Time-Dependent Probability of Exceeding a Target Level of Recovery in Resilience Analysis

Fabrizio Nocera[a], Paolo Gardoni[b], Gian Paolo Cimellaro[c]

[a] *PhD Student, Department of Civil and Environmental Engineering, MAE Center: Creating a Multi-Hazard Approach to Engineering, University of Illinois at Urbana-Champaign, 205 N. Mathews Ave., Urbana, IL 61801. E-mail:* fnocera@illinois.edu

[b] *Professor, Department of Civil and Environmental Engineering, MAE Center: Creating a Multi-Hazard Approach to Engineering, University of Illinois at Urbana-Champaign, 205 N. Mathews Ave., Urbana, IL 61801. E-mail:* gardoni@illinois.edu

[c] *Professor, Department of Structural and Geotechnical Engineering, DISEG, Politecnico di Torino, Corso Duca Degli Abruzzi 24, Turin, Italy 10129. E-mail:* gianpaolo.cimellaro@polito.it

Abstract

The resilience of a system is generally defined in terms of its ability to withstand external perturbation(s), adapt, and rapidly recover. This paper introduces a probabilistic formulation to predict the recovery process of a system given past recovery data, and estimate the probability of reaching or exceeding a target value of functionality at any time. A Bayesian inference is used to capture the changes over time of model parameters as recovery data become available during the work progress. The proposed formulation is general and can be applied to continuous recovery processes such as those of economic or natural systems, as well as to discrete recovery processes typical of engineering systems. As an illustration of the proposed formulation, two examples are provided. The paper models the recovery of a reinforced concrete bridge following seismic damage, as well as the population relocation after the occurrence of a seismic event when no data on the duration of the recovery are available a priori.

Keywords: Decision Support, Recovery, Resilience, Resilience Metrics, Probability, Reliability Analysis.

Introduction

Civil infrastructure enables the conveyance of goods, services, and resources to communities (Corotis 2009; Ellingwood et al. 2016; Gardoni et al. 2016). Past disasters continue to show the vulnerability of civil infrastructure to natural and anthropogenic hazards and highlight the significance of risk mitigation and management (Murphy and Gardoni 2006; Gardoni et al. 2016). Buildings, bridges, and other structures and infrastructure may experience extreme natural events, such as floods, earthquakes, hurricanes, and anthropogenic hazards, such as accidents and terrorist attacks, which may lead to significant damage making infrastructure networks inoperative (Gardoni and LaFave 2016). Past disasters stressed the importance of being prepared and to be able to recover in a short period (e.g., Bruneau et al. 2003; McAllister 2013; Caverzan and Solomos 2014).

The concept of resilience has gained relevance in the last fifteen years as a desirable feature for communities (Bruneau et al. 2003; McAllister 2013; Caverzan and Solomos 2014; Ellingwood et al. 2016; Guidotti et al. 2016, 2017; Sharma et al. 2018; Gardoni 2018). The relatively recent interest in resilience has resulted in several definitions of the concept of resilience and several approaches to measuring resilience across several application domains. In general, resilience is defined as the ability of systems to recover after a disturbance to the pre-disturbance state or a new (improved) state (e.g., Bruneau et al. 2003; Cimellaro et al. 2010a; Bocchini et al. 2012). The U.S. Presidential Policy Directive 21 (PPD 21) defines resilience as *the ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents*. A review of the current state of the research in community resilience can be found in Koliou et al. (2018). Going beyond the engineering domain, Doorn et al. (2018) explored how philosophical and social science considerations can be incorporated into a multidisciplinary definition of resilience to account for social justice. The choice of a defined recovery curve plays a key role in resilience analysis in terms of quantifying the resilience of a system. A recovery curve describes the behavior of a system as a function of time following the impact of a hazard as the system recovers to achieve a desired state (of functionality or of reliability.) In absence of disrupting shocks during the recovery phase, the recovery curve is, in general, a non-decreasing and time-dependent function. Different studies have attempted to model and define the recovery curve of engineering systems subject to a hazard (e.g., Cimellaro et al. 2010b; Decò et al. 2013; Titi et al. 2015). Recovery curves are usually assumed based on qualitative attributes, such as the preparedness of the society, that influence the recovery process. As such, they i) are not based on the actual physics of the recovery process, ii) do not account for the underlying uncertainties, and iii) are not able to incorporate additional information as it becomes available (such as ongoing progress of the work or increased resource availability, which affect the recovery models and reduce the uncertainty involved.) As a result, models of recovery typically only provide crude approximations and not accounting for the underplaying uncertainties makes it not possible to estimate the probability of reaching or exceeding a target percentile of interest of the ultimate desired state (e.g., a target value of functionality or reliability). To overcome these limitations, Sharma et al. (2018) proposed a mathematical formulation for resilience analysis that models the recovery curves based on the actual work plan of activities involved in the recovery process.

Once a recovery curve is defined, there is a need to define a metric or a set of metrics of recovery that distinctively characterize the recovery curve. A typical resilience metric has been defined as the integral of the recovery curve over a specified interval of time (Bruneau and Reihnorn 2007; Cimellaro et al. 2010a; Bonstrom and Corotis 2016). However, such metrics do not uniquely and fully characterize a recovery curve. Sharma et al. (2018) defined a set of resilience metrics in analogy with the moments of a random variable to quantify the resilience of a system. The Sharma et al.'s metrics i) are intuitive because of their analogy with the moments of a random variable, and ii) define a complete set of partial descriptors that uniquely and fully characterize a recovery curve.

This paper contributes to the literature in resilience analysis. In particular, this paper proposes a probabilistic formulation to predict a recovery process of a system, and then estimate the probability of reaching or exceeding a target value of functionality (or reliability) of the system at any given time as the system recovers. The proposed formulation uses Sharma et al.'s resilience metrics obtained from historical recovery data to predict possible recovery processes along with their likelihood, as well as to estimate the probability of reaching or exceeding a desired level of recovery by a desired time. The proposed formulation can be applied to systems in different fields, i.e., economical, natural, and engineering systems. The proposed formulation first defines the joint probability density function (PDF) of resilience metrics that captures the underlying uncertainties. Then, parametrized recovery curves are introduced to model the time-varying recovery process, and the joint PDF of model parameters is obtained as a function of the joint PDF of the resilience metrics. The joint PDF of the model parameters defines the variability in the possible recovery curves, which is used to estimate the probability of reaching or exceeding a target value of functionality by conducting a reliability analysis (Ditlevsen and Madsen 1996; Gardoni 2017). A Bayesian inference is also proposed to include possible information from the field while the work for the recovery is in progress. We use field data to update the predicted recovery curve such that the recovery curve is updated to reflect the advancement of the actual recovery in the field. Thanks to the Bayesian updating, the uncertainties in the recovery process diminish while more data become available. The main benefits of the proposed formulation are that the estimates of the recovery curve can be simply defined as a function of resilience metrics and the modeling can take advantage of data collected both before and during the recovery process. We illustrate the proposed formulation considering the recovery of a typical reinforced concrete (RC) bridge following a seismic damage, and the population relocation after the occurrence of a seismic event when no data on the duration of the recovery are available a priori.

Following this introduction, Section 2 gives a brief review of mathematical formulations for resilience analysis. Section 3 presents the proposed probabilistic formulation. Section 4 illustrates the proposed formulation considering the recovery of an example bridge following a seismic damage. Finally, Section 5 uses the propose approach considering the population relocation after a seismic event.

Review of Mathematical Formulations for Resilience Analysis

The resilience analysis of engineering systems plays a key role in mitigation planning and allocation of resources in pre- and post-disruption scenarios (Ellingwood et al. 2016). Resilience of a system is, in general, defined as its ability to maintain or promptly resume a level of functionality or performance after a disruption. What is promptly enough is usually defined based on the owner's, customers' or, more generally, societal needs. A performance measure (e.g., the system functionality), typically indicated as $Q(t)$, can be used to describe the system state as a function of time t (Cimellaro et al. 2010a; Bocchini et al. 2012; Bonstrom and Corotis 2016; Sharma et al. 2018). An external shock, such as a natural or anthropogenic event (a shock), might reduce $Q(t)$ instantaneously. Such reduction is typically a function of the intensity of the shock, the system design specifications (which define the system robustness at $t=0$, e.g., Bai et al. 2009), and the system state immediately before the shock (which reflect the deterioration of a system over time and also define the system robustness at time t, e.g., Kumar and Gardoni 2014; Kumar et al. 2015; Jia and Gardoni 2018a,b). After a shock, the recovery process starts to retrieve the system functionality to a desired level, which may be below, same or better than the pre-disruption value (Ayyub 2014, 2015).

Resilience, independently from the field of application, consists of four properties (Bruneau et al. 2003; Tierney and Bruneau 2007): 1) robustness as the ability to withstand a given level of stress or demand without suffering degradation or loss of function or, if a degradation occurs, the residual level of $Q(t)$; 2) resourcefulness, interrelated to the ability to diagnose and prioritize issues and to initiate solutions by identifying and monitoring all resources; 3) redundancy, defined as the extent to which the system and other elements satisfy and sustain functional requirements in the event of disturbance; and 4) rapidity as the ability to recover in a timely manner to limit losses and avoid future disruptions. These four properties define the resilience of a system and characterize the recovery process.

Recovery curves capture the changes in system functionality over time and defined how the system state improves to achieve a desired value of functionality at the end of the recovery process. Different studies have attempted to quantify the resilience of a system based on the shape of the recovery curves. As a first attempt to quantify the resilience of a system, Bruneau et al. (2003) proposed to measure the resilience as the area underneath the recovery curve. Chang and Shinozuka (2004) assessed the resilience as the probability that the time needed for the recovery due to a performance loss after a disruption would be less than a predefined threshold. Garbin (2007) outlined an approach to quantitatively measure the resilience of a network as the percentage of links damaged and the percentage of nodes damaged versus a network performance measure. Bruneau and Reinhorn (2007) proposed metrics for measuring resiliency based on the expected degradation in the quality of an infrastructure by quantifying robustness, redundancy, resourcefulness, and rapidity to recovery. While these contributions show the importance of quantifying resilience in an objective and formal way, the metrics they define only provide partial information about the actual resilience and might not be able to distinguish among different resilience levels (as noted in Sharma et al. 2018). Uniquely and fully characterizing the resilience of the system requires capturing all of the relevant characteristics of the recovery curve. Consequently, a single metric cannot represent a curve and capture all of its attributes.

Sharma et al. (2018) showed that the existing metrics are not able to uniquely and fully characterize recovery curves with different shapes and might not be able to capture the difference in the resilience levels. To address this issue, they developed a complete set of resilience metrics able to fully describe the recovery process and capture the differences in the shapes of different recovery curves. Sharma et al.'s resilience metrics are analogous to the partial descriptors commonly adopted in probability and statistics (e.g., mean, standard deviation and higher moments of a random variable.) The recovery curve $Q(t)$, which Sharma et al. (2018) call the *cumulative resilience function* (CRF) in analogy with the *cumulative distribution function* (CDF) of a random variable, represents the overall recovery process as a function of time. If the CRF is a continuous and differentiable function of the time, it is possible to describe the instantaneous rate of recovery as the *resilience density function* (RDF) q defined as the time derivative of the CRF (in analogy with the definition of the *probability density function* (PDF) of a random variable). If the CRF is not continuous and differentiable, it is possible to define a *resilience mass function* (RMF) that describes the instantaneous change of the recovery occurring as a step-wise function (in analogy with the probability mass function (PMF) of a random variable).

Based on these definitions, Sharma et al. (2018) introduced a set of resilience metrics to capture the specific characteristics of the recovery process in analogy to the moments of random variables. In analogy to the mean and standard deviation of a random variables, Sharma et al. (2018) defined the *center of resilience* ρ and the *resilience bandwidth*, χ as two fundamental partial descriptions. The definition of these metrics is general and can be systematically extended to higher order metrics to fully characterize any $Q(t)$. The metric ρ defines where the recovery curve is centered with respect to the time of the initial shock. In addition, Sharma et al. (2018) also introduced the resilience quantile, ρ_ω, which is the time instant corresponding to the ω^{th} $(0 \le \omega \le 1)$ quantile of the CRF. Mathematically, the recovery quantile can be written as $\rho_\omega := \min\{t \in [0, T_R] : \omega \le [Q(t)/Q(T_R)]\}$, where T_R is the recovery time (i.e., the time needed to reach a desired final level of $Q(t)$.) The metric χ gives the breath of the recovery process, small values represent a situation in which a significant percentage of the recovery process is completed over a short period concentrated around ρ. By contrast, a large value of χ captures a recovery process spread over a prolonged period of time. To further characterize the recovery curve, Sharma et al. (2018) also introduced the skewness of the recovery, ψ. If $\psi = 0$ the recovery progress is symmetric about ρ (i.e., the recovery process has the same pace before and after ρ.) If $\psi < 0$, the process is slower during the initial phases (i.e., in the interval $[0, \rho]$) and then it becomes faster over the next period $(\rho, T_R]$, which is the most typical case for recovery processes that include a lengthy planning phase in the post-disruption period. If planning is done ahead of the disruptive event as a pre-disruption planning and preparation, then $\psi > 0$. In this case, the recovery progress picks up quickly and the relative most time-consuming portion is the completing of the repairs/reconstruction (i.e., faster in the interval $[0, \rho]$, and slower in the interval, $(\rho, T_R]$). Finally, to uniquely and fully characterize the recovery curve, Sharma et al. (2018) also introduced higher order partial descriptors (in analogy with higher order moments or a random variable). However, in most cases, ρ and χ are sufficient to characterize a recovery process. Based on Sharma et al. (2018), we can write the center of resilience as

$$\rho := \frac{\int_0^{T_R} \tau q(\tau)\, d\tau}{\int_0^{T_R} q(\tau)\, d\tau} \tag{1}$$

Likewise, we can write the resilience bandwidth as

$$\chi^2 := \frac{\int_0^{T_R} (\tau - \rho)^2 q(\tau)\, d\tau}{\int_0^{T_R} q(\tau)\, d\tau} \tag{2}$$

Finally, as a generalization, the n^{th} recovery moment can be written as

$$\rho^{(n)} := \frac{\int_0^{T_R} \tau^n q(\tau)\, d\tau}{\int_0^{T_R} q(\tau)\, d\tau} \tag{3}$$

Proposed Probabilistic Formulation

This section explains the proposed probabilistic formulation to develop recovery curves accounting for the relevant uncertainties and estimate the probability of reaching or exceeding a target level of functionality at any time.

Work progress for civil structures and infrastructure typically advances continuously, or near-continuously, over time (Klinger and Susong 2006; Gardoni et al. 2007), whereas, the system state changes only at completion of a group of activities (Sharma et al. 2018). As a result, the functionality of a system typically changes in a step-wise fashion with discrete increments at the completion of each group of activities. Besides civil structures and infrastructure, or more generally, engineering systems, the recovery might be a continuous function of time when we deal with the restoration of natural systems, such as the recovery and resilience of tropical forests (Cole et al. 2014;. van Leeuwen 2008), or the Gross Domestic Product (GDP) as a monetary measure of the market value of all final goods and services produced in a period to quantify the economic performance of a whole country or region. The proposed methodology is general and allows to estimate processes described either by discrete or continuous recovery curves. The proposed formulation has the following four steps: Step 1: Obtaining the joint PDF of the Sharma et al.'s resilience metrics, Step 2: Obtaining the joint PDF of the model parameters of the recovery curve, Step 3: Obtaining point and predictive estimates of the recovery curve and confidence bounds, Step 4: Estimating the probability of reaching or exceeding a target percentile of interest of the ultimate desired state, and Step 5: Updating the model parameters as new data become available.

Obtaining the joint PDF of the resilience metrics

The first step of the proposed formulation consists in collecting historical recovery data for the system of interest and with them obtaining estimates of the statistics (means, standard deviations and correlation coefficients) and marginal PDFs of Sharma et al.'s resilience metrics (reviewed in Section 2). Based on the obtained statistics and marginal PDFs, we can then construct the joint PDF of the resilience metrics using a Nataf formulation (Liu and Der Kiureghian 1986). Let $f_{\mathrm{P}}(\rho)$, $f_{\mathrm{X}}(\chi)$, up to $f_{\mathrm{P}^{(n)}}(\rho^{(n)})$ be the marginal PDFs of Sharma et al.'s resilience metrics, and let r_{ij} be the estimated correlation coefficients between the i^{th} and the j^{th} resilience metric. Following the Nataf formulation, the joint PDF of the resilience metrics is

$$f_{\mathrm{P}}\left(\boldsymbol{\rho}\right)=f_{\mathrm{P}}\left(\rho\right)f_{\mathrm{X}}\left(\chi\right)\ldots f_{\mathrm{P}^{(n)}}\left(\rho^{(n)}\right)\frac{\varphi_n\left(\mathbf{z},\mathbf{R}'\right)}{\varphi\left(z_1\right)\varphi\left(z_2\right)\ldots\varphi\left(z_n\right)} \tag{4}$$

where $z_i=\Phi^{-1}[F_{\mathrm{P}_i}(\rho_i)]$, $\varphi(\cdot)$ is the standard normal PDF, $\varphi_n(\mathbf{z},\mathbf{R}')$ is the n-dimensional standard normal PDF with correlation matrix $\mathbf{R}'$. The elements r_{ij}' in the correlation matrix $\mathbf{R}'$ are obtained based on the correlation coefficients r_{ij} through the integral

$$\begin{aligned} r_{ij} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{\rho^{(i)}-\mu_i}{\sigma_i}\right)\left(\frac{\rho^{(j)}-\mu_j}{\sigma_j}\right)f_{\rho^{(i)}}\left(\rho^{(i)}\right)f_{\rho^{(j)}}\left(\rho^{(j)}\right) \\ &\quad\times\frac{\varphi_2\left(z_i,z_j,\rho_{ij}'\right)}{\varphi\left(z_i\right)\varphi\left(z_j\right)}d\rho^{(i)}d\rho^{(j)} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{\rho^{(i)}-\mu_i}{\sigma_i}\right)\left(\frac{\rho^{(j)}-\mu_j}{\sigma_j}\right)\varphi_2\left(z_i,z_j,r_{ij}'\right)dz_i dz_j \end{aligned} \tag{5}$$

In case historical recovery data for the system of interest are not available, one can choose a distribution that either reflect some degree of judgement and experience, or a distribution with minimal information (i.e., a noninformative distribution as usually done in Bayesian inference), to reflect the fact that little or no information is available a priori. In addition, the Bayesian inference discussed later in Section 3.5, can be used to update the state of knowledge every time new knowledge becomes available (i.e., recovery data are collected as the recovery unfolds) (Box and Tiao 1992)

Obtaining the joint PDF of the model parameters of the recovery curve

The second step consists in introducing parametrized recovery curves to describe the recovery process over time. In general, the functional form of the selected parametrized recovery curve may affect the time-varying recovery process of a general performance measure. However, one can choose the parametrized recovery curve based on engineering judgement and experience of the problem. In addition, one can use flexible functional forms, such that the recovery curve can be updated as the actual recovery progresses and data become available. Example of parametrized recovery curves can be found in Gardoni et al. (2007) and Ayyub (2015). A parametrized CRF describes the time-varying recovery process of a general performance measure in the following form:

$$\mathrm{T}\left[Q(\boldsymbol{\Theta},\tau)\right]=Q(\boldsymbol{\theta},\tau)+\sigma\varepsilon \tag{6}$$

where, $\mathrm{T}[\cdot]$ is a transformation function, $\boldsymbol{\Theta}=(\boldsymbol{\theta},\sigma)$; $\boldsymbol{\theta}=(\theta_1,\theta_2,...)$ is a vector of unknown model parameters associated with Q, that needs to be estimated; and $\sigma\varepsilon$ is an additive model error term of Q (additivity assumption), in which σ is the standard deviation of the model error, assumed not to depend on τ (homoskedasticity assumption), and ε is a standard normal random variable (normality assumption). The additivity, normality and homoskedasticity assumptions typically can be satisfied using an appropriate variance stabilizing transformation from the parametrized family of transformations introduced by Box and Cox (1964). We then define the joint PDF of the unknown model parameters $\boldsymbol{\Theta}$ based on the joint PDF of the resilience metrics (Hogg et al. 2012; Ang and Tang 2006). Let the set $(\rho,\chi,...,\rho^{(n)})$ have a jointly continuous distribution with PDF $f_{\mathbf{P}}(\rho,\chi,...,\rho^{(n)})$ on a defined support set C. According to the definition of the resilience metrics, the resilience metrics are a function of the model parameters $\boldsymbol{\theta}=(\theta_1,...,\theta_n)$ in the support set D, such that $\rho=k_1(\theta_1,...,\theta_n)$, $\chi=k_2(\theta_1,...,\theta_n)$, up to $\rho^{(n)}=k_n(\theta_1,...,\theta_n)$, where the generic i^{th} function $k_i(\theta_1,...,\theta_n)$ represents the expression of the i^{th} resilience metric $\rho^{(i)}$ based on Equations (1)-(3), after introducing a parametrized recovery curve according to Equation (6). We first evaluate the $n\times n$ Jacobian given by

$$\mathrm{J}=\begin{bmatrix} \dfrac{\partial\rho}{\partial\theta_1} & \dfrac{\partial\rho}{\partial\theta_2} & \cdots & \dfrac{\partial\rho}{\partial\theta_n} \\ \dfrac{\partial\chi}{\partial\theta_1} & \dfrac{\partial\chi}{\partial\theta_2} & \cdots & \dfrac{\partial\chi}{\partial\theta_n} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial\rho^{(n)}}{\partial\theta_1} & \dfrac{\partial\rho^{(n)}}{\partial\theta_2} & \cdots & \dfrac{\partial\rho^{(n)}}{\partial\theta_n} \end{bmatrix} \tag{7}$$

Then, let us consider two subsets of the supports, respectively named A and B, where B denotes the mapping of A under a one-to-one transformation. Due to the conservation of the probability the event $\{(\rho,\chi,...,\rho^{(n)})\in A\}$ is equivalent to the event $\{(\theta_1,\theta_2,...,\theta_n)\in \mathrm{B}\}$. Therefore, we can write

$$\mathbb{P}\left[\left(\theta_1,\theta_2,...,\theta_n\right)\in \mathrm{B}\right]=\mathbb{P}\left[\left(\rho,\chi,...,\rho^{(n)}\right)\in \mathrm{A}\right]=\int\cdots\int_A f_{\mathbf{P}}\left(\rho,\chi,...,\rho^{(n)}\right)d\rho\, d\chi...d\rho^{(n)} \tag{8}$$

We change variables of integration by writing $\theta_1=h_1(\rho,\chi,...,\rho^{(n)})$, $\theta_2=h_2(\rho,\chi,...,\rho^{(n)})$, up to $\theta_n=h_n(\rho,\chi,...,\rho^{(n)})$ such that

$$\begin{aligned}&\int\cdots\int_A f_{\mathbf{P}}\left(\rho,\chi,...,\rho^{(n)}\right)d\rho\, d\chi...d\rho^{(n)}=\\&\int\cdots\int_B f_{\boldsymbol{\theta}}\left[k_1(\theta_1,\theta_2,...,\theta_n),k_2(\theta_1,\theta_2,...,\theta_n),...,k_n(\theta_1,\theta_2,...,\theta_n)\right]\left|J\right|d\theta_1 d\theta_2...d\theta_n\end{aligned} \tag{9}$$

Therefore, for every set $\mathrm{B}\subseteq \mathrm{D}$, we can write

$$\begin{aligned}&\mathbb{P}\left[\left(\theta_1,\theta_2,...,\theta_n\right)\in \mathrm{B}\right]=\\&\int\cdots\int_B f_{\boldsymbol{\theta}}\left[k_1(\theta_1,\theta_2,...,\theta_n),k_2(\theta_1,\theta_2,...,\theta_n),...,k_n(\theta_1,\theta_2,...,\theta_n)\right]\left|J\right|d\theta_1 d\theta_2...d\theta_n\end{aligned} \tag{10}$$

We conclude that the joint PDF of interest $f_{\boldsymbol{\theta}}(\theta_1,\theta_2,...,\theta_n)$ is

$$f_{\boldsymbol{\theta}}(\theta_1,\theta_2,...,\theta_n)=\begin{cases} f_{\boldsymbol{\theta}}\left[k_1(\theta_1,\theta_2,...,\theta_n),...,k_n(\theta_1,\theta_2,...,\theta_n)\right]\left|J\right| & \left(\theta_1,\theta_2,...,\theta_n\right)\in \mathrm{D} \\ 0 & \text{elsewhere}\end{cases} \tag{11}$$

Eq. (11) represents the state of knowledge on the model parameters $\boldsymbol{\theta} = \left(\theta_1, ..., \theta_n\right)$. We can now derive the expected recovery curve and the related uncertainties based on the distribution of the parameters in Eq. (6).

Obtaining point and predictive estimates of the recovery curve and confidence bounds

Different estimates of the recovery curves can be obtained depending on how we treat the model parameters. Following Gardoni et al. (2002), we can obtain point estimates or predictive estimates. A point estimate of the recovery curve is obtained using a point estimate of $\hat{\boldsymbol{\Theta}}$, in place of $\boldsymbol{\Theta}$. In general, the mean value of $\boldsymbol{\Theta}$ or the maximum likelihood estimate (MLE) $\boldsymbol{\Theta}_{\text{MLE}}$ can be used. However, the point estimate does not incorporate the epistemic (statistical) uncertainties in the model parameters $\boldsymbol{\Theta}$. To incorporate these uncertainties, we need to consider $\boldsymbol{\Theta}$ as random variable. The predictive estimate of the recovery curve is then the expected value of the recovery curve over the space of the model parameters, i.e.,

$$\tilde{Q}(\tau) = \int Q(\boldsymbol{\Theta}, \tau) f(\boldsymbol{\Theta}) d\boldsymbol{\Theta} \tag{12}$$

This estimate incorporates the epistemic uncertainties in the model parameters $\boldsymbol{\Theta}$. In addition, we can construct probability bounds on the recovery curve using the PDF of the model parameters, as illustratively shown in Figure 1.

Estimating the probability of reaching or exceeding a target percentile of interest of the ultimate desired state

Once we obtained $Q(\boldsymbol{\Theta}, \tau)$, we can estimate the probability of reaching or exceeding a target value of Q by reliability analysis (Ditlevsen and Madsen 1996; Gardoni 2017). We can write a limit-state function $g(\boldsymbol{\Theta}, \tau)$ as

$$g(\mathbf{\Theta},\tau)=Q(\mathbf{\Theta},\tau)-Q_T \tag{13}$$

where Q_T is a level of performance we desire to reach or exceed, expressed as a percentile of the ultimate desired state, Q_∞. Mathematically, we can write the probability that the recovery process is above Q_T at a time τ, $H(\mathbf{\Theta},\tau)$, as

$$H(\mathbf{\Theta},\tau)=1-\mathbb{P}\left[g(\mathbf{\Theta},\tau)\leq 0 \middle| \tau\right] \tag{14}$$

Figure 2 shows a conceptual representation of $Q(\mathbf{\Theta},\tau)$ and the corresponding $H(\mathbf{\Theta},\tau)$ over time. Following Gardoni et al. (2002), we can construct a point estimate of $H(\tau)$, a predictive estimate as well as confidence bounds as previously proposed for the recovery curve. Hence, we define the point estimate of the probability that the recovery process is above Q_T at a time τ using a point estimate of $\hat{\mathbf{\Theta}}$, in place of $\mathbf{\Theta}$, whereas the predictive estimate $\tilde{H}(\tau)$ is defined taking the expected value of the quantity of interest over the space of the model parameters, in the same way as previously showed for the recovery curve. Furthermore, we obtain confidence bounds on the estimate in Eq. (14). We can define the reliability index as

$$\beta(\mathbf{\Theta},\tau)=\Phi^{-1}\left[H(\mathbf{\Theta},\tau)\right] \tag{15}$$

where $\Phi^{-1}(\cdot)$ indicates the inverse of the standard normal CDF. Following Gardoni et al. (2002), the variance of $\beta(\mathbf{\Theta},\tau)$ can be estimates as

$$\sigma_\beta^2(\tau)\approx\nabla_{\mathbf{\Theta}}\beta(\tau)\mathbf{\Sigma}_{\mathbf{\Theta\Theta}}\nabla_{\mathbf{\Theta}}\beta(\tau)^T \tag{16}$$

where $\nabla_{\boldsymbol{\Theta}}\beta(\tau)$ is the gradient of $\beta(\boldsymbol{\Theta},\tau)$ evaluated at the mean value and $\boldsymbol{\Sigma}_{\boldsymbol{\Theta\Theta}}$ is the estimated covariance matrix. The gradient vector $\nabla_{\boldsymbol{\Theta}}\beta(\tau)$ is obtained by performing a FORM (First-Order Reliability Method) analysis (Ditlevsen and Madsen 1996). Therefore, we obtain

$$\left\{\Phi\left[-\tilde{\beta}(\tau)-\sigma_{\beta}(\tau)\right],\Phi\left[-\tilde{\beta}(\tau)+\sigma_{\beta}(\tau)\right]\right\} \tag{17}$$

as one standard deviation bounds, where $\tilde{\beta}(\tau)=\Phi^{-1}[\tilde{H}(\tau)]$. The bounds represent approximately 15% and 85% probability levels.

Updating the model parameters as new data become available

Finally, Bayesian inference can be used to update the model parameters $\boldsymbol{\Theta}$ combining existing information with new information as it might become available during the actual recovery process (Gardoni et al. 2007). Steps 3 and 4 can then be repeated to obtain updated recovery curves and updated probabilities of reaching or exceeding a desired level Q_T. Mathematically, we can write the posterior distribution $f''(\boldsymbol{\Theta})$ that includes the updated status of knowledge about $\boldsymbol{\Theta}$ as (Box and Tiao 1992)

$$f''(\boldsymbol{\Theta}\,|\,\mathbf{Q})=\kappa L(\boldsymbol{\Theta}\,|\,\mathbf{Q})f'(\boldsymbol{\Theta}) \tag{18}$$

where $L(\boldsymbol{\Theta}\,|\,\mathbf{Q})$ is the likelihood function that contains the objective information on $\boldsymbol{\Theta}$ in a set of observations, $f'(\boldsymbol{\Theta})$ is the prior distribution, reflecting the state of knowledge about $\boldsymbol{\Theta}$ prior to obtaining the observations $\mathbf{Q}=(Q_1,\ldots,Q_m)$, and $\kappa=\left[\int L(\boldsymbol{\Theta}\,|\,\mathbf{Q})f(\boldsymbol{\Theta})d\boldsymbol{\Theta}\right]^{-1}$ is a normalizing factor.

The prior distribution includes the status of knowledge based on previous experiences, engineering judgments, and/or past data. The likelihood function is proportional to the conditional probability of observing the recorded data $\mathbf{Q}=(Q_1,\ldots,Q_m)$ for given values of the parameters $\boldsymbol{\Theta}$. In general, the likelihood function permits to include lower, upper and equality data (see Gardoni et al. 2002). A lower bound datum is defined as an observation of Q that is larger than a certain value Q_i at time τ; an upper bound datum is defined as an observation that is smaller than a certain value Q_i at time τ; an equality datum is defined as the value of Q recorded at time τ. Following Gardoni et al. (2002), the likelihood function can be written as

$$\begin{aligned} L(\boldsymbol{\theta},\sigma) \propto & \prod_{\substack{\text{equality}\\\text{data}}} \mathbb{P}[\sigma\varepsilon_i = \mathrm{Q}_i - \mathrm{Q}(\boldsymbol{\theta},\tau)] \\ & \times \prod_{\substack{\text{lower bound}\\\text{data}}} \mathbb{P}[\sigma\varepsilon_i > \mathrm{Q}_i - \mathrm{Q}(\boldsymbol{\theta},\tau)] \\ & \times \prod_{\substack{\text{upper bound}\\\text{data}}} \mathbb{P}[\sigma\varepsilon_i < \mathrm{Q}_i - \mathrm{Q}(\boldsymbol{\theta},\tau)] \end{aligned} \tag{19}$$

Based on the normality assumption, we can then write

$$\begin{aligned} \mathrm{L}(\boldsymbol{\theta},\sigma) \propto & \prod_{\substack{\text{equality}\\\text{data}}} \left\{ \frac{1}{\sigma}\varphi\left[\frac{Q_i - Q(\boldsymbol{\theta},\tau)}{\sigma}\right]\right\} \\ & \times \prod_{\substack{\text{lower bound}\\\text{data}}} \left\{ \Phi\left[-\frac{Q_i - Q(\boldsymbol{\theta},\tau)}{\sigma}\right]\right\} \\ & \times \prod_{\substack{\text{upper bound}\\\text{data}}} \left\{ \Phi\left[\frac{Q_i - Q(\boldsymbol{\theta},\tau)}{\sigma}\right]\right\} \end{aligned} \tag{20}$$

where $\varphi(\cdot)$ is the standard normal PDF, and $\Phi(\cdot)$ is the standard normal CDF.

Eqs. (15)-(20) can be used every time additional information is available to update the model parameters. For instance, when a set of samples $\mathbf{Q}_1$ is available, we can write

$$f''(\boldsymbol{\Theta}\,|\,\mathbf{Q}_1) \propto L(\boldsymbol{\Theta}\,|\,\mathbf{Q}_1)\,f'(\boldsymbol{\Theta}) \tag{21}$$

Then, let us suppose another set of samples $\mathbf{Q}_2$ is available and this is independent from the previous one, we can update the posterior PDF evaluated in Eq. (21) such that

$$f'''(\boldsymbol{\Theta} \mid \mathbf{Q}_1, \mathbf{Q}_2) \propto L(\boldsymbol{\Theta} \mid \mathbf{Q}_1) L(\boldsymbol{\Theta} \mid \mathbf{Q}_2) f'(\boldsymbol{\Theta}) \propto L(\boldsymbol{\Theta} | \mathbf{Q}_2) f''(\boldsymbol{\Theta} | \mathbf{Q}_1) \tag{22}$$

Generally, if n independent set of observations are available, we can write

$$\begin{aligned} & f^{(k+1)}(\boldsymbol{\Theta} \mid \mathbf{Q}_1, \ldots, \mathbf{Q}_k) \propto L(\boldsymbol{\Theta} \mid \mathbf{Q}_k) f^{(k)}(\boldsymbol{\Theta} | \mathbf{Q}_1, \ldots, \mathbf{Q}_{\text{k-1}}) \\ & k = 2, \ldots, n \end{aligned} \tag{23}$$

Field measurements can often be inexact and include measurement errors (Gardoni et al. 2002; Murphy et al. 2011). Following Gardoni et al. (2002), measurement errors can be incorporated in the updating process. To incorporate the measurements errors in the updating process, we assume that $Q_i = \hat{Q}_i + e_{Q_i}$ is the true value of the i^{th} observation, where $\hat{Q}_i$ represents the measured value and e_{Q_i} is the measurement error. We also assume that e_{Q_i} has zero mean, which reflects that the measurements have been corrected from any systematic errors, and variance s_i^2, which represents the uncertainties inherent in the measurements. For the equality data we have $\hat{Q}_i + e_{Q_i} = Q(\boldsymbol{\theta}, \tau) + \sigma\varepsilon_i$, for the lower bound data we have $\hat{Q}_i + e_{Q_i} < Q(\boldsymbol{\theta}, \tau) + \sigma\varepsilon_i$, and for the upper bound data we have $\hat{Q}_i + e_{Q_i} > Q(\boldsymbol{\theta}, \tau) + \sigma\varepsilon_i$. Therefore, the conditions for the three type of data can be, respectively, written as $\sigma\varepsilon_i - e_{Q_i} = \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$, $\sigma\varepsilon_i - e_{Q_i} > \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$, and $\sigma\varepsilon_i - e_{Q_i} < \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$. The left-hand sides of these expressions are a normal random variable with zero mean and variance $\hat{\sigma}(\boldsymbol{\theta}, \sigma) = \sigma^2 + s_i^2$. Hence, in presence of measurement errors, the likelihood function is

$$
\begin{aligned}
\mathrm{L}(\boldsymbol{\theta},\sigma) \propto & \prod_{\substack{\text{equality}\\ \text{data}}} \left\{ \frac{1}{\hat{\sigma}(\boldsymbol{\theta},\sigma)} \varphi\left[\frac{\hat{Q}_i - Q(\boldsymbol{\theta},\tau)}{\hat{\sigma}(\boldsymbol{\theta},\sigma)} \right] \right\} \\
& \times \prod_{\substack{\text{lower bound}\\ \text{data}}} \left\{ \Phi\left[-\frac{\hat{Q}_i - Q(\boldsymbol{\theta},\tau)}{\hat{\sigma}(\boldsymbol{\theta},\sigma)} \right] \right\} \\
& \times \prod_{\substack{\text{upper bound}\\ \text{data}}} \left\{ \Phi\left[\frac{\hat{Q}_i - Q(\boldsymbol{\theta},\tau)}{\hat{\sigma}(\boldsymbol{\theta},\sigma)} \right] \right\}
\end{aligned}
\tag{24}
$$

Example 1: Recovery curves for an example bridge

This section presents the proposed formulation considering the recovery process of a typical RC bridge subject to seismic excitations. The first example demonstrates the application of the formulation in a realistic case related to civil structures in support of risk and resilience analysis.

We previously discussed that for civil structures the work progress is a continuous, or near-continuous, function, whereas a discrete function describes the performance indicators (e.g., functionality) with jumps when a group of activities is completed. This section illustrates the proposed formulation applied to a RC bridge. Figure 3 shows the configuration of the considered (single column, single bent) testbed bridge from Kumar and Gardoni (2014a) and Jia et al. (2017). Following the proposed formulation, we obtain the estimates of the first two resilience metrics to describe the recovery process of the selected engineering system. Figure 4 shows the pair (ρ, χ) used in this example, and their correlation. Based on the data in Figure 4, we assume that both ρ and χ follow a lognormal distribution, whose parameters are listed in Table 1. Then, based on the estimated coefficient of correlation and the marginal PDFs we can construct the joint PDF of the resilience metrics as described in Section 3.1. Next, we introduce a parametrized recovery curve to describe the changes of a selected performance measure over time. The performance indicator considered in this example is the reliability index β. Moreover, in this example we assume that there is only one recovery step that restores the reliability of the bridge, as described in Sharma et al. (2018). Consequently, we consider the recovery curve in the following form:

$$Q_1(\mathbf{\Theta},\tau)=\begin{cases}\theta_1 & \tau<\theta_2\\ Q_\infty & \tau\geq\theta_2\end{cases} \tag{25}$$

where θ_1 is the residual reliability index after the occurrence of the hazard, and before the completion of the recovery; and θ_2 is the time at which the reliability index reaches the ultimate desired value Q_∞. To model the reliability, we consider the reliability-based resilience metrics coming from previous analyses. Thus, we do not need to model the occurrence of earthquake mainshock-aftershocks sequence and their impact on structural properties because the resilience metrics capture all these information. Based on the definition of the resilience metrics in Eqs. (1) and (2), the parameters θ_1 and θ_2 can be written as a function of the resilience metrics. Specifically, Figure 5 shows the RMF of the adopted parametrized curve. Following the proposed methodology, we compute the joint PDF of the model parameters and the corresponding expected recovery process in terms of the reliability index β. We observe that the number of resilience metrics needed to adopt the formulation is at least equal to the number of the model parameters of the selected parametrized recovery curve. Therefore, considering the possibility of having a drop in the functionality, during the recovery process due to aftershocks, would require implying higher resilience metrics. Next, we estimate the probability of reaching or exceeding a target value of functionality at any time setting, for instance, $Q_T=3.5$.

Initial estimate of the recovery curve and corresponding probability of exceeding the target value of functionality

As previously discussed, in this example we assume that there is only one recovery step that restores the reliability of the bridge. Nevertheless, we can also estimate the behavior of the system toward the desired value of the functionality at the end of the recovery in terms of the mean value of the different probable recovery curves. Figure 6 shows the expected changes of the instantaneous reliability index over time. Adopting the reliability-based definitions for the damage state proposed in Sharma et al. (2018), the initial damage level is moderate. Figure 7 shows the probability of exceeding the target value of Q_T. The figure also presents the confidence band due to the statistical uncertainty in $\mathbf{\Theta}$. Based on the expected initial value of the reliability index, we can observe that the probability of exceeding the target value of functionality, $Q_T = 3.5$, at time $\tau = 25$ days, is equal to 0.5. The observed result matches the results provided in Sharma et al. (2018), where the expected value of the time to recover is approximately 26 days when the initial damage level is moderate.

Updated estimate of the recovery curve and corresponding probability of exceeding the target value of functionality

We assume that after the occurrence of the hazard we collect data on the state of damage for the first 10 days, and then we update the model parameters. In the presented example, we assume that inspection data are collected after the occurrence of the hazard. Specifically, we assume that a qualitative description of the damage state indicates moderate damage following the definition in ATC-38 (ATC 2000) and Bai et al. (2009) (i.e., “Repairable structural damage has occurred. The existing elements can be repaired in place, without substantial demolition or replacement of elements”.) Then, the qualitative definition of the damage state is mapped into a reliability-based definition in terms of the corresponding reliability index β (i.e., $1.5 \le \beta \le 2.5$) following in Sharma et al. (2018). As a result, we obtain the new expected changes in the reliability index and the corresponding time-varying probability of exceeding the same target value of functionality, as shown in Figure 8 and 9. Figure 8 shows the expected changes of the reliability index over time, after updating the model parameters based on the observed data. First, we can observe that the recovery process follows the observed data in terms of its mean; then, the

Bayesian inference also reduces the relevant uncertainties. The probability of exceeding the target value of functionality reflects both the effects of the Bayesian inference.

Example 2: Population relocation after a seismic event

The second example shows the application of the formulation in a scenario where historical recovery data are not available. In this example, we consider the population relocation of the city of Seaside, OR, after the occurrence of an earthquake originated from the Cascadia Subduction Zone. We consider a seismic event of magnitude $M_W = 7.0$, located 25 km southwest of the city. Since no data are available, we consider a noninformative PDF of the first resilience metric ρ in the form $f_{\mathrm{P}}(\rho) = 1/\rho,\ \rho > 0$, which reflects the fact that little is known a priori. We consider a parametrized S-shape recovery curve proposed in Gardoni et al. (2007) in the following form:

$$Q_2(\mathbf{\Theta}, \tau) = 1 - Q_R + \left(Q_\infty - Q_R\right)\left(\frac{\tau}{\theta_1}\right)^2 \left[3 - 2\left(\frac{\tau}{\theta_1}\right)\right] \quad \tau \le \theta_1 \tag{26}$$

where Q_R represents the percentage of population dislocation at time t_{0^+} (i.e., after the occurrence of the seismic event), Q_∞ represents the percentage of the population that relocates at the end of the recovery, and θ_1 is the time at which the recover ends.

Ground Motion Prediction Equations (Boore and Atkinson, 2008) are used to obtain maps of the seismic intensity measure at the residential building location. Next, we perform a building damage analysis using different fragility functions (e.g., HAZUS-MH (FEMA 2015), and Steelman et al. 2007). Then, we estimate the initial percentage of population dislocation due to structural damage using a logistic regression model (Lin 2009). For the purpose of this example, we assume that the entire population returns to their homes at the end of the recovery (i.e., $Q_\infty = 100\%$.)

Based on the definition of the resilience metric in Eq. (1), the parameter θ_1 can be written as a function of the resilience metric ρ. Therefore, we obtain the PDF $f_{\theta_1}(\theta_1)$ according to Eq. (11), as well as the corresponding estimate of Q_2 over time (shown in Figure 10(b).) More generally, Q_∞ could also be taken as a parameter (i.e., $Q_\infty = \theta_2$). In this case, we would obtain the joint PDF $f_{\boldsymbol{\theta}}(\theta_1, \theta_2)$ again using Eq. (11) given ρ and χ.

After the occurrence of the seismic event, recovery activities start to retrieve structures and infrastructure functionality, thereby we can observe the population returning to their homes. For the purpose of this example, we assumed that data on the population relocation are available at given time-steps. The relocation data at different times can be used to obtain the corresponding values of Q_2 (shown by dots in Figure 10(b).) Using these values of Q_2, we obtain the new expected value of Q_2 as a function of time, as shown in Figure 10(b). In particular, we can observe that the uncertainties in the initial estimate in Figure 10(a) reflect the fact that little is known in terms of the duration of the recovery. In Figure 10(b) the confidence band is significantly smaller around the mean line indicating that the values of Q_2 used to update the mean prediction also reduce the prediction uncertainty.

Finally, Figure 11 shows the probability that the population dislocation is higher than 25% of the total population (i.e., $Q_T = 0.25$) before and after we update the recovery curve, including the confidence band due to the statistical uncertainty in $\boldsymbol{\Theta}$. We can see that the information used to update the model parameter can also adjust the prediction in terms of the probability of exceeding a target level of functionality, as well as reduce the prediction uncertainty.

Conclusions

The paper proposed a formulation to (i) predict the recovery curves that define the recovery of engineering systems subject to a hazard, and (ii) estimate the probability of reaching or exceeding a target value of a selected performance indicator at any given time. The formulation uses the resilience metrics defined in Sharma et al. (2018), which quantify the resilience of systems and form a complete set of partial descriptors that characterize the recovery curve of the system of interest. To evaluate the recovery process of an engineering system, the paper proposed to use the probability density function (PDF) of the resilience metrics, defined based on historical data, to obtain the PDF of the model parameters that define the recovery curve. The proposed formulation incorporates the Bayesian inference to update the estimates of the unknown parameters when additional information is available. The paper illustrated the implementation of the proposed formulation by predicting the recovery of a single-bent, single-column reinforced concrete (RC) bridge subject to seismic damage, and the population relocation after the occurrence of a seismic event when no data on the duration of the recovery are available a priori. The proposed formulation is general and suited to applications such as risk analysis and mitigation, and resilience-based design.

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References

Boore, D.M., and Atkinson, G.M. (2008). “Ground-motion prediction equations for the average horizontal component of PGA, PGV, and 5%-damped PSA at spectral periods between 0.01 s and 10.0 s.” *Earthquake Spectra* 24(1), 99-138

Ang, A.H-S., and Tang W.H. (2006). *Probability Concepts in Engineering. Emphasis on Applications in Civil & Environmental Engineering*, John Wiley & Sons, Inc.

ATC. (2000). Database on the performance of structures near strong-motion recordings: 1994 Northridge earthquake. Redwood City, CA: Applied Technology Council.

Ayyub, B.M. (2014). “Systems resilience for multihazard environments: Definition, metrics, and valuation for decision making.” *Risk Analysis*, 34(2), 340–355.

Ayyub, B.M. (2015). “Practical resilience metrics for planning, design, and decsion making.” *Journal of Risk Uncertainty in Engineering Systems, Part A: Civil Engineering*, 1(3), 04015008.

Bai, J.-W., Hueste, M.B.D., and Gardoni, P., (2009). “Probabilistic assessment of structural damage due earthquakes for buildings in Mid-America,” ASCE Journal of Structural Engineering, 135 (10), 1155-1163.

Bocchini, P., Decò, A., and Frangopol, D.M. (2012). “Probabilistic functionality recovery model for resilience analysis.” *Bridge Maintenance, Safety, Management, Resilience and Sustainability*, Biodini and Frangopol (Eds), 1920–1927.

Bonstrom, H., and Corotis, R.B. (2016). “First-Order Reliability Approach to Quantify and Improve Building Portfolio Resilience.” *Journal of Structural Engineering*, 142(8), C4014001.

Box, G.E.P., and Tiao, G.C. (1992). *Bayesian inference in statistical analysis*. John Wiley & Sons.

Bruneau, M., Chang, S.E., Eguchi, R.T., Lee, G.C., O’Rourke, T.D., Reinhorn, A.M., Shinozuka, M., Tierney, K., Wallace, W.A., and von Winterfeldt, D. (2003). “A Framework to Quantitatively Assess and Enhance the Seismic Resilience of Communities.” *Earthquake Spectra*, 19(4), 733–752.

Bruneau, M., and Reinhorn, A. (2007). “Exploring the Concept of Seismic Resilience for Acute Care Facilities.” *Earthquake Spectra*, 23(1), 41–62.

Caverzan, A., and Solomos, G. (2014). “Review on resilience in literature and standards codes for critical built-inrastructures.” *JRC Science and Policy Report*.

Chang, S.E., and Shinozuka, M. (2004). “Measuring Improvements in the Disaster Resilience of Communities.” *Earthquake Spectra*, 20(3), 739–755.

Cimellaro, G.P., Reinhorn, A.M., and Bruneau, M. (2010a). “Seismic resilience of a hospital system.” *Structure and Infrastructure Engineering*, 6(1), 127–144.

Cimellaro, G.P., Reinhorn, A.M., and Bruneau, M. (2010b). “Framework for analytical quantification of disaster resilience.” *Engineering Structures*, 32(11), 3639–3649.

Cole, E.S.L., Bhagwat, S.A., and Willis, K.J. (2014). “Recovery and resilience of tropical forests after disturbance.” *Nature Communications*, 5(1), 59–65.

Collins English Dictionary Complete and Unabridged edition 12th edition. (2014)

Corotis, R. (2009). “Societal issues in adopting life-cycle concepts within the political system.” *Structure and Infrastructure Engineering*, 5:3906.

Decò, A., Bocchini, P., and Frangopol, D.M. (2013). “A probabilistic approach for the prediction of seismic resilience of bridges.” *Earthquake Engineering & Structural Dynamics*, 42(10), 1469–1487.

Ditlevsen, O., and Madsen, H.O. (1996). *Structural reliability methods*, Wiley, New York.

Doorn, N., Gardoni, P., Murphy, C. (2018). “A multidisciplinary definition and evaluation of resilience: the role of social justice in defining resilience,” *Sustainable and Resilient Infrastructure,* DOI: 10.1080/23789689.2018.1428162.

Ellingwood, B.R., Cutler, H., Gardoni, P., Peacock, W.G., van de Lindt, J.W., and Wang, N. (2016). “The Centerville Virtual Community: a fully integrated decision model of interacting physical and social infrastructure systems.” *Sustainable and Resilient Infrastructure*, 1(3–4), 95–107.

FEMA (2015). *Hazus 2.1 technical and user’s manuals*, available at https://www.fema.gov/media-library/assets/documents/24609 (last accessed 21 February 2018)

Garbin, D. (2007). “Moving beyond critical infrastructures protection to disaster resilience.” *CIP Program Discussion Paper Series.* George Mason University school of Law.

Gardoni, P., Der Kiureghian, A., and Mosalam, K.M. (2002). “Probabilistic capacity models and Fragility estimates for reinforced concrete columns based on experimental observations.” *Journal of Engineering Mechanics*, 128(10), 1024–1038.

Gardoni, P., Mosalam, K.M., and Der Kiureghian, A. (2003). “Probabilistic seismic demand models and fragility estimates for RC bridges.” *Journal of Earthquake Engineering*, 7(Special Issue 1), 79–106.

Gardoni, P., Reinschmidt, K.F., and Kumar, R. (2007). “A probabilistic framework for Bayesian adaptive forecasting of project progress.” *Computer-Aided Civil and Infrastructure Engineering*, 22(3), 182–196.

Gardoni, P., and LaFave, J.M. (Eds.) (2016). *Multi-hazard Approaches to Civil Infrastructure Engineering*, Springer International.

Gardoni, P., Murphy, C., Rowell, A. (Eds.) (2016). *Societal Risk Management of Natural Hazards*, Springer.

Gardoni, P. (Ed.) (2017). *Risk and Reliability Analysis: Theory and Applications*. Springer International Publishing.

Gardoni, P., (Ed.) (2018). *Handbook of Sustainable and Resilient Infrastructure*, Routledge.

Guidotti, R., Chmielewski, H., Unnikrishnan, V., Gardoni, P., McAllister, T., and van de Lindt, J.W. (2016). “Modeling the resilience of critical infrastructure: the role of network dependencies.” *Sustainable and Resilient Infrastructure*, 1(3–4), 153–168.

Guidotti, R., Gardoni, P., and Chen, Y. (2017). “Network reliability analysis with link and nodal weights and auxiliary nodes.” *Structural Safety*, 65, 12–26.

Hogg, R. V., McKean, J.W., and Craig, A. T. (2012). *Introduction to Mathematical Statistics*. 7th Edition, Prentice Hall College Div.

Jia, G., Tabandeh, A., and Gardoni, P. (2017). “Life-Cycle Analysis of Engineering Systems: Modeling Deterioration, Instantaneous Reliability, and Resilience.” *Risk and Reliability Analysis: Theory and Applications*, P. Gardoni, (Ed.), Springer International Publishing.

Jia, G., and Gardoni, P. (2018a). “State-dependent stochastic models: A general stochastic framework for modeling deteriorating engineering systems considering multiple deterioration processes and their interactions,” *Structural Safety,* 72, 99-110.

Jia, G., and Gardoni, P. (2018b). “Stochastic life-cycle analysis: Renewal-theory life-cycle analysis with state-dependent deterioration stochastic models,” *Structure and Infrastructure Engineering* (under review).

Klinger, M., and Susong, M. (2006). *The construction project : phases, people, terms, paperwork, processes*. American Bar Association.

Koliou, M., van de Lindt, J.W. McAllister, T., Ellingwood, B.R., Dillard, M., and Cutler, H. (2018). “State of the research in community resilience: progress and challenges.” *Sustainable and Resilient Infrastructure,* DOI: 10.1080/23789689.2017.1418547.

Kumar, R., and Gardoni, P. (2014). “Renewal theory-based life-cycle analysis of deteriorating engineering systems.” *Structural Safety*, 50, 94–102.

Kumar, R., Cline, D., and Gardoni, P., (2015). “A stochastic framework to model deterioration in engineering systems.” *Structural Safety*, 53, 36-43.

Lin, Y.S. (2009). Development of algorithms to estimate post-disaster population dislocation - a research-based approach. Ph.D. Thesis, Texas A&M University.

McAllister, T. (2013). *Developing Guidelines and Standards for Disaster Resilience of the Built Environment: A Research Needs Assessment*. Gaithersburg, MD.Murphy, C., Gardoni, P., and Harris, C.E., (2011). "Classification and moral evaluation of uncertainties in engineering modeling," *Science and Engineering Ethics*, 17 (3), 553-570.

PPD-21. (2013, February 12). Presidential policy directive/ PPD-21 – Critical infrastructure security and resilience. Washington, DC: The White House.

Sharma, N., Tabandeh, A., and Gardoni, P. (2018). “Resilience analysis: a mathematical formulation to model resilience of engineering systems.” *Sustainable and Resilient Infrastructure,* 3(2), 49-67.

Steelman, J., Song, J., and Hajjar, J.F. (2007). “Integrated Data Flow And Risk Aggregation For Consequence-Based Risk Management Of Seismic Regional Losses.” MAE Center. Retrieved from http://mae.cee.illinois.edu/publications/reports/Report_Jan_07.pdf

Tierney, K, and Bruneau, M. (2007). “Conceptualizing and measuring resilience.” *TR News 250*.

Titi, A., Biondini, F., and Frangopol, D.M. (2015). “Seismic resilience of deteriorating concrete structures.” *Structures Congress*.

Van Leeuwen, W. J. D. (2008). “ Monitoring the effects of forest restoration treatments on post-fire vegetation recovery with MODIS multitemporal data.” *Sensors,* 8(3), 2017-2042.

Figures

Figure 1. 90% Confidence bounds on the estimate of the recovery curve

Figure 2. Conceptual representation of probability of reaching or exceeding a target value of functionality at any time

Figure 3. The considered RC bridge (adapted from Jia et al. 2017)

Figure 4. Correlation between the adopted resilience metrics

Figure 5. RMF of the adopted parametrized recovery curve

Figure 6. Mean value and corresponding 95% confidence band of the recovery curve

Figure 7. Probability of exceeding the selected value of $Q_T = 3.5$

Figure 8. Mean value and corresponding 95% confidence band of the recovery curve after updating the model parameters

Figure 9. Probability of exceeding the selected value of $Q_T = 3.5$ after updating the model parameters

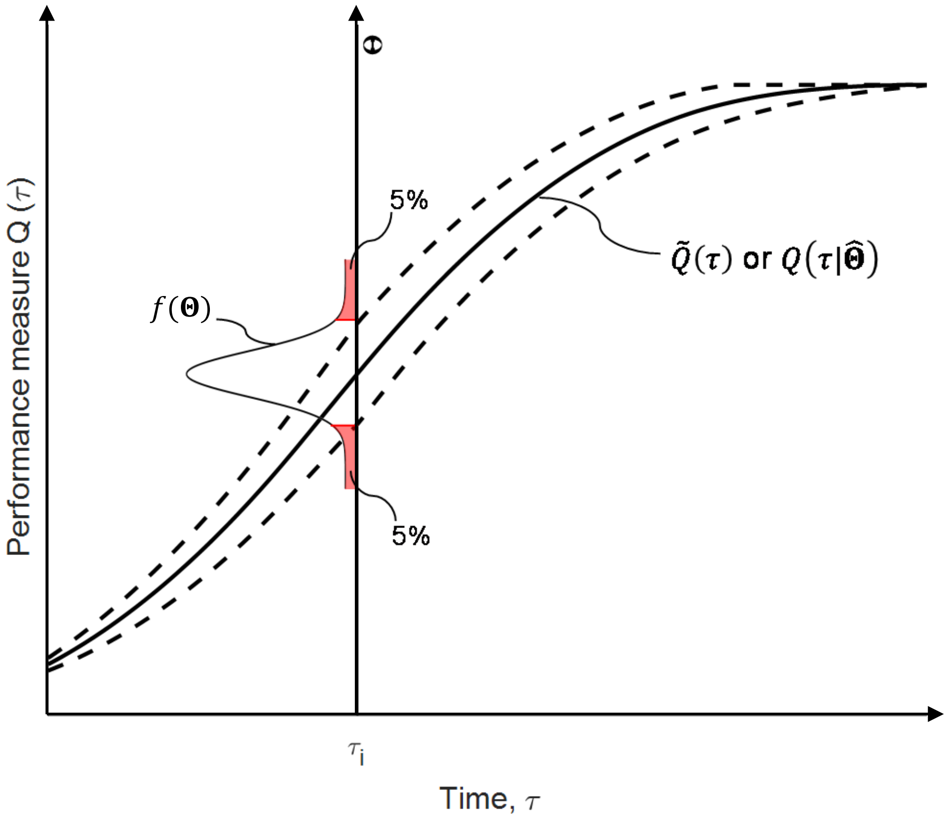
Figure 10. Mean value and corresponding 95% confidence band of the population dislocation recovery curve (a) before and (b) after updating the model parameter

Figure 11. Probability of population dislocation (i.e,, $Q_T = 25\%$) (a) before and (b) after updating the model parameter

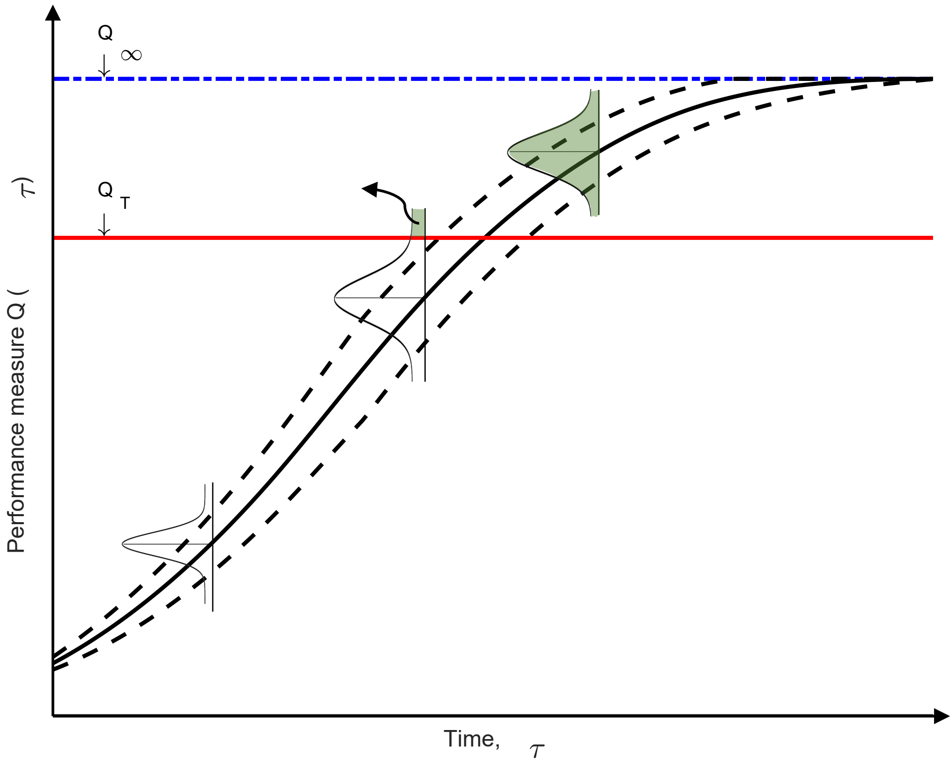
Tables

Table 1. Distribution parameters of the resilience metrics

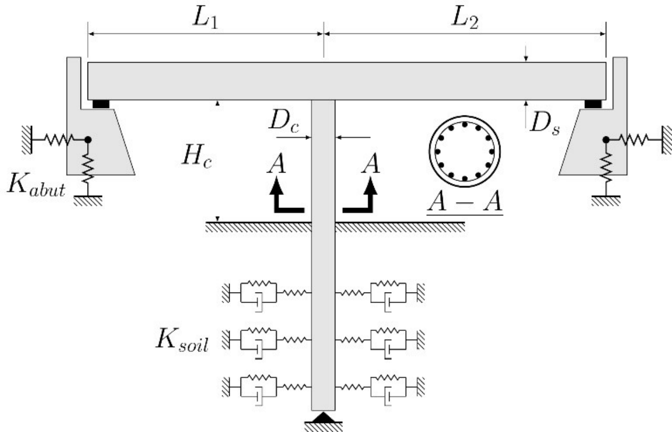
Resilience metric	λ	ξ
ρ	–0.94	0.22
χ	1.14	0.20

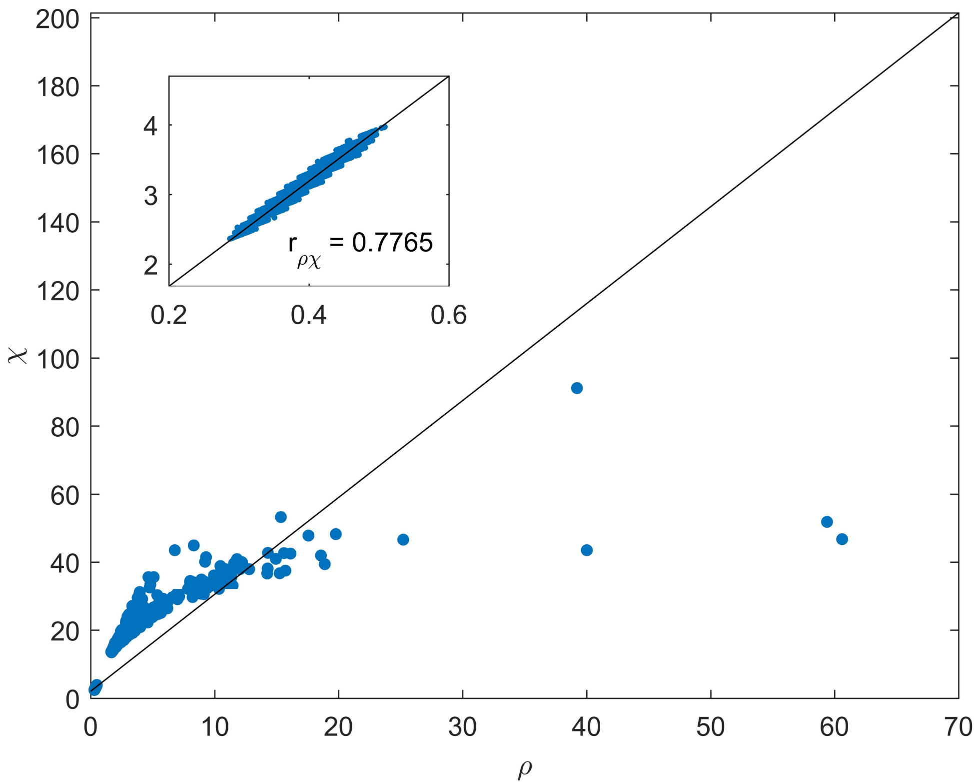


Θ
5%
$\tilde{Q}(\tau)$ or $Q(\tau|\hat{\mathbf{\Theta}})$
$f(\mathbf{\Theta})$
5%
Performance measure Q (τ)
τ_i
Time, τ

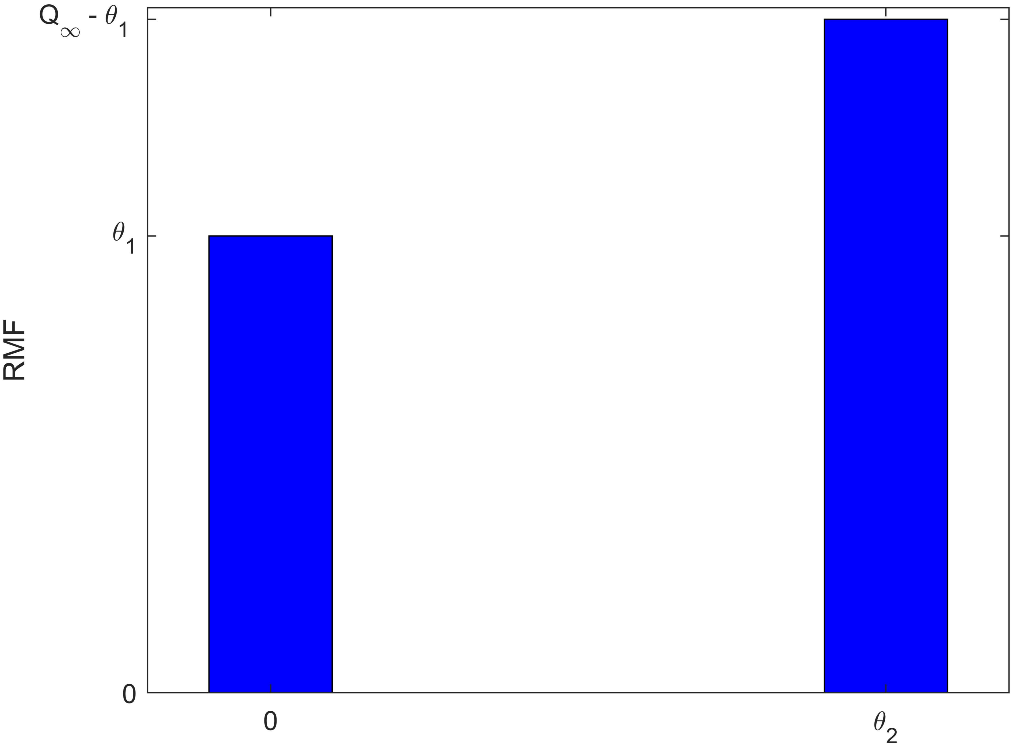


Q ∞
Q T
Performance measure Q (τ)
Time, τ

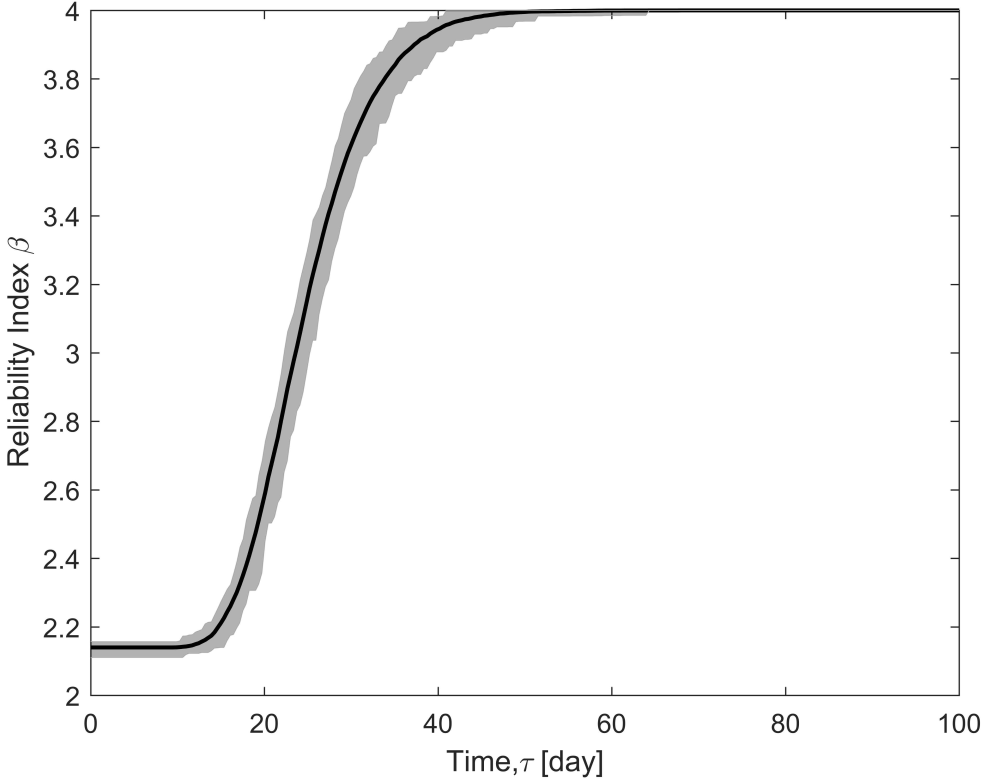

L_1
L_2
D_c
D_s
H_c
A
A
$A - A$
K_{abut}
K_{soil}



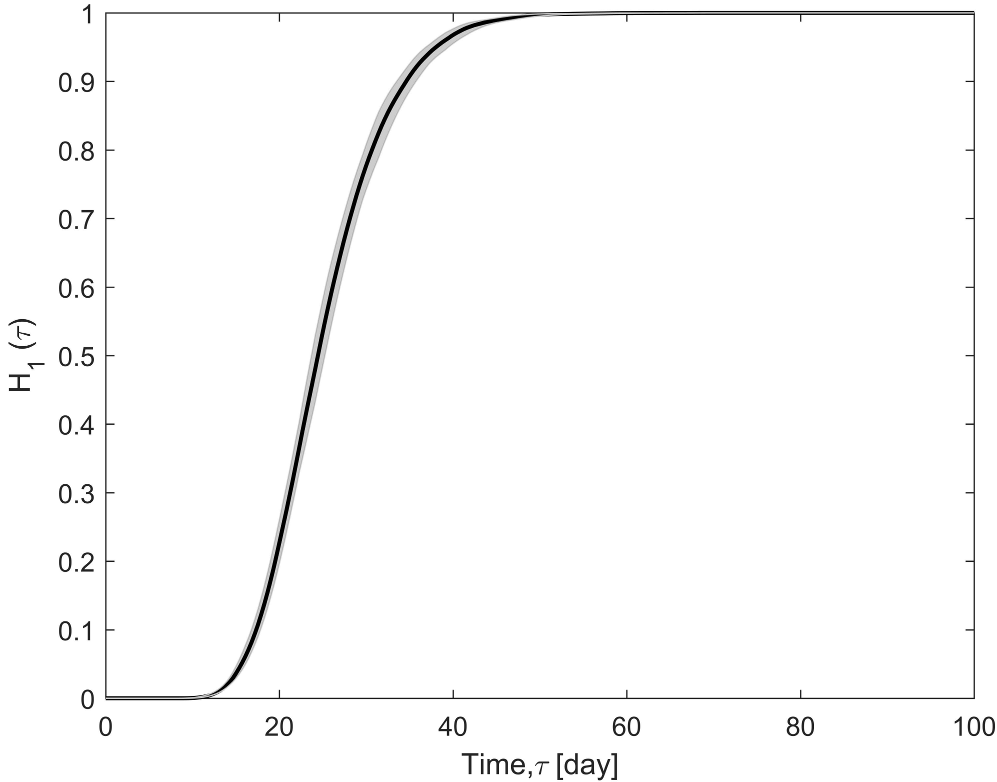
r_{ρχ} = 0.7765
0.2
0.4
0.6
2
3
4
χ
ρ
0
10
20
30
40
50
60
70
0
20
40
60
80
100
120
140
160
180
200



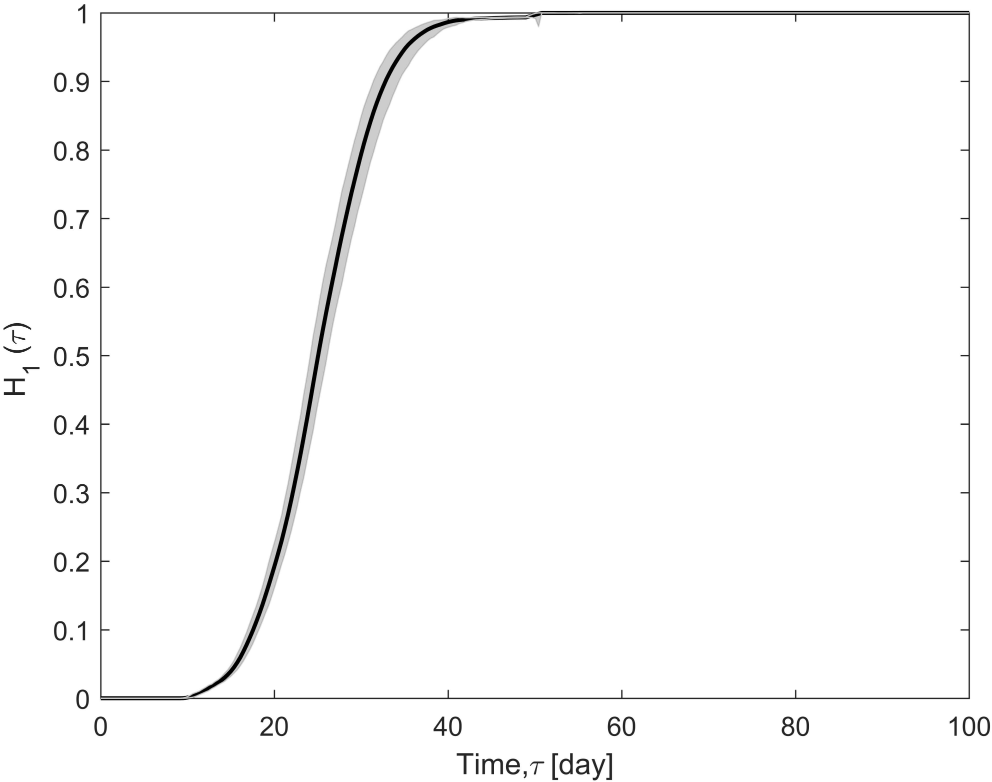
$Q_\infty - \theta_1$
θ_1
0
RMF
0
θ_2



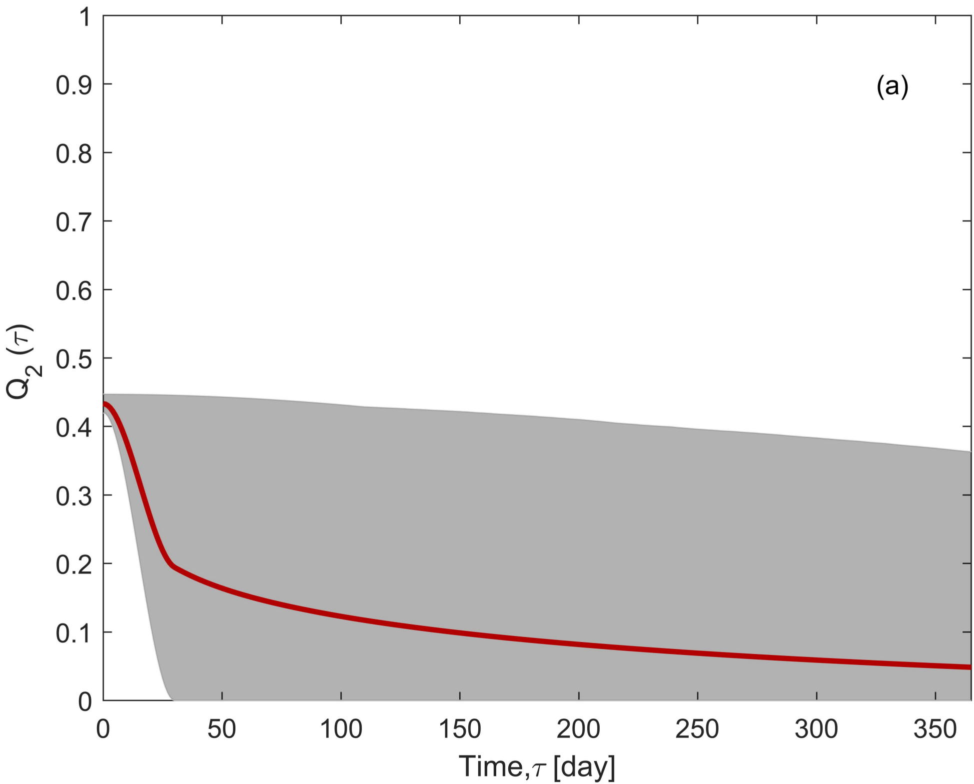
Reliability Index β
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
0
20
40
60
80
100
Time, τ [day]



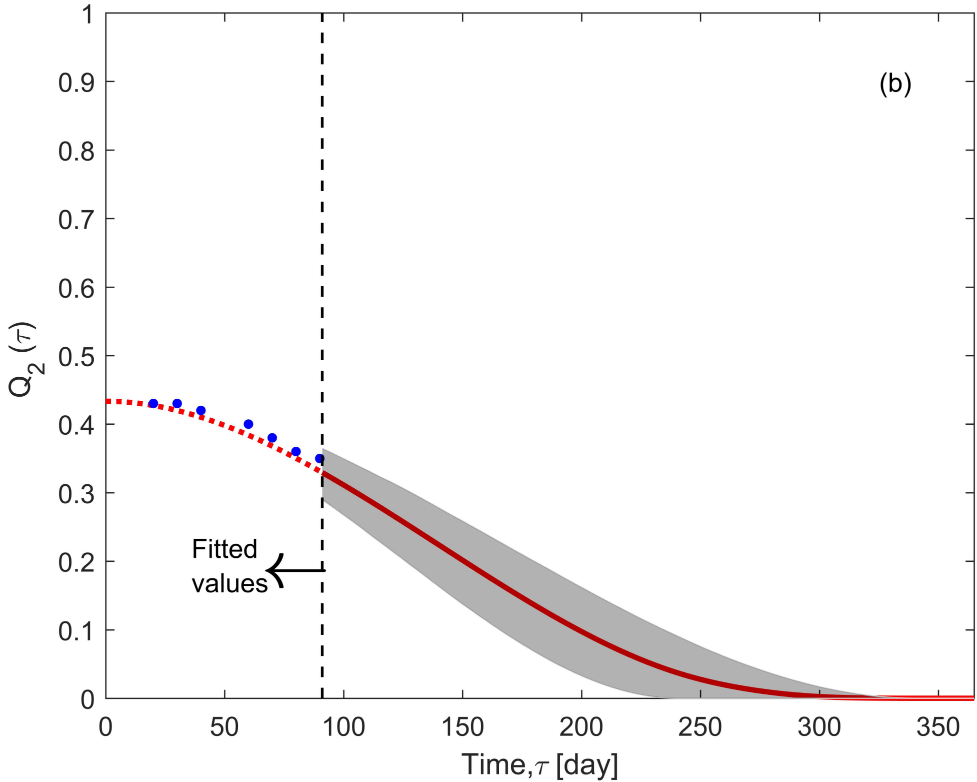
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
$H_1(\tau)$
Time, τ [day]



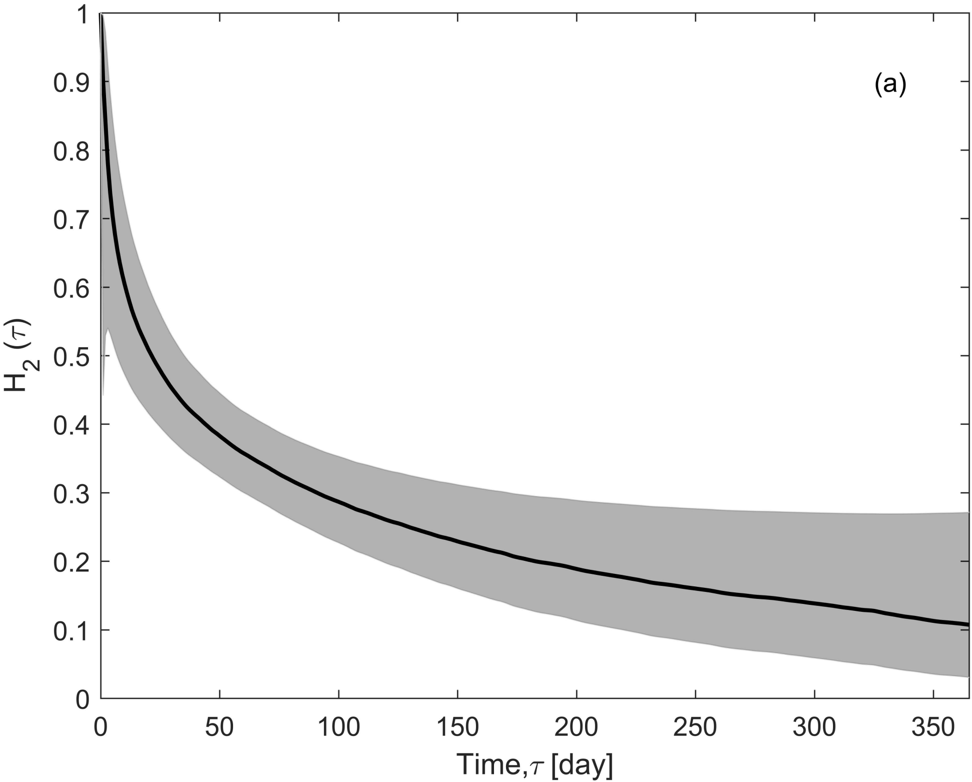
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$H_1(\tau)$
0
20
40
60
80
100
Time, τ [day]



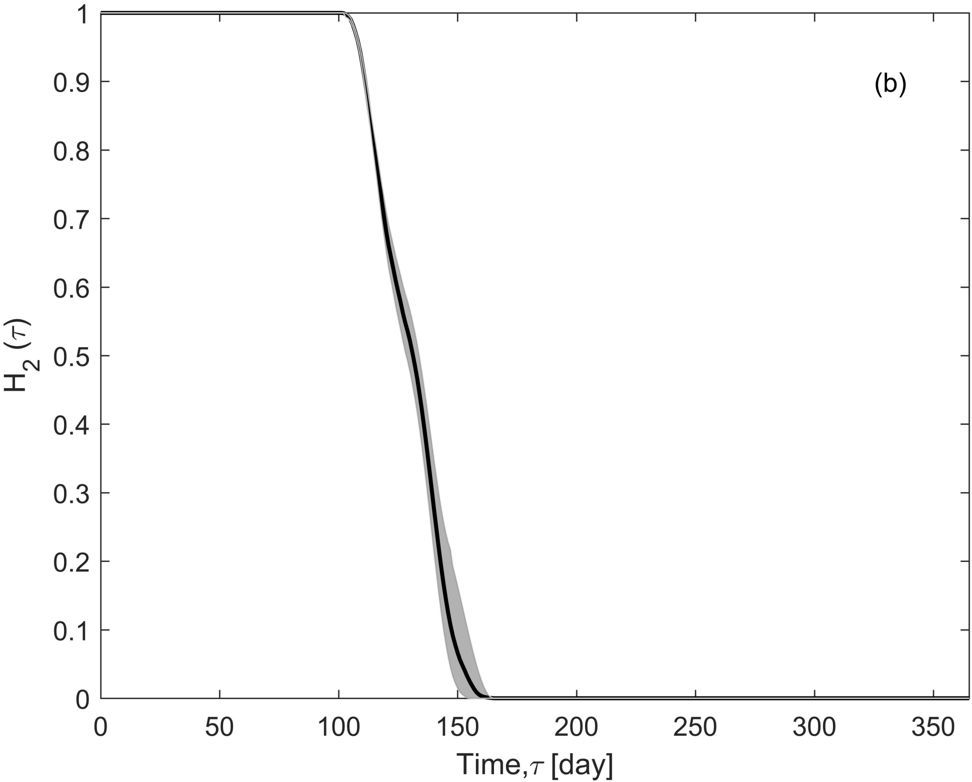
(a)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$Q_2(\tau)$
0
50
100
150
200
250
300
350
Time, τ [day]



(b)
Fitted values
Q_2 (τ)
Time, τ [day]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50
100
150
200
250
300
350



(a)
$H_2(\tau)$
Time, τ [day]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50
100
150
200
250
300
350



(b)
H2 (τ)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
300
350
Time,τ [day]