the concatenated reduced magnetic flux Φ_1/Φ_0 for different bias voltages eV_b/Δ_I. b) Flux-to-current transfer functions $\partial I/\partial \Phi_1$. The numerical analysis was performed imposing the particular condition of $\Delta\Phi = \Phi_1 - \Phi_2 = 0.5\,\Phi_0$.
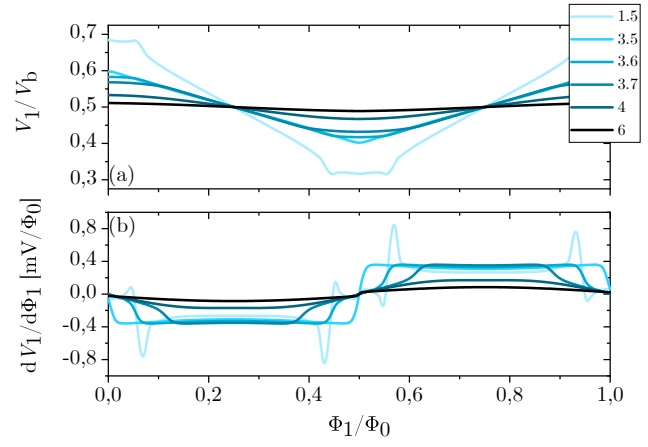

Fig. 4. a) Modulation in the two facing SQUIPT configuration of the voltage drop across the first junction normalized on the bias voltage V_1/V_b as a function of the concatenated reduced magnetic flux Φ_1/Φ_0 for different bias voltages eV_b/Δ_I. b) Flux-to-voltage transfer functions $\partial V_1/\partial \Phi_1$.

of the filled branch of the other side (*resonance*). In this intermediate bias regime the SQUIPT acts as a flux-to-current transducer with a high sensitivity confined in two narrow flux intervals, whose positions are determined by V_b. Eventually, in the low bias regime the current flow is strongly-suppressed, recovering a flux-independency.

The curves in Fig. 3a show the behaviour of the current flowing across each of the junctions in the two facing SQUIPTs configuration as a function of the concatenated magnetic flux Φ_1 for different bias conditions. The corresponding flux-to-current transfer functions $\partial I/\partial \Phi_1$ are presented in Fig. 3b. Similarly to the previous case, three bias regimes can be appreciated. This time however, in the intermediate regime the transfer functions present less pronounced peaks, located in correspondence of the contemporaneous achievement of the resonance condition across the two junctions, condition promoted by the floating voltage of the island. Thus, the proposed configuration can act as a flux-to-current transducer with a lower sensitivity with respect the single SQUIPT voltage biased configuration. Unlike this latter, the voltage biased couple of facing SQUIPTs can act also as a flux-to-voltage transducer, where the physical measurand is, for instance, the voltage drop across the first junction V_1. This voltage can be experimentally evaluated with a tunnel probe inducing in the floating island a probing current negligible with respect I.

The curves in Fig. 4a represent the voltage drop across the first junction, normalized on V_b, as a function of the normalized magnetic flux Φ_1/Φ_0, for different voltage bias conditions. For the highest bias values the conductances of the two identical junctions are comparable, and the voltage drops across them tend to the so-called *symmetrical bias* condition ($V_1 = V_2 = V_b/2$), independently from the concatenated magnetic flux. As in the case of the current response, for

the lowest bias values, the behaviour of the voltage drops is strongly flux-dependent. The displacement of the curves from $V_1/V_b = 0.5$ is maximum for multiples of $\Phi_0/2$, values for which the conductances are the most unbalanced, due to the maximum difference of the two gap amplitudes. Vice versa, for odd multiples of $\Phi_0/4$, regardless from the imposed bias voltage, the displacement is null, since the conductances of the two junctions are equal, being in this situation $\tilde{\Delta}_{NW,1} = \tilde{\Delta}_{NW,2}$. The corresponding flux-to-voltage transfer functions $\partial V_1/\partial \Phi_1$ are shown in Fig.4b, where it can be noticed that in the intermediate voltage regime these curves have two plateaus, each of which can cover almost completely a flux semi-period when properly biased (i.e., for $eV_b/\Delta_I = 3.5$ in this simulation). In this condition the transducer exhibit an almost constant sensitivity on the overall magnetic flux period.

III. CONCLUSION

In this work the behaviour as a magnetic flux transducer of two facing fully superconducting SQUIPTs connected by means of a floating island was investigated. Similarly to the case of a voltage biased single SQUIPT, the flux-to-current transfer functions of the proposed configuration show sharp peaks, the positions of which are determined by the bias conditions. In this situation the sensitivity of the transducer is limited to narrow intervals in the magnetic flux periodicity. Differently from the voltage biased single SQUIPT, the double SQUIPT concept enables the possibility of a flux-to-voltage transduction, resulting in a sensitivity extension to the overall magnetic flux period. The mesoscopic nature of the island connecting the two facing SQUIPTs in this latter configurations offers several opportunities like active on-chip cooling [13]–[15] or energy filtered transport across tunnel junctions [16] that can overcome the magnetic flux sensitivity limits even further respect state-of-the-art SQUIPTs.

REFERENCES

[1] F. Giazotto, J. T. Peltonen, M. Meschke, and J. P. Pekola, "Superconducting quantum interference proximity transistor," *Nature Physics Letters*, **6**, pp. 254–259, April 2010. DOI: 10.1038/nphys1721.

[2] W. Belzig, C. Bruder, and G. Schön, "Local density of states in a dirty normal metal connected to a superconductor," *Physical Review B*, **54**, 9443, 1996. DOI:10.1103/PhysRevB.54.9443.

[3] B. Pannetier, and H. Courtois, "Andreev Reflection and Proximity effect," *Journal of Low Temperature Physics*, **118**, pp. 599–615, 2000. DOI: 10.1023/A:1004635226825.

[4] P. Virtanen, A. Ronzani, and F. Giazotto, "Spectral Characteristics of a Fully Superconducting SQUIPT," *Physical Review Applied*, **6**, 054002, 2016. DOI: 10.1103/PhysRevApplied.6.054002.

[5] N. Ligato, G. Marchegiani, P. Virtanen, E. Strambini, and F. Giazotto, "High operating temperature in V-based superconducting quantum interference proximity transistors," *Scientific Reports*, **7**, 8810, 2017. DOI: 10.1038/s41598-017-09036-0.

[6] E. Enrico, and F. Giazotto, "Superconducting Quantum Interference Single-Electron Transistor," *Physical Review Applied*, **5**,064020, 2016. DOI:0.1103/PhysRevApplied.5.064020.

[7] E. Enrico, E. Strambini, and F. Giazotto, "Phase-driven charge manipulation in Hybrid Single-Electron Transistor," *Scientific Reports*, **7**, 13492, 2017. DOI: 10.1038/s41598-017-13894-z.

[8] R.C. Dynes, J. P. Garno, G. B. Hertel, and T. P. Orlando, "Tunneling Study of Superconductivity near the Metal-Insulator Transition," *Physical Review Letters*, **53**, pp.2437–2440, 1984. DOI:10.1103/PhysRevLett.53.2437

[9] J. P. Pekola, V. F. Maisi, S. Kafanov, N. Chekurov, A. Kemppinen, Y. A. Pashkin, O.Saira, M. Möttönen, and J.S. Tsai, "Environment-Assisted Tunneling as an Origin of the Dynes Density of States," *Physical Review Letters*, **105**,026803, 2010. DOI: 10.1103/PhysRevLett.105.026803

[10] A. Ronzani, C. Altimiras, and F. Giazotto, "Highly Sensitive Superconducting Quantum-Interference Proximity Transistor," *Physical Review Applied*, **2**, 024005, 2014. DOI:10.1103/PhysRevApplied.2.024005

[11] E. Enrico, L. Croin, E. Strambini, and F. Giazotto, "On-chip tuning of electrodes quasiparticles population in fully superconducting SQUISETs," *arXiv:1901.01248*, 2019

[12] N. A. Court, A. J. Ferguson, and R. G. Clark, "Energy gap measurement of nanostructured aluminium thin films for single Cooper-pair devices," *Superconductor Science and Technology*, **21**, 015013, 2007. DOI: 10.1088/0953-2048/21/01/015013

[13] F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, "Opportunities for mesoscopics in thermometry and refrigeration: Physics and applications," *Review of Modern Physics*, **78**, 217, 2006. DOI: 10.1103/RevModPhys.78.217

[14] J.P. Pekola, F. Giazotto, and O. P. Saira, "Radio-Frequency Single-Electron Refrigerator," *Physical Review Letter*, **98**, 037201 , 2007. DOI: 10.1103/PhysRevLett.98.037201

[15] S. Kafanov, A. Kemppinen, Yu. Pashkin, M. Meschke, J.S. Tsai, and J. P. Pekola, "Single-Electronic Radio-Frequency Refrigerator," *Physical Review Letter*, **103**, 120801, 2009. DOI: 10.1103/PhysRevLett.103.120801

[16] P. Bhadrachalam, R. Subramanian, V. Ray, L. C. Ma, W. Wang, J. Kim, K. Cho, and S. J. Koh, "Energy-filtered cold electron transport at room temperature," *Nature Communications*, **5**, 4745, 2014. DOI: 10.1038/ncomms5745