

EXPERIMENTAL ANALYSIS OF THE INFLUENCE OF DEFECTS ON BEARING ROLLING BALLS ON NOISE AND VIBRATIONS

*Original*

EXPERIMENTAL ANALYSIS OF THE INFLUENCE OF DEFECTS ON BEARING ROLLING BALLS ON NOISE AND VIBRATIONS / Sesana, Raffaella; PESSOLANO FILOS, Irene; Brusa, Eugenio; Pagliassotto, Stefano. - ELETTRONICO. - (2020), pp. 1-12. (Intervento presentato al convegno IRF 2020 tenutosi a Funchal, Madeira PT nel 6-10 settembre 2020).

*Availability:*

This version is available at: 11583/2819412 since: 2020-06-25T16:04:23Z

*Publisher:*

INEGI-Instituto de Ciência e Inovação em Engenharia Mecânica e Gestão Industrial

*Published*

DOI:

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

(Article begins on next page)

PAPER REF: 17284

## **EXPERIMENTAL ANALYSIS OF THE INFLUENCE OF DEFECTS IN BEARING ROLLING BALLS ON NOISE AND VIBRATIONS**

**Raffaella Sesana<sup>1(\*)</sup>, Irene Pessolano Filos<sup>1</sup>, Eugenio Brusa<sup>1</sup>, Stefano Pagliassotto<sup>2</sup>**

<sup>1</sup>DIMEAS, Politecnico di Torino, Torino, Italy

<sup>2</sup>TN Product Development, Tsubaki Nakashima Italy, Pinerolo, Torino, Italy

(\*)*Email: raffaella.sesana@polito.it*

### **ABSTRACT**

This study analyzes the impact of surface defect on rolling ball on the bearing vibration and noise, in working conditions. It resorts to an experimental approach, based on the DOE. Artificial defects were obtained on different sets of balls, assembled in axial bearings, which were tested at constant spin speed, on a dedicated test rig. The test rig was expressly designed to detect the ball vibration, with respect to its geometric and physical characteristics, and to quantify the impact on bearing noise and vibration, as well as their mutual interaction.

**Keywords:** ball bearing, damage, vibration, DOE, fault detection, diagnosis.

### **INTRODUCTION**

Rolling elements play an important role on the performance of rotating machinery and in literature are described as the main source of non-linearity in dynamic behavior [1], [2]. Surface defects are responsible for those dynamic phenomena, as well as internal clearance, unbalance, and preloading conditions. Many parameters of the ball bearing affect that behavior as stiffness, damping, and number of rolling elements. Bearing vibration has been deeply investigated, in the literature. A wide overview on causes, monitoring techniques, and modeling is presented in [3]. Vibration can be assumed as a damage and failure indicator, both in machines and in bearing components. The vibration analysis is a well-known technique for the component damage detection [2], [4]-[6]. It is known that vibration in bearing generates a periodic signal, due the finite number of rolling bodies [7]. For instance, in [8] a balanced rotor-bearing system without defects, the vibration peak amplitudes are detected, and related to frequency, to associate the signal harmonics to the number of rolling elements undergoing the cyclic loading in rotation. In [9], it is stated that bearing vibration associated to the rolling elements motion is randomly cyclic-stationary. This property is a bright indicator of an incipient fault, and it can be exploited in bearing diagnosis.

It is known that bearing contributes to the machine vibration and noise [4]. Despite a main characteristic behavior exhibited by the ball bearing in operation, to be associated to a certain radial and axial stiffness, it is known that slight differences in rolling elements and in raceways, zone by zone, might affect the overall dynamic response of the bearing, and lead to detect a vibration, referred to as conformity vibration [10]. Bearing vibration is usually increased by the local or distributed defects [11].

Distributed defects are generally due to manufacturing, assembling and to abrasive wear [1], [2]. Local defects include cracks, deformations, and scratches on rolling bodies. The pitting phenomenon is the main cause of damage in rolling bearing [5], [12].

Many papers, in the literature, investigate how the vibration spectrum is related to the nature and position of defect. A screening on the analytical models for estimating the role of many factors on the bearing vibration has been even performed [13], [14], [15]. For instance, in [16] the internal radial clearance, surface waviness, and off-size balls are considered. In [17] and [18], a dynamic model predicts the vibration response of a ball bearing, due to some localized surface defects on races, by describing the role of defect edge topographies, when the ball hits them, according to theory of Hertz, on the elastic contact. A similar analysis is carried out in [19]. Local defects are typically detected, on the frequency spectrum, assuming that an impulse is generated by the rolling body passing on the defected area. Almost all papers focus on scratches, generally artificially obtained. More details about the defect localization in the inner or outer ring, or in balls through the vibration spectrum analysis are proposed in [20]. A single defect, i.e. a single point fault, is used, being created by means of the EDM process, having dimensions of 25x25  $\mu\text{m}$ .

When the rolling elements are specifically investigated, the attention of the literature is focused on the effect of off-sizing balls [15], [16]. Several investigations are available, based on both the experimental and numerical activities, to identify the most relevant characteristics of balls which have a brighter impact on the bearing noise and vibration. In [21], an analytical modelling technique investigates the nonlinear vibration analysis of a bearing system assembled on a rotating machine. Bearings used in those tests exhibit some off-sizing and an imperfect roundness, measured by a roundness measuring instrument. The proposed analysis is based on the spectral analysis of bearing vibration and focuses on the ball waviness, more than on the ovalization.

In this paper, an experimental research activity is documented. It focuses on the effect of surface defects of rolling balls on the vibration frequency in association with the interaction between several factors, which affect the bearing dynamic response.

However, few studies have investigated the presence of surface scratches and vibration behavior. Currently, there are no data on different kind of surface defects on balls, such as geometry, plastic deformation or ovality. The original aim of this research is to investigate the influence of different kinds of surface defects of balls, not only scratches, and predicting the effect on vibration, related to the kind and the extension of defects. This paper proposes a new methodology by resorting to the DOE.

The original approach consists in the analysis of the effect of different defect on vibration velocity, without spectral analysis.

A preliminary identification of the main geometric properties of ball, which have a stronger impact on bearing noise and vibration, respectively, is performed. An evaluation of the relative interaction between some ball characteristics, resulting in an evident vibration, is then completed. A straight application of the Design of Experiment (DOE) technique to improve the bearing performance, and to reduce the time of experimental testing is finally promoted.

## **SET-UP OF RELEVANT PARAMETERS FOR BEARING FAULT DETECTION**

The Design of Experiment (DOE) technique is herein applied to evaluate the effect of physical and geometrical characteristics of rolling spheres on bearing vibration. The paper is aimed at focusing on what are the most influent parameters on the bearing system vibrational response. Therefore, some artificial defects have been created on balls, which have been then assembled into a bearing. The bearing has been set-up in a test rig, and then loaded and rotated at constant speed to extract its vibrational response. A similar experimental procedure has been recently

applied in [22], but defects have been created only on the races of rings. Nevertheless, no detail is reported on the defect kind and about the number of tests. Moreover, the interaction between factors is out of that investigation, although the effect of defects is analyzed by means of the spectrum analysis. In some other paper, like in [23], the single taper or single ball bearing with a single defect obtained by means of EDM is introduced, although the effect of the defect dimensions is not investigated.

In [11], a scratch on the ball is obtained by means of grinding and its dimensions vary from 20 to 71.5  $\mu\text{m}$  in extension and is up to 800  $\mu\text{m}$  in circumferential width. In [24], an analytical model is compared with some literature data, but only single scratch defects, with given dimensions are investigated.

In all the above mentioned studies, the spectral analysis identifies the effect of defect on vibration, but there is no specific investigation related to defect type, to its dimensions and to the interaction between defects.

To proceed with the DOE, it is worth noticing that bearing noise and vibration depend on assembling quality, lubrication, static and dynamic loading conditions, material hardness, ball geometry, ball surface properties, etc. In this research activity only some of those characteristics are considered, while the other ones are assumed to be stable and not varying in testing. Particularly, investigation focuses on the following parameters:

- geometry (ovality -  $V_{Dws}$ ), measured as the difference between the diameter of the smallest sphere circumscribed and the largest inscribed one, for given profile;
- dimensional scatter ( $V_{Dwl}$ ), conceived as the difference between the mean diameters of the largest and the smallest ball, in the sample, respectively;
- local plastic deformation;
- scratch properties.

The ISO 3290-1 Standard provides a significant reference for this purpose. The profile parameters are defined to classify the manufactured balls in quality grades: nominal ball diameter  $D_w$  (the diameter identifying the ball dimension), average ball diameter  $D_{wm}$  (the arithmetic average between the smallest and largest diameters of a single ball), average diameter of a ball sample  $D_{wml}$  (that is the arithmetic average between the average diameters of the largest and the smallest balls in the sample), ball gauge  $S$  (the allowed value of the difference between the average diameter of the ball sample and the nominal ball diameter).

The bearing investigated in this study is an angular contact manufactured by FAG. Its characteristics are listed in Table 1. Bearings have been disassembled and washed, employing ultrasound systems. The balls are made with 100Cr6 steel, having nominal diameter (6 mm), but different dimensions. There are two different qualities, according to the ISO 3290 Standard: G3 ( $V_{Dws} = 0,05 \mu\text{m}$ ) and G5 ( $V_{Dws} = 0,120 \mu\text{m}$ ).

Table 1 - Tested angular contact bearing characteristics

Pitch diameter [mm]	Ball diameter [mm]	Contact angle [°]	Number of balls
22	6	40	9

The profiles of balls have been measured by means of a MWA 160 SKF Steyr roundness meter, with 0.01  $\mu\text{m}$  of sensitivity. Examples of measurements are shown in Figure 1. The diameter

of balls is measured and is classified in three sets of samples, according to dimensional scatter  $V_{Dwl}$ : Sample 1 (0,050  $\mu\text{m}$ ), Sample 2 (0,175  $\mu\text{m}$ ) and Sample 3 (0,300  $\mu\text{m}$ ).

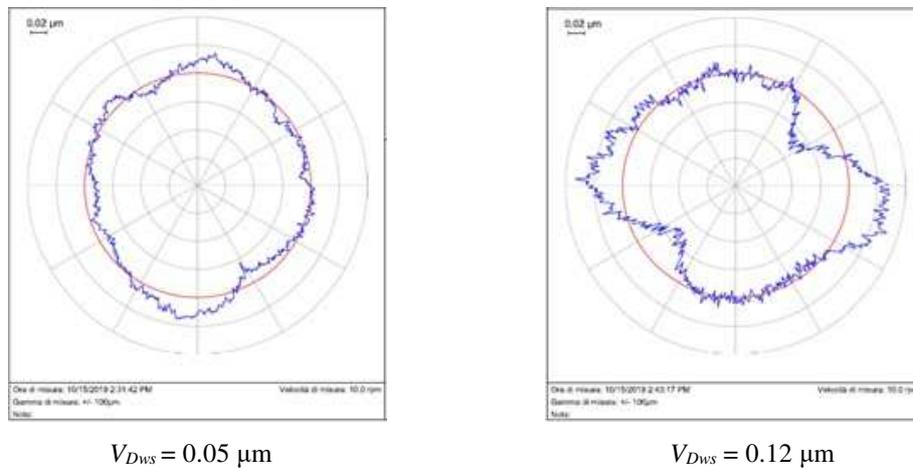


Fig. 1 - Profile measurements for two balls. Quality G3 (left) and G5 (right)

The local plastic deformation *LPD* is a defect due to a collision between balls, which generally occurs during the manufacturing process. In this research, to generate a controlled *LPD*, in testing each ball is made colliding by falling from a defined height against another ball, thus obtaining a controlled energy collision. The *LPD* is measured by means of a laser interferometer [25] Trioptics  $\mu\text{Phase}$  1000, with  $1000 \times 1000$  px resolution and 1 nm sensitivity. The *LPDs* are classified basing on deformation depth, that is the difference between the highest peak and the lowest valley. The surface extension is not taken into account. Three levels of *LPD* are then obtained: Level 1, with no plastic deformation; Level 2 ( $LPD < 0.3 \mu\text{m}$ ) corresponding to 700 mm falling height; Level 3 ( $LPD = 0.3 \div 0.6 \mu\text{m}$ ) corresponding to 1000 mm falling height. In Figure 2, an example of measurement is reported.

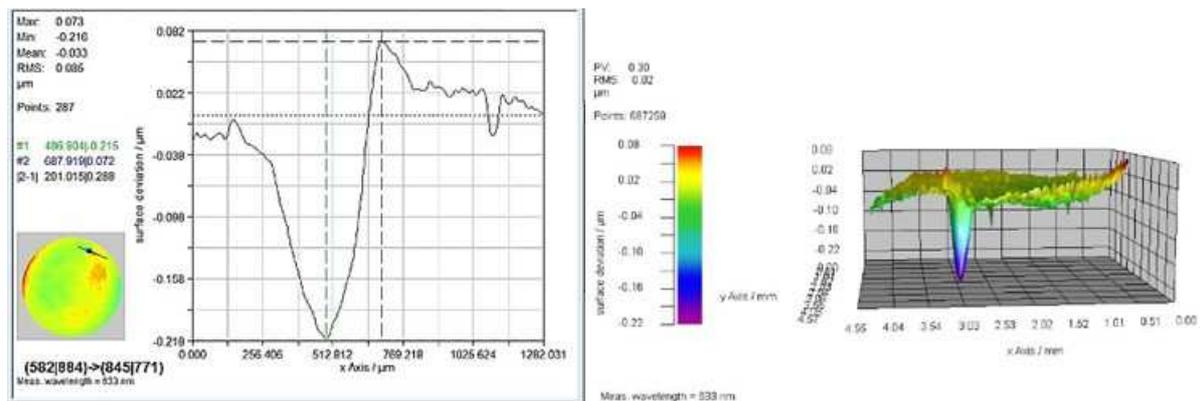


Fig. 2 - *LPD* measurements (example of the defined level 2)

In real operation, the contact stress between balls is influenced by surface roughness and this effect requires to be estimated to avoid any underestimation of damage risk, noise and vibration. In this research, some scratches, whose roughness has been controlled, are obtained using a file, and then they are observed employing a microscope and measured with a roughness tester. The parameter  $R_t$  [ $\mu\text{m}$ ] has been selected to quantify the factor scratch. Scratches are classified

according to three levels: Level 1 ( $R_t = 0.1 \div 0.39 \mu\text{m}$ , no scratch), Level 2 ( $R_t = 0.4 \div 0.59 \mu\text{m}$ ), Level 3 ( $R_t = 0.6 \div 0.9 \mu\text{m}$ ). In Figure 3, an example of observed scratch is reported.

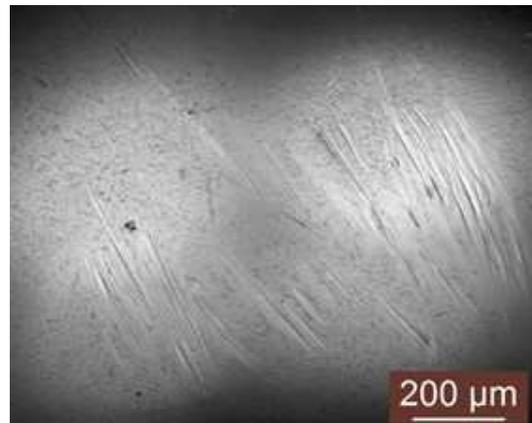


Fig. 3 - Microscope observation of a scratch on the ball surface

### VIBRATION MEASUREMENT IN THE BEARING

In the last years, many instruments have been developed to measure and process in line and off-line vibration data, to investigate on bearing vibration phenomena [26]–[31]. Noise and vibration in bearings can be classified in four categories: structural (related to cages, races, etc.), manufacturing (related to geometric imperfections of inner and outer rings and rolling bodies), handling related (related to local defects and contamination, generating irregular noise), other causes (for instance, due to lubricant).

It is known that collisions with deformations due to surface defects in rings and in rolling elements generate vibration. Neglecting sliding phenomena, the frequency of vibration signal components allows detecting where the defect is located; in diagnostics, the so-called *BPF<sub>i</sub>* (*Ball Passing Frequency i*) frequencies are defined, according to position of defect. Particularly, the transitory impulsive signal is generated at some typical frequencies, which depend on bearing angular speed and geometry [25]. If the defect is located in the bearing cage, the corresponding frequency is defined as *FTF* (train or cage frequency). If it applies to the inner ring, it is the *BPFI* (Ball Pass Frequency Inner race); when it does to the outer ring, the corresponding frequency is *BPFO* (Ball Pass Frequency Outer race); while the effect on the rolling body is corresponding to the *BSF* (Ball Spin Frequency) [32]. Those frequencies are usually and deeply used in machinery monitoring, where bearings are sentry of occurring damage, as in [33].

Apart of spectrum analysis, measuring the vibration velocity provides useful information of defect kind and defect extension on rolling bearing elements, especially when bearing rotates at constant spin speed.

In this activity, vibration is acquired by the Schaeffler MGG11-MC500 noise testing system, allowing to measure the vibration, in case of bearing rotating at constant speed. During the test, the inner ring rotates with shaft (i.e. at 1800 rpm), while an axial load (100 N) is applied to the outer ring. The SG4 inductive velocity sensor, located above the bearing, acquires the radial speed of vibration in  $\mu\text{m/s}$ ; the signal is acquired and filtered through some dedicated software.

To detect the defect, the vibration of a commercial radial axial ball bearing is here investigated. The relative sliding between races and balls is not taken into account. The main parameters affecting the frequency of collisions are the relative rotational speed of rings and the number of rolling bodies. The signal generated by a defect present in the bearing cannot be assumed as

periodic, with ring revolution, if the sliding value is high. If the sliding value is low, the signal can be recognized as stationary.

## BEARINGS TESTS AND ANALYSIS

The implementation of the DOE has been performed through a testing activity. Bearings assembled with defected samples of balls underwent axial load and shaft rotation and vibration velocity in radial direction was acquired. Each acquisition lasted 2 seconds, with 3 repetitions for each sample. The acquisition has been performed with the same pair of rings. The input parameters are ovality ( $V_{Dws}$ ), scratch, plastic deformation ( $LPD$ ), dimensional scatter ( $V_{Dwl}$ ). Each parameter has 3 levels, except for  $V_{Dws}$  which has only 2. Up to 36 tests have been performed, and 9 balls for each test have been used, up to 324 balls tested. The output parameter of this activity is the amplitude of vibration velocity wave, at BSF. The DOE variation list is described in Table 2.

Table 2 - DOE variation list

Test number	( $V_{Dws}$ ) [ $\mu\text{m}$ ]	Scratch ( $R_r$ ) [ $\mu\text{m}$ ]	LPD [ $\mu\text{m}$ ]	$V_{Dwl}$ [ $\mu\text{m}$ ]	Vibration velocity [ $\mu\text{m/s}$ ]
T 1	0,05	0,05	0	0,05	2,3
T 2	0,05	0,05	0,265	0,175	3,2
T 3	0,05	0,05	0,413	0,3	3,2
T 4	0,05	0,05	0	0,05	2,1
T 5	0,05	0,05	0,282	0,175	4,1
T 6	0,05	0,05	0,402	0,3	4,1
T 7	0,05	0,05	0	0,05	1,8
T 8	0,05	0,05	0,316	0,175	3,7
T 9	0,05	0,05	0,407	0,3	3,2
T 10	0,05	0,43	0	0,05	3,2
T 11	0,05	0,84	0,139	0,175	3,8
T 12	0,05	0,64	0,474	0,3	4,9
T 13	0,05	0,6	0	0,05	3,0
T 14	0,05	0,85	0,259	0,175	4,0
T 15	0,05	0,6	0,325	0,3	3,6
T 16	0,05	0,5	0	0,05	4,2
T 17	0,05	0,76	0,219	0,175	2,8
T 18	0,05	0,85	0,416	0,3	3,7
T 19	0,12	0,05	0	0,05	3,5
T 20	0,12	0,05	0,289	0,175	4,2
T 21	0,12	0,05	0,447	0,3	3,0
T 22	0,12	0,05	0	0,05	3,2
T 23	0,12	0,05	0,223	0,175	4,2
T 24	0,12	0,05	0,547	0,3	4,3
T 25	0,12	0,05	0	0,05	3,6
T 26	0,12	0,05	0,288	0,175	3,7

T 27	0,12	0,05	0,461	0,3	4,2
T 28	0,12	0,77	0	0,05	4,7
T 29	0,12	0,81	0,359	0,175	4,3
T 30	0,12	0,89	0,42	0,3	5,1
T 31	0,12	0,47	0	0,05	4,7
T 32	0,12	0,71	0,296	0,175	5,2
T 33	0,12	0,52	0,322	0,3	4,5
T 34	0,12	0,55	0	0,05	3,6
T 35	0,12	0,6	0,259	0,175	4,6
T 36	0,12	0,66	0,331	0,3	5,0

In Table 2, the measured vibration velocities are reported in column 6. Each value is the average of three repetitions. These results are processed by means of ANOVA with four factors, three levels and with repetitions. A preliminary test to determine if the association between the output (vibration velocity) and each factor in the model was statistically significant, by comparing the P-value of each factor with the level of significance to evaluate the null hypothesis. The selected level of significance is 0.05 and the resulting P-values are reported in Table 3; they show that the factors can be significant.

Table 3 - P-values for the investigated factors

	$V_{Dws}$	Scratch	$LPD$	$V_{Dwl}$
P-value	0.000	0.000	0.001	0.043

A further confirmation of the models comes from the so-called residual plot reported in Figure 4. The points are randomly distributed, meaning that the proposed model is adequate.

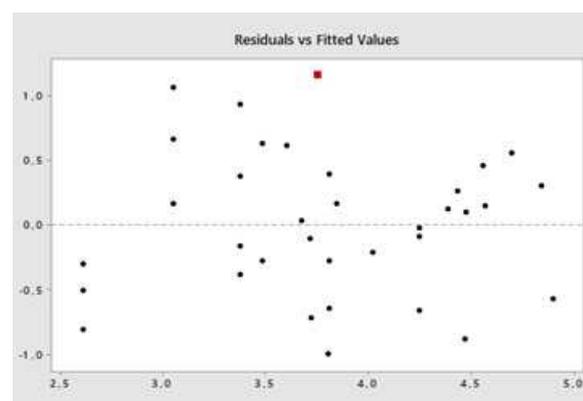


Fig. 4 - Residual fitted value obtained by the DOE

A more detailed analysis can give indications on what are the factors affecting the vibrations and the possible effect of their interactions.

In Figure 5 the main effect plots are reported, that is the variation of the output in function of the variation of each factor. It shows that scratch, ovality ( $V_{Dws}$ ) and  $LPD$  have a relevant influence on vibration, while the dimension scatter ( $V_{Dwl}$ ) appears to be less influent on the system dynamic response.

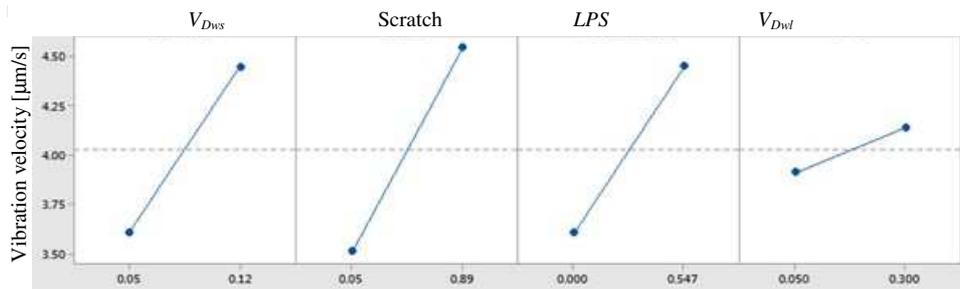


Fig. 5 - Main effects plot obtained by the DOE

In Figure 6, the effect of combination of the analyzed factors on vibration is reported. An interesting result is related to the case of combination of dimension scatter  $VDwl$  and LPD. For a bearing without LPD, increasing the dimension scatter, the vibration velocity remains constant. By converse, when balls have a maximum level of LPD, the increment of  $VDwl$  causes an increasing vibration. The interaction between scratch and LPD shows a similar effect. For ball without scratches, increasing LPD, the vibration increment is steep, while if balls have the highest level of scratch, the increment of vibration is less steep with the LPD depth.

An additional result can be appreciated. The DOE analysis allows to identify a suitable combination of optimal factors to limit the noise and vibration, for what concerns geometry, scratch, plastic deformation, and  $V_{Dwl}$ . As an example, the top 5 combinations which minimize and maximize vibration are reported in Table 4.

This result agrees with those found in [9], where it is demonstrated that not only the presence, but also the severity of damage can be detected, by means of the vibration spectrum analysis. In [22] the experimental investigation and vibration spectrum analysis show that for constant speed and constant load, defects located on inner and outer rings generate amplitudes of vibration and of accelerations which increase with increasing defect sizes. The FFT analyzers result as a powerful tool for health monitoring for rolling bearings.

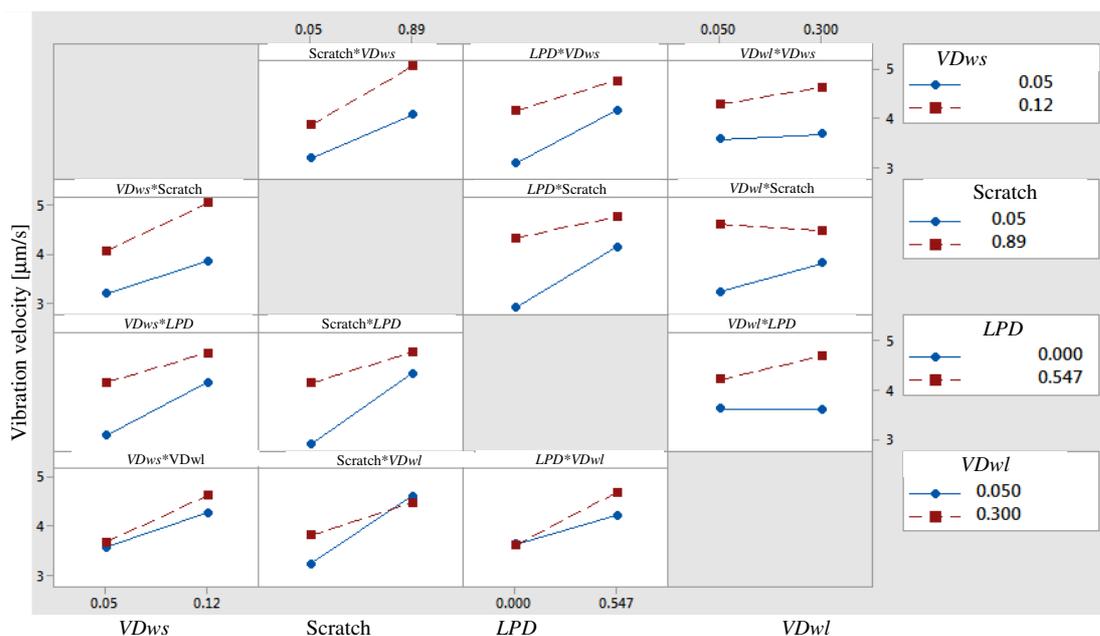


Fig. 6 - Interaction between factors according to the DOE

Table 4 - predicted vibration velocities for optimal and worst factors combinations

	$V_{Dws}$ [ $\mu\text{m}$ ]	Scratch [ $\mu\text{m}$ ]	$LPD$ [ $\mu\text{m}$ ]	$V_{Dwl}$ [ $\mu\text{m}$ ]	Predicted vibration velocity [ $\mu\text{m/s}$ ]
vibration minimizing combinations	0,05	0,43	0	0,050	2,86931
	0,05	0,05	0,265	0,175	3,11717
	0,05	0,05	0,282	0,175	3,14752
	0,05	0,50	0	0,175	3,17514
	0,05	0,05	0,316	0,175	3,20822
vibration maximizing combinations	0,12	0,89	0,420	0,175	5,15645
	0,12	0,81	0,359	0,050	4,72992
	0,12	0,66	0,331	0,175	4,72613
	0,12	0,52	0,322	0,175	4,54484
	0,12	0,71	0,296	0,050	4,49943

## CONCLUSIONS

The research aims to point out which kind and level of the surface damage in balls, in a bearing, have a higher impact on noise and vibrations by analyzing the vibration speed amplitude variation. The findings reported here shed new light on finding the best combination between the input factors (ovality, scratch, local plastic deformation, dimensional scatter) and the measured output, that is the vibration velocity and then the noise, by means of Design Of Experiment and experimental testing. In particular an ANOVA, with 4 factors, 3 levels and repetitions, was performed.

Artificial defects were obtained on one ball per sample and the samples including one defected ball were assembled in radial axial bearings. The bearings were tested on a test ring, under axial loading, constant rotational speed and radial vibrations were measured.

This new approach should help to improve predictions of the impact of surface defects on balls.

The scratch, the geometry, and the plastic deformation resulted the most influencing factors, while the dimension scatters affect less the noise and the vibration phenomena.

Moreover, the analyses showed that despite the dimension scatter influence seems to be low if considered alone, it results to be more effective if in combination with other factors.

The best and worst combinations of the factors affecting bearing vibrations were simulated.

An original approach, different from traditional spectral analysis of vibration, was implemented to investigate the effect of different kind of defects and the possible interactions. In particular the contribution of the different kind of defects and the influence of the extension of the defects is highlighted by means of this approach.

## REFERENCES

- [1] Tandon N, Choudhury A, "Review of vibration and acoustic measurement methods for the detection of defects in rolling element bearings," *Tribol. Int.*, vol. 32, no. 8, pp.469-480, 1999, doi: 10.1016/S0301-679X(99)00077-8.

- [2] Tandon N, Choudhury A, "A theoretical model to predict the vibration response of rolling bearings in a rotor bearing system to distributed defects under radial load," *J. Tribol.*, vol. 122, no. 3, pp.609-615, 2000, doi: 10.1115/1.555409.
- [3] Sharma A, Upadhyay N, Kankar PK, Amarnath M, "Nonlinear dynamic investigations on rolling element bearings: A review," *Adv. Mech. Eng.*, vol. 10, no. 3, pp.1-15, 2018, doi: 10.1177/1687814018764148.
- [4] Ngai KW, Ng CF, "Structure-borne noise and vibration of concrete box structure and rail viaduct," *J. Sound Vib.*, vol. 255, no. 2, pp.281-297, 2003, doi: 10.1006/jsvi.2001.4155.
- [5] Sharma A, Amarnath M, Kankar PK, "Feature extraction and fault severity classification in ball bearings," *JVC/Journal Vib. Control*, vol. 22, no. 1, pp.176-192, 2016, doi: 10.1177/1077546314528021.
- [6] Sharma A, Amarnath M, Kankar PK, "Novel ensemble techniques for classification of rolling element bearing faults," *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 39, no. 3, pp.709-724, 2017, doi: 10.1007/s40430-016-0540-8.
- [7] C. S. Sunnersjö, "Rolling bearing vibrations-The effects of geometrical imperfections and wear," *J. Sound Vib.*, vol. 98, no. 4, pp.455-474, 1985, doi: 10.1016/0022-460X(85)90256-1.
- [8] Sunnersjö CS, "Varying compliance vibrations of rolling bearings," *J. Sound Vib.*, vol. 58, no. 3, pp.363-373, 1978, doi: 10.1016/S0022-460X(78)80044-3.
- [9] Hayata Y, Antoni J, "Cyclic spectral analysis of rolling-element bearing signals: Facts and fictions," *Chem. Pharm. Bull.*, vol. 304, no. 3-5, p.2091, 2007, doi: 10.1016/j.jsv.2007.02.029.
- [10] El-Thalji I, Jantunen E, "A summary of fault modelling and predictive health monitoring of rolling element bearings," *Mech. Syst. Signal Process.*, vol. 60-61, pp.252-272, Aug. 2015, doi: 10.1016/j.ymssp.2015.02.008.
- [11] Igarashi T, Hamada H, "Studies on the Vibration and Sound of Defective Rolling Bearings," *Bull. JSME*, vol. 25, pp.8, 1982.
- [12] Orhan S, Aktürk N, Çelik V, "Vibration monitoring for defect diagnosis of rolling element bearings as a predictive maintenance tool: Comprehensive case studies," *NDT E Int.*, vol. 39, no. 4, pp.293-298, Jun. 2006, doi: 10.1016/j.ndteint.2005.08.008.
- [13] Jang GH, Jeong SW, "Nonlinear excitation model of ball bearing waviness in a rigid rotor supported by two or more ball bearings considering five degrees of freedom," *J. Tribol.*, vol. 124, no. 1, pp.82-90, 2002, doi: 10.1115/1.1398289.
- [14] Jang GH, Jeong SW, "Stability analysis of a rotating system due to the effect of ball bearing waviness," *J. Tribol.*, vol. 125, no. 1, pp.91-101, 2003, doi: 10.1115/1.1504090.
- [15] Harsha SP, "The effect of ball size variation on nonlinear vibrations associated with ball bearings," *Proc. Inst. Mech. Eng. Part K J. Multi-body Dyn.*, vol. 218, no. 4, pp.191-210, 2004, doi: 10.1243/1464419043541455.

- [16]Lynagh N, Rahnejat H ,Ebrahimi M, Aini R, “Bearing induced vibration in precision high speed routing spindles,” *Int. J. Mach. Tools Manuf.*, vol. 40, no. 4, pp.561-577, 2000, doi: 10.1016/S0890-6955(99)00076-0.
- [17]Liu J, Shao Y, “A new dynamic model for vibration analysis of a ball bearing due to a localized surface defect considering edge topographies,” *Nonlinear Dyn.*, vol. 79, no. 2, pp.1329-1351, 2014, doi: 10.1007/s11071-014-1745-y.
- [18]Liu J, Shao Y, Lim TC, “Impulse vibration transmissibility characteristics in the presence of localized surface defects in deep groove ball bearing systems,” *Proc. Inst. Mech. Eng. Part K J. Multi-body Dyn.*, vol. 228, no. 1, pp.62-81, 2014, doi: 10.1177/1464419313514572.
- [19]Niu L , Cao H , He Z, Li Y, “Dynamic Modeling and Vibration Response Simulation for High Speed Rolling Ball Bearings With Localized Surface Defects in Raceways,” *J. Manuf. Sci. Eng.*, vol. 136, no. 4, pp.1-16, Aug. 2014, doi: 10.1115/1.4027334.
- [20]Rafsanjani A, Abbasian S, Farshidianfar A, Moeenfard H, “Nonlinear dynamic modeling of surface defects in rolling element bearing systems,” *J. Sound Vib.*, vol. 319, no. 3–5, pp.1150-1174, Jan. 2009, doi: 10.1016/j.jsv.2008.06.043.
- [21]Su Y-T, Lin M-H, Lee M-S, “The Effects of Surface Irregularities on Roller Bearing Vibrations,” *J. Sound Vib.*, vol. 165, no. 3, pp.455-466, Aug. 1993, doi: 10.1006/jsvi.1993.1270.
- [22]Kondhalka G, Diwakar E, “Effect of various defects in roller bearings and ball bearings on vibration,” *Int. J. Innov. Technol. Explor. Eng.*, vol. 8, no. 12, pp.5137-5141, 2019, doi: 10.35940/ijitee.L2763.1081219.
- [23]Ma J, Li JC, “Detection of localised defects in rolling element bearings via composite hypothesis test,” *Mech. Syst. Signal Process.*, vol. 9, no. 1, pp.63-75, Jan. 1995, doi: 10.1006/mssp.1995.0005.
- [24]Tandon N, Choudhury A, “An Analytical Model for the Prediction of the Vibration Response of Rolling Element Bearings Due to a Localized Defect” *J. Sound Vib.*, vol. 205, no. 3, pp.275-292, Aug. 1997, doi: 10.1006/jsvi.1997.1031.
- [25]Interferometry US, Norgia M, Donati S, “A Displacement-Measuring Instrument,” vol. 52, no. 6, pp.1765-1770, 2003.
- [26]Antoni J, Bonnardot F, Raad A, El Badaoui M, “Cyclostationary modelling of rotating machine vibration signals,” *Mech. Syst. Signal Process.*, vol. 18, no. 6, pp.1285-1314, Nov. 2004, doi: 10.1016/S0888-3270(03)00088-8.
- [27]McCormick AC, Nandi AK, “Cyclostationarity in Rotating Machine Vibrations 1 Introduction 2 Wide-sense Cyclostationarity,” *Mech. Syst. Signal Process.*, vol. 12, no. 2, pp.225-242, 1998.
- [28]Antoniadis I, Glossiotis G, “Cyclostationary analysis of rolling-element bearing vibration signals,” *J. Sound Vib.*, vol. 248, no. 5, pp.829-845, 2001, doi: 10.1006/jsvi.2001.3815.

- [29] Randall RB, Antoni J, Chobsaard S, “The relationship between spectral correlation and envelope analysis in the diagnostics of bearing faults and other cyclostationary machine signals,” *Mech. Syst. Signal Process.*, vol. 15, no. 5, pp.945-962, 2001, doi: 10.1006/mssp.2001.1415.
- [30] C. Capdessus, M. Sidahmed, and J. L. Lacoume, “Cyclostationary processes: application in gear faults early diagnosis,” *Mech. Syst. Signal Process.*, vol. 14, no. 3, pp.371-385, 2000, doi: 10.1006/mssp.1999.1260.
- [31] Antoni J, Randall RB, “Differential diagnosis of gear and bearing faults,” *J. Vib. Acoust. Trans. ASME*, vol. 124, no. 2, pp.165-171, 2002, doi: 10.1115/1.1456906.
- [32] Su YT, Lin SJ, “On initial fault detection of a tapered roller bearing: Frequency domain analysis,” *J. Sound Vib.*, vol. 155, no. 1, pp.75-84, 1992, doi: 10.1016/0022-460X(92)90646-F.
- [33] Brusa E, Lemma L, Benasciutti D, “Vibration analysis of Sendzimir cold rolling mill and bearing fault detection”, *Proc. ImechE - Part C: J. Mech.Eng. Science*, 224, 2010, pp.1645-1654, doi:10.1243/09544062JMES1540.