A Concept for a Leaky Wave Antenna Oscillator With Second Order Degeneracy

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***Abstract*— We exploit a second order exceptional point of degeneracy (EPD) to enhance frequency purity and oscillation stability in oscillators. The EPD we consider in this paper is a regular band edge (RBE) that exists in lossless and gainless periodic waveguides. We present an example of single-ladder oscillator that may act as a leaky wave antenna (LWA). The oscillator we develop has 8 unit-cells that form a resonant cavity and an active element is used to compensate for the losses and start the oscillation. We show that the oscillation frequency is in proximity of the RBE frequency thanks to the second order degeneracy.**

Keywords— Regular band edge (RBE), microstrip resonator, microstrip oscillators, leaky wave antenna (LWA), circuit theory.

I. Introduction

The aim of this article is twofold: we first highlight how a second order EPD, specifically an RBE, is modelled and obtained in a one-dimensional periodic structure, second we present an example of a microstrip oscillator featuring an RBE that provides both a pure oscillation spectrum, suitable for leaky wave radiation. An EPD is a condition where two or more eigenmodes are fully degenerate. The order of degeneracy is equal to the number of coalescing eigenvectors and eigenvalues.

The RBE is a second order degeneracy in a lossless and gainless structure [1] and it corresponds to a band edge between pass and stop bands. Since the wave number in a "single mode waveguide" at an RBE is either $k = 0$ or π/d, the Floquet mode propagating in the structure forms a standing wave that can be employed to make a resonant cavity.

In the vicinity of an RBE, waves travelling in a waveguide experience a dramatic reduction of group velocity, hence they exhibit enhanced quality factor Q, high density of states, etc. Therefore, they are suited for applications like sensing [2], high-power electron beams [3], RF or microwave oscillators and filters [4], optical resonators or cavities [5,6], ultra-narrow beam LWAs [7], etc.

II. Description of RBEs in periodic structures

The wave propagation in a periodic microstrip transmission line is described using the transmission matrix method as in [7,8]. We define the two-dimensional system state vector that comprises the voltage and current in the transmission line as $\mathbf{\Psi}(x) = [V(x),\ I(x)]^T$, such that $\mathbf{\Psi}(x+d) = \underline{\mathbf{T}}(x+d, x)\,\mathbf{\Psi}(x)$, where d is the period, and $\underline{\mathbf{T}}$ is the unit-cell transfer matrix. In case of an infinitely long periodic structure, solutions are periodic eigenvectors represented by complex time harmonic functions. Assuming a time harmonic evolution $e^{j\omega t}$, the state vector at each n-th node satisfies the property $\mathbf{\Psi}(n) = \mathbf{\Psi}(n-1)e^{-jkd}$, where $k = \beta - j\alpha$ is the complex propagation constant and $n = 1,2,\ldots$. The matrix $\underline{\mathbf{T}}$ is a 2x2 diagonalizable matrix, whose eigenvalues are $e^{\pm jkd}$, representing backward (plus sign) and forward (minus sign) propagating eigenmodes, respectively. The RBE condition is verified when both eigenvalues and eigenvectors are degenerate, at frequency f_e. Under this assumption, $\underline{\mathbf{T}}$ becomes defective, i.e., it is no more diagonalizable, and it is similar to a Jordan block [8]. Therefore, at the RBE, the 2x2 similarity matrix $\underline{\mathbf{U}}$, which is a matrix containing the eigenvectors defined as in [8], becomes singular, i.e., $\det\{\underline{\mathbf{U}}(f_e)\}=0$. This means that two eigenvectors coalesce, i.e., become degenerate.

Perfect degeneracy condition is not satisfied for periodic waveguides that have losses like material loss or leaky wave radiation loss [9]. In case of finite-length structure, the actual resonant frequency $f_{e,r}$ is close to f_e and it is asymptotically approximated to $f_{e,r} \approx f_e \pm \delta(\pi/Nd)^2$, where $\delta = 2\pi(\partial^2 f/\partial k^2)|_{k=0}$ and N is the number of unit cells [1]. The finite-length structure forms a resonant cavity, where the guided mode is therefore a quasi-standing mode.

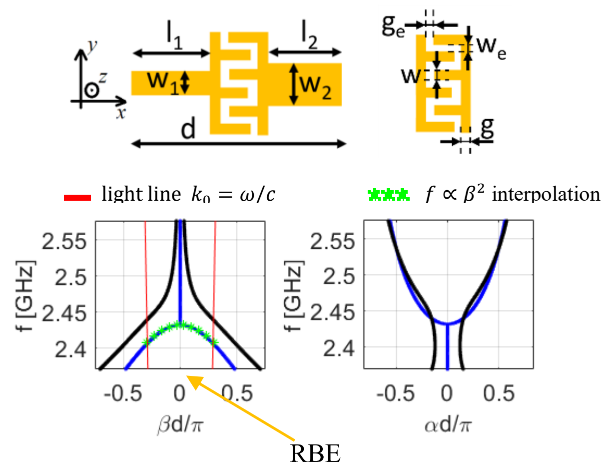

Fig. 1. (Top) Sketch of the designed unit-cell. (Bottom) Propagation constant β (left) and attenuation constant α (right) using Method of Moments full-wave simulations implemented in Keysight ADS for both lossless unit-cell (blue) and lossy case that accounts for conduction, dielectric and radiation losses (black).

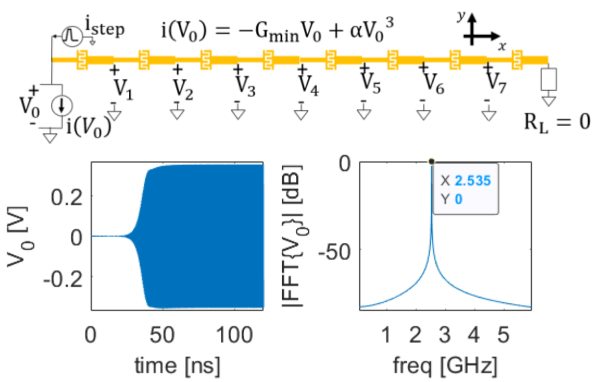

Fig. 2. Sketch of the ladder oscillator radiating as a LWA. (Bottom) Voltage waveform at the input node, V_0, in: (left) time domain and (right) normalized spectrum of the voltage evaluated by applying the FFT in a time window from 80 to 200 ns.

III. RBE OSCILLATOR

The proposed design aims to realize an RBE in a microstrip distributed single-ladder oscillator, at $f_{e,r}$, with $k(f_{e,r})d \approx 0$. To accomplish this task, we propose the asymmetrical unit cell design in Fig. 1 (top): d = 18 mm, w_1 = 0.7 mm, w_2= 1.62 mm, $L_1 = L_2$ = 8.088 mm, w = 0.2 mm, g = 0.1mm, g_e= 0.163 mm, w_e= 0.25 mm. The planar structure is based on a dielectric substrate (Duroid 6010) with height h= 0.127 mm, over a copper metal sheet ($\sigma = 5.8 * 10^7$ S/m). Figure 1 (bottom) shows that the dispersion diagram fits to $(f_e - f)/f_e \sim 0.11(\beta d)^2$ near the RBE and that degeneracy is partially lost when losses are introduced (conduction, dielectric and radiation loss). The proposed oscillator consists of a series cascade of $N = 8$ unit cells of total length $L = 144$ mm on a ground plane as shown in Fig. 2 (top). The active device is placed at the beginning of the cascaded 8 unit cells, and the other end of the structure is terminated in a load R_L. The active element is modelled as a shunt ideal non-linear voltage controlled current source, whose output characteristic is given by: $i_s = -g_s v_s + \alpha v_s^3$, where the value of α is chosen to be equal to $g_s/3$. This element is responsible for loss compensation, in order to satisfy the Kurokawa oscillation condition, i.e. $g_s \geq G_{min}$, where G_{min} = 5.8 mS is the resonator driving point admittance (input admittance at $f_{e,r}$), necessary to start-up oscillation. A current step generator is used to start-up the self-oscillation and a short circuit is used as load R_L= 0 on the right. The voltage spectrum appears rather pure, with oscillation frequency of $f_{e,r}$ = 2.53 GHz.

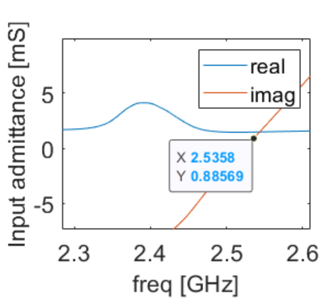

Fig. 3. Ladder oscillator input admittance around the RBE frequency.

The proposed framework would be valuable for realizing a new class of active LWAs based on the RBE, i.e., a 2nd order EPD.

IV. CONCLUSIONS

We have shown that an RBE realized in microstrip transmission line is useful to realize oscillators utilizing the 2nd order degeneracy condition. The spectral purity associated to such oscillators has been demonstrated through full-wave simulations. In fact, the oscillation frequency is in proximity of the RBE frequency. The second order degeneracy at the RBE is responsible for the single frequency of oscillation. The proposed oscillator is very promising for realizing a leaky wave antenna oscillator based on a RBE.

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