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A new method for evaluating stony debris flow rainfall thresholds: the Backward Dynamical Approach

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ABSTRACT

Debris flow rainfall thresholds aim to provide a level of rainfall duration and average intensity above which the probability of a debris flow occurrence is significant. Estimating reliable thresholds for use in early warning systems has proved to be a challenging task. In fact the methodologies currently available in the literature are not entirely satisfactory since they provide thresholds unlikely low. The goal of the present research is exploring new paths aimed at improving the reliability of rainfall thresholds. A possible weak point of the literature approaches is the way the duration and the average intensity pertaining to a debris flow is determined. Up to now, these values are evaluated using only the characteristics of the hyetograph associated to a debris-flow event. In the present paper, we propose a new methodology based on volumetric relations deriving from a simplified description of the dynamic of a stony debris flow: by using these relations, from a measure of the deposited volume it is possible to estimate backwardly the volume of rain that caused the deposition; then, from this last value and the knowledge of the relevant hyetograph, it is possible to reconstruct the duration and the average intensity. Application of this new technique to a sample study area allowed us to prove the feasibility of the method and, to some extent, its capabilities: with respect to a classical literature method, the new approach produces an higher threshold and a smaller characteristic duration scales. Finally, strengths and weakness of the method have been evaluated thoroughly.

1. Introduction

In alpine regions, debris flows are rather common and widespread phenomena that produce considerable damages to houses and infrastructures. In the last decades, steps forward have been made in the field of mathematical and numerical modelling of these phenomena by using either a mono-phase or a two-phase approach (see e.g. Iverson, 1997; Takahashi, 1978; Brufau et al., 2000; Pirulli and Sorbino, 2008; Armanini et al., 2009; Pudasaini, 2012; Liu et al., 2017, among others) and even tools aimed at simplifying back-analysis of past events, evaluating forward scenarios of possible events and drawing up hazard maps (Iverson, 2014; Chen et al., 2016; Rosatti et al., 2018, among others) are now available. Nevertheless, a reliable forecast concerning the possibility that a storm may produce a debris flow is still hard, even just in a probabilistic framework. Attempts aiming at this goal have led to what is known in the literature as the *rainfall threshold* approach (Caine, 1980; Guzzetti et al., 2007). According to this approach, indicating with D the duration of a rainfall event pertaining to a debris flow and with I the relevant average rainfall intensity, the rainfall threshold is a function that splits the I-D domain in two fields: one located above the threshold and one located below. If a storm is foreseen to have a couple (I, D) falling in the upper field, it is expected that a debris flow may occur during that storm. On the contrary, if the foreseen couple does not exceed the threshold, it is expected that the storm should not generate a debris-flow.

Two main components are necessary to set up a rainfall threshold starting from an historical series of rainfalls related to debris-flow events:

- a method for evaluating the amount of rain strictly relevant to each debris-flow event in term of cumulated volume of precipitation V, averaged intensity I and duration D (we stress here that only two of the previous variables are independent);

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- a statistical approach that allows to identify a given occurrence probability threshold starting from a series of intensity-duration couples, namely (I_k, D_k) with $k = 1, \ldots, N$ and N is the total number of registered events in a given area.

In the literature, there are a few papers facing the rainfall threshold topic (see Section 2 and Section 4 for specific references). They differ (sometimes slightly) in the method used to evaluate the first or the second component of the procedure described above, but they share the same hydrological perspective, based on the assumption that the proper amount of water pertaining to a debris flow event can be estimated solely on the analysis of the rain hyetograph in a suitable time interval around the debris-flow occurrence time. Unfortunately, as stressed by some authors (Staley et al., 2013; Nikolopoulos et al., 2014), thresholds obtained so far with these approaches are unlikely low and many events with rain exceeding the threshold do not give rise to any debris-flow occurrence. Therefore, rainfall thresholds are seldom used as an effective tool in debris-flow warning systems.

The goal of the research underlying this paper is exploring new paths aimed at improving the reliability of rainfall thresholds. We started our work analyzing the possible weak point of the classical literature approach and it seemed to be the way the rain volume pertaining to a debris flow is determined. Then, we have looked for an alternative methodology for determining this volume involving, in some way, not only the forcing of the phenomenon (represented by the rain hyetograph), but also the dynamics of the debris flow. The novel approach we have developed starts from the knowledge of the volume occupied by the sediments deposited in an event (typically measured by regional agencies); then, the liquid volume (i.e. the rain) responsible of this mass movement is back reconstructed by using a simplified dynamical description of the phenomenon. Finally, the duration of the storm giving rise to the estimated volume (and the relevant average intensity as well) is identified from the analysis of the rain hyetograph. We have called this method the Backward Dynamical based Approach (BDA). A proof of concept is presented in the paper by applying the BDA and a literature method to a sample study area.

The structure of the paper is the following. In Section 2, we give a synthetic overview of the classical literature approaches used to estimate the rain relevant to a debris-flow event, with particular focus on the Critical Duration Method (CDM), and we identify a possible intrinsic limit of these approaches. Section 3 is dedicated to the detailed description of the BDA theoretical framework while, in Section 4, a literature method for evaluating a threshold relation from an I-D series, namely the frequentist method, is briefly presented. A sample study area and the relevant rainfall thresholds obtained by using BDA and CDM are presented in Section 5. Discussion of the result is reported in Section 6. Conclusions end the paper.

2. Estimate of the rain relevant to a debris-flow event: the classical literature approach

In the literature, the rain relevant to a debris-flow event is assumed to be the volume of water poured into a basin from the early beginning of the storm system, indicated with t_{init} (time measured respect an arbitrary reference), up to the debris-flow triggering time t_{trig} (see e.g. Lazzari et al., 2013). Assuming a uniform distribution of the precipitation over the basin, the volume V can be evaluated in the following way:

$$V = A_b \int_{t_{init}}^{t_{trig}} i(t)\, dt \tag{1}$$

where $i(t)$ is the measured hyetograph and A_b is the area of the basin. The rainfall duration D is then defined as the difference between two characteristic times, namely the triggering and the initial time:

$$D = t_{trig} - t_{init}. \tag{2}$$

Finally, the average intensity

$$I = \frac{1}{D} \int_{t_{init}}^{t_{trig}} i(t)\, dt \tag{3}$$

can be obtained dividing the volume V by the duration D and the area of the basin A_b:

$$I = \frac{V}{DA_b} \tag{4}$$

Therefore, considering a given hyetograph, the procedure to work out a couple (I, D) depends on the determination of the two characteristic times. Their practical determination, despite their simple definition, is not easy at all. In fact, the triggering time is seldom a piece of data known with precision. In most cases, only the date of a debris-flow occurrence is available in an event report. In these situations, the triggering time is assumed the instant in which the storm reaches the maximum rainfall intensity, i.e. $t_{trig} \equiv t_{max}$ (Iadanza et al., 2016). Other approaches (see e.g. Lazzari et al., 2013) assume this time as the last rainy measurement of the day.

Even more complex is the determination of the beginning of the rainfall event. In the literature there are many methods developed for this purpose and can be divided into two categories. The first collects the methods in which t_{init} is estimated based on empirical and subjective rules, valid only for the specific sites for which they have been developed. The second refers to objective methods based on a probabilistic approach. Despite several works use the first way (e.g. Nikolopoulos et al., 2014; Zhuang et al., 2015; Bel et al., 2017), the methods belonging to the second category (e.g. Bonta, 2001, 2003; Bonta and Nayak, 2008; Iadanza et al., 2016) seem more attractive because of their generality and possibility to be applied to different study areas.

2.1. *A specific approach for the determination of t_{init}: the Critical Duration Method*

The Critical Duration Method (CDM) (Restrepo-Posada and Eagleson, 1982; Bonta and Rao, 1988) aims at identifying independent storm systems singling out a characteristic time, the Critical Duration (CD), such that if the time between two rainy periods is longer than this value, the two rainy periods can be considered belonging to independent storm systems. A synthetic summary of the statistical procedure for the evaluation of the CD is reported in the following Section (for a more detailed description, we refer the reader to the original papers mentioned above). Since the statistical characteristics of the storms change with the seasons, monthly values are commonly determined.

Once the CD has been estimated, t_{init} is determined as the first dry instant before t_{trig}, whose time distance respect to the previous rainy instant is equal or longer than CD. An example of CDM application is reported in Fig. 1, where the triggering time has been considered as the t_{max} of the event.

2.1.1. *A synthetic summary of the CDM*

A time sequence of rainfall records consists of rainy periods separated by non-rainy intervals. When two rainy periods are separated by a time interval of the order of minutes (or possibly even some hours), they surely cannot be considered independent, but generated by the same meteorological system. On the contrary, when the two rainy periods are separated by a long dry interval, they can be considered caused by independent weather systems. The CDM identifies, by using a statistical approach, the characteristic time interval that allows distinguishing independent rainstorms. CD is defined as the minimum dry period which separates two stochastically independent rainy periods.

The main hypothesis in the CDM, verified by Restrepo-Posada and Eagleson (1982), is that the rainfall event durations are much shorter

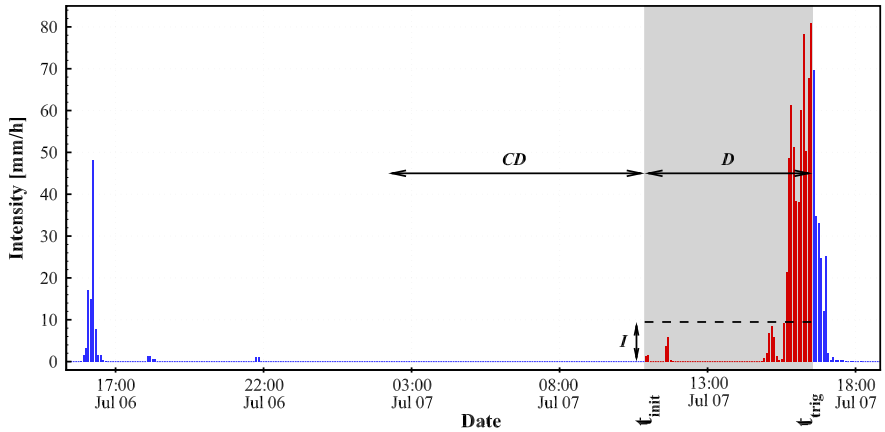

Fig. 1. Example of determination the rain relevant to a real debris flow. Here, $t_{trig} \equiv t_{max}$, t_{init} is obtain by using the CDM, $D = t_{trig} - t_{init}$ and I is the event-averaged rain intensity.

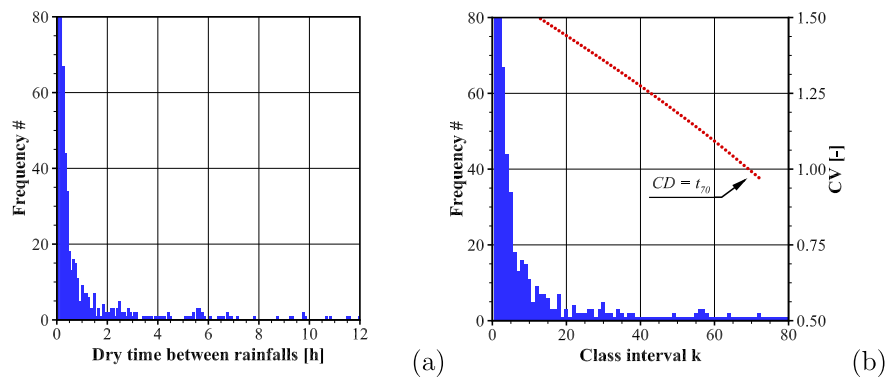

Fig. 2. Histogram of dry periods between rainfalls (a) as a function of time and (b) as a function of the classes used for the evaluation of CV. In the two plot are reported the same amount of data.

than the interstorm (dry) periods. Under this hypothesis, as proposed by Bonta and Rao (1988), the occurrence of storms can be assimilated to a random Poisson process, whose interstorm period distribution can be approximated with an exponential function of this type:

$$f(t) = \alpha e^{-\alpha t} \tag{5}$$

where α is the reciprocal of the mean time between storms and t is the dry-period duration. It is useful to recall that the variation coefficient CV, given by the ratio of the standard deviation s to the expected value E, is unitary. This feature can be used to single out the CD as the time beyond which the histogram of the dry periods can be well approximated with the exponential distribution. The procedure to get the CD is the following:

1. Subdivide a time axis in time steps long as the gauge sampling interval δt. Each interval is defined by:

   $$[(n-1)\delta t, n\delta t], \quad n = 1, \dots N$$

   where the central value is $(n - 1/2)\delta t$. Consider the number of occurrence of dry periods whose durations fall inside each interval. Build the relevant histogram (see Fig. 2(a)). It is worth noting that some intervals may be empty.
2. Call "classes" the *non-empty* time intervals of the previous histogram, number them from 1 to K and associate to each class the central time of the relevant interval. Build the histogram of the occurrence as a function of the classes (see Fig. 2(b)). In this case, the histogram has a value associated to each class.
3. Consider the sequence given by:

   $$CV_k = \frac{s_{[k,N]}}{E_{[k,N]}}, \quad k = 1, \dots K \tag{6}$$

   where $E_{[k,N]}$ and $s_{[k,N]}$ are, respectively, the expected value and the standard deviation for the dataset composed by the values included in the class interval $[k, N]$
4. Call $\widetilde{k}$, the first value of the sequence such that

   $$CV_{\widetilde{k}} = \frac{s_{[\widetilde{k},N]}}{E_{[\widetilde{k},N]}} \leq 1 \tag{7}$$

5. The value of the critical duration is defined as the time corresponding to the $\widetilde{k}$-th class, namely

   $$CD = t_{\widetilde{k}} \tag{8}$$

An example of CD determination is shown in Fig. 2 where the bars represent the histogram of dry-periods while the dotted lines are the values of CV_k.

According to Iadanza et al. (2013), the minimum number of years of data necessary to correctly estimate CD is 6. In this study we have used 10 years of data.

2.2. The possible weak point of the classical approaches

As we hinted at the beginning of the paper, the methods present in the literature produce unlikely low rainfall thresholds. The reasons of this behaviour may be multiple and connected both either to the first

or to the second component of the procedure depicted in the Introduction. Nevertheless, we think that this feature is mainly due to a not entirely appropriate estimate of the rain relevant to debris-flow events or, according to eq. (1), a not entirely appropriate definition of the characteristic times.

Although the triggering time presents significant uncertainties that can affect the reliability of the threshold, we think the assumption that the initial time, defined as the early beginning of the storm system during which a debris-flow event occurs, is conceptually the main weak point of the classical approaches. In fact, this choice leads to include in the duration D even all the dry periods occurring *within* a given storm system, with a consequent average intensity I that can be non-significant or non-representative of the real intensity that caused a debris-flow (see Fig. 1). Moreover, with the given definition of t_{init}, the water considered relevant to a debris-flow event is not only the water directly involved in the mass movement, but also the water that leads to the pre-condition for a debris-flow event, i.e. the saturation of the soil at least for a depth equal to the eroded layer.

The reason of this basic assumption can be due to a couple of factors. Firstly, the rainfall threshold approach and the given definition of t_{init} were originally proposed for generic hydrogeological phenomena, namely landslides and debris-flows together. For landslides, the water responsible of the phenomenon is probably all the water occurred before its triggering, while the water rained after that time plays a negligible role. Therefore, the assumption seems quite reasonable. On the contrary, in debris flows, the water volume flowing *after* the initiation of the flow plays an important role, since it determines the actual volume of the debris flow (as it will be explained in the next Section). Therefore, because of the differences of the two phenomena, an undistinguished application of the approach to landslides and to debris flows does not appear completely appropriate. The second factor, in a pure hydrological framework, is finding some features of the measured hyetograph that can be correlated to the debris flow itself to evaluate quantitatively the relevant rain. The hydrological beginning of a storm system, independently of the actual way it is determined, is therefore not only the natural choice but also perhaps even the only practical choice that can be done to characterize the starting of the rain relevant to a debris flow.

3. Estimate of the rain relevant to a debris-flow event: the Backward Dynamical Approach

Our perspective for estimating the (I, D) couple relevant to a debris-flow event is quite different from the perspective used so far in the literature. Hereafter we present a detailed description of the rigorous framework leading to the BDA, along with all the assumptions necessary to define it.

Let us indicate with V_{rain}^{DF}, the volume of rain directly embroiled in the mass movement. Assuming a uniform distribution of the precipitation over the basin, this volume can be expressed in similar way to eq. (1) as:

$$V_{rain}^{DF} = A_b \int_{t_1}^{t_2} i(t)\, dt \tag{9}$$

where t_1 and t_2 are two time limits related to the debris-flow duration and A_b is the area of the basin. We assume that:

1. V_{rain}^{DF} can be back reconstructed from the knowledge of V_{dep}, i.e. from the volume occupied by the mixture that stops in a deposition fan during the event. The relation between the two volumes, that we call "BDA relation", can be expressed formally as

$$V_{rain}^{DF} = f(V_{dep}) \tag{10}$$

and can be obtained considering a simplified dynamical description of the debris-flow phenomenon, to be specified further on.

2. The time limits t_1 and t_2 are conceptually connected to t_{trig} and to the debris flow duration but, in general, not to t_{init}. Since the triggering time and the duration of a debris flow event is rarely available, following Iadanza et al. (2016), we assume $t_{trig} \equiv t_{\max}$ while, for the time interval, we assume that it is symmetrically distributed around $t_{\max}$, namely:

$$[t_1, t_2] = [t_{\max} - \Delta t, t_{\max} + \Delta t] \tag{11}$$

where Δt is an unknown value.

The duration and average intensity of an event can be determined by using the previous three equations in the following way:

i. considering eq. (10) and (11), eq. (9) can be rewritten as

$$f(V_{dep}) = A_b \int_{t_{\max}-\Delta t}^{t_{\max}+\Delta t} i(t)\, dt \tag{12}$$

where the only unknown is Δt. This equation can be solved by means of a trial and error method;

ii. the rainfall duration, that we will indicate with $\hat{D}$ in order to distinguish from the classical duration D, is equal to the length of the time interval used in the integral:

$$\hat{D} = 2\Delta t \tag{13}$$

iii. considering the previous two expressions, the average intensity

$$\hat{I} = \frac{1}{\hat{D}} \int_{t_{\max}-\Delta t}^{t_{\max}+\Delta t} i(t)\, dt$$

can be rewritten as:

$$\hat{I} = \frac{f(V_{dep})}{A_b\, \hat{D}} \tag{14}$$

Rainfall thresholds will be calculated using couples of values $(\hat{I}, \hat{D})$ instead of (I, D). Consequences of this new point of view on threshold evaluations are analysed further in the paper (see Section 6).

In order to make the previous procedure effective, it is now necessary to specify the BDA relation. This is the subject of the following Sections.

3.1. Basic assumptions for the BDA relation

The BDA relation is based on a series of assumptions and approximations that, even if apparently rough, are quite reasonable and can give, at least, the right order of magnitude of the rain estimate. We list here all the assumed hypotheses and some motivations of their reasonableness.

- We refer only to stony debris flows, i.e. flows in which silt or clay does not affect the overall behaviour of the mixture.
- The concentration of sediments in the bed c_b is constant everywhere. This assumption is generally accepted in debris-flow dynamics (Takahashi, 2007).
- When a debris flow occurs, the soil is commonly completely saturated (e.g. Hungr et al., 2001). Therefore, we assume that the basin has been saturated by the rain foregoing the event, up to the level interested to the erosive stage. This condition is reasonable even if nobody has ever verified it in real cases. From this assumption and the previous ones, it follows that along the debris-flow path the pointwise water content in the terrain is everywhere $(1 - c_b)$.
- Since the characteristic time scale of a debris-flow event is much smaller than the time scale of the infiltration process, we assume that during a debris-flow occurrence, all the rainfall transforms into runoff.

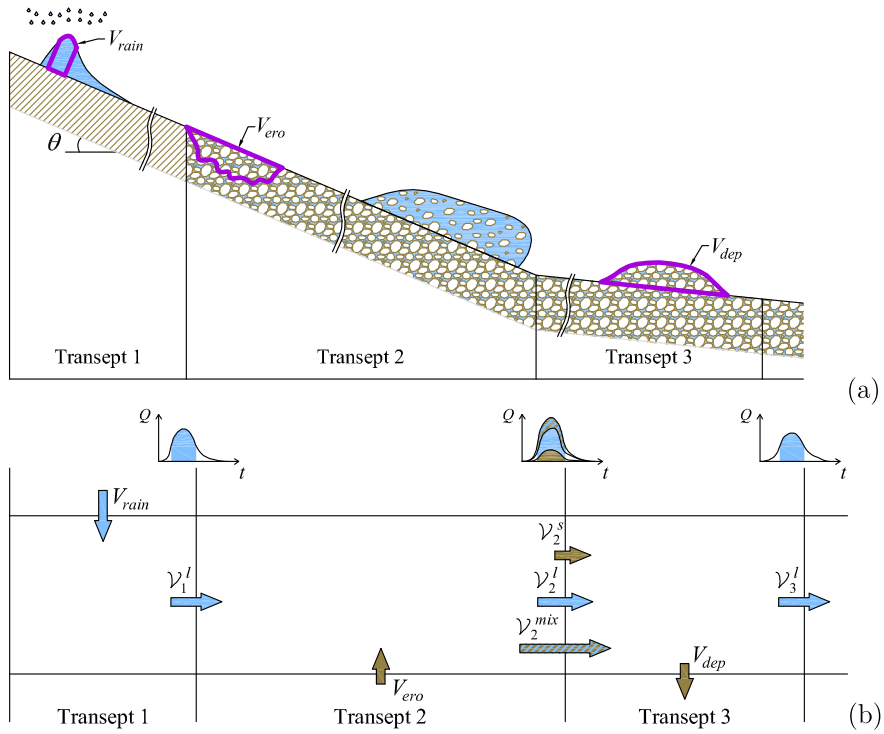

Fig. 3. Sketch of the conceptual scheme of a debris-flow dynamics: (a) Lagrangian point of view; (b) Eulerian point of view. Meanings of the quantities can be found in the text.

- The volume of sediments surveyed in the field after an event is the major part of the sediment involved in the debris-flow. This can be accepted if the deposition fan is a piece of territory with average slope significantly smaller than the slope in the flowing part and the volume of small-size sediments carried away with the water is negligible. This situation commonly occurs if the basin has a well-defined deposition fan.
- The reach just upstream the deposition fan is characterized by an average slope $i_f = \tan\theta$, where θ is the angle that the bed forms with an horizontal reference direction, and by a length long enough to allow the debris flow to be in uniform flow condition, i.e. with a volumetric solid concentration that, according to Takahashi (1978), for stony debris flows can be expressed as

$$c = \frac{\tan\theta}{\Delta(\tan\psi - \tan\theta)} \tag{15}$$

where $\Delta = (\rho^s - \rho^l)/\rho^l$ is the sediment relative submerged density, ρ^l and ρ^s are, respectively, the liquid and solid density while ψ is the internal friction angle. Considering that debris flows reach the uniform flow condition in rather short lengths, this assumption seems to be applicable in most cases.

3.2. Conceptual scheme of a stony debris-flow dynamics

Besides the previous assumptions, the BDA relation is based on a schematisation of a debris-flow dynamics. Obviously, real dynamics may be much more complex than the simple conceptual scheme herein reported. Nevertheless, we think that it is reasonably representative of a large set of real conditions.

For sake of simplicity, let us consider a long plane bed with unit width and constant slope i_f. An upper non-erodible transept followed by a lower erodible and saturated one characterizes the plane bed. At the end of the plane bed, there is a quasi-horizontal reach (see Fig. 3(a)).

The phenomena developing in the three transepts are:

Transept 1: Runoff formation. Rain gives a significant contribution only in this transept.

Transept 2: Erosion of the bed material and formation of a debris-flow characterized by a concentration given by eq. (15).

Transept 3: Deposition of all the sediments with saturation water entrapment. The rest of the water flow away outside the deposition fan.

In Fig. 3(a) we have sketched the phenomena occurring in the three transepts. The picture can be considered also a superimposition of three snapshots taken at three subsequent times (Lagrangian description) in the case the three phenomena occur disjointedly. Actually, it is very likely that they occur simultaneously in different position of the flow field but, in order to make the plot clearer, we preferred to represent the disjointed case.

3.3. Quantitative volumetric description of the debris-flow dynamics

In order to obtain a quantitative volumetric description of the conceptual debris-flow dynamics, it is useful turning to the one-dimensional Eulerian description of the flow sketched in Fig. 3(b). Here we report the detailed derivation of the relations valid in each transept, along with the assumptions necessary to obtain them. More complex relations can be obtained relaxing one or more hypothesis, but this generalization is left for a future work.

The debris-flow phenomenon can be described starting from the partial differential equations expressing the mass balances in a two-phase, depth-averaged framework in which there is no lag between the phases (see e.g. Armanini et al., 2009; Rosatti and Begnudelli, 2013). More specifically, the mass-balance equation of the liquid phase and the solid one can be written as:

$$\frac{\partial}{\partial t}\left[(1-c)h + (1-c_b)z_b\right] + \frac{\partial}{\partial x}\left[(1-c)uh\right] = S_l \tag{16a}$$

$$\frac{\partial}{\partial t}\left[ch + c_b z_b\right] + \frac{\partial}{\partial x}\left[cuh\right] = 0 \tag{16b}$$

where h is the mixture depth, c is the solid volumetric concentration, z_b is the mobile-bed elevation, u the depth-averaged velocity and finally S_l is the source term for the liquid phase. Here, the constant densities

of each phase has been simplified. Since no source for the solid phase is present, the relevant term is null. Momentum balance of the solid phase is also accounted for by mean of the uniform flow relation expressed by eq. (15).

The relations necessary for our approach are derived from the integration of the mass balance equations, namely eq. (16a) and (16b), along each transept in space, and throughout the duration of the debris flow in time. Rearranging some terms, these equations become:

$$\int_{t_k^{ini}}^{t_k^{end}}\int_{X_k^{ini}}^{X_k^{end}}\left(\frac{\partial}{\partial t}((1-c)h)+\frac{\partial}{\partial x}((1-c)uh)\right)dxdt=$$
$$\int_{t_k^{ini}}^{t_k^{end}}\int_{X_k^{ini}}^{X_k^{end}}\left(-(1-c_b)\frac{\partial z_b}{\partial t}+S_l\right)dxdt \tag{17a}$$

$$\int_{t_k^{ini}}^{t_k^{end}}\int_{X_k^{ini}}^{X_k^{end}}\left(\frac{\partial}{\partial t}(ch)+\frac{\partial}{\partial x}(cuh)\right)dxdt=-\int_{t_k^{ini}}^{t_k^{end}}\int_{X_k^{ini}}^{X_k^{end}}c_b\frac{\partial z_b}{\partial t}dxdt \tag{17b}$$

where the k subscript refers to the k-th transept, X_k^{ini}, X_k^{end} are the x-coordinate respectively of the transept initial and ending point and t_k^{ini}, t_k^{end} are the initial and ending times of the debris-flow. Performing some formal integrations, they can be rewritten as:

$$\left[\int_{X_k^{ini}}^{X_k^{end}}(1-c)h\,dx\right]_{t_k^{ini}}^{t_k^{end}}+\left[\int_{t_k^{ini}}^{t_k^{end}}(1-c)uh\,dt\right]_{X_k^{ini}}^{X_k^{end}}=$$
$$-(1-c_b)\left[\int_{X_k^{ini}}^{X_k^{end}}z_b\,dx\right]_{t_k^{ini}}^{t_k^{end}}+\int_{t_k^{ini}}^{t_k^{end}}\int_{X_k^{ini}}^{X_k^{end}}S_l\,dxdt \tag{18a}$$

$$\left[\int_{X_k^{ini}}^{X_k^{end}}ch\,dx\right]_{t_k^{ini}}^{t_k^{end}}+\left[\int_{t_k^{ini}}^{t_k^{end}}cuh\,dt\right]_{X_k^{ini}}^{X_k^{end}}=-c_b\left[\int_{X_k^{ini}}^{X_k^{end}}z_b\,dx\right]_{t_k^{ini}}^{t_k^{end}} \tag{18b}$$

in which we have used the notation that $[\mathcal{X}]_a^b$ represents the difference of the generic quantity $\mathcal{X}$ evaluated in b minus the same quantity evaluated in a.

The first term of each equation represents the time variation of respectively the liquid and the solid volume flowing inside the k-th transept. These terms can be neglected under the assumption that the flowing volumes are equal at the initial and at the ending time inside each transept. This may not be true in some cases, but we think that in the framework of a conceptual scheme, this approximation is acceptable.

The second term of each equation represents the difference of the fluxes integrated in time (namely, the volumes) that leave and enter the k-th transept in the given time interval. Indicating with:

$$\left[\int_{t_k^{ini}}^{t_k^{end}}(1-c)uh\,dt\right]_{X_k^{end}}=\mathcal{V}_k^l \quad\text{and}\quad \left[\int_{t_k^{ini}}^{t_k^{end}}cuh\,dt\right]_{X_k^{end}}=\mathcal{V}_k^s$$

the outflow volumes of the liquid and the solid phase in the k-th transept, these second terms can be written, respectively, as:

$$\begin{cases}\mathcal{V}_k^l-\mathcal{V}_{k-1}^l\\ \mathcal{V}_k^s-\mathcal{V}_{k-1}^s\end{cases}$$

in which the inflow volumes of the k-th transept are expressed as outflow volumes of the $(k-1)$-th one.

The first term on the right hand side of each equation represents the volumes of liquid and solid released or stored in the bed because of the bed evolution. Since c_b is constant in time and space, they can be written as a function of the volume of bed variation in the transept, defined as:

$$\left[\int_{X_k^{ini}}^{X_k^{end}}z_b\,dx\right]_{t_k^{ini}}^{t_k^{end}}=V_k^{bed}$$

namely:

$$\begin{cases}(1-c_b)\,V_k^{bed}\\ c_bV_k^{bed}\end{cases}$$

The last term in the liquid mass-balance equation represents the source term for the liquid phase, i.e. the possible volume of rain. We indicate it as:

$$\int_{t_k^{ini}}^{t_k^{end}}\int_{X_k^{ini}}^{X_k^{end}}S_l\,dxdt=V_k^l$$

Considering all the previous expressions, eq. (18a-18b) can be rewritten in the following compact way:

$$\mathcal{V}_k^l-\mathcal{V}_{k-1}^l=-\left(1-c_b\right)V_k^{bed}+V_k^l \tag{19a}$$
$$\mathcal{V}_k^s-\mathcal{V}_{k-1}^s=-c_bV_k^{bed} \tag{19b}$$

Since the bed variation and the source term are not present in each transept, it is useful to write explicitly for each reach the relevant equations according to the assumed conceptual scheme.

Transept $k=1$: here we have no upstream input therefore $\mathcal{V}_0^{l,s}=0$; we indicate with V_{rain} the liquid source V_1^l; finally, since no bed variation is present, namely $V_1^{bed}=0$, then, the equations for this transept become:

$$\mathcal{V}_1^l=V_{rain} \tag{20a}$$
$$\mathcal{V}_1^s=0 \tag{20b}$$

Transept $k=2$: indicating with $-V_{ero}=V_2^{bed}$ the bed volume variation connected to the erosion (with $V_{ero}\geq 0$), we can write:

$$\mathcal{V}_2^l-\mathcal{V}_1^l=(1-c_b)V_{ero} \tag{21a}$$
$$\mathcal{V}_2^s=c_bV_{ero} \tag{21b}$$

In the outflow section of this transept, according to the assumptions, the concentration is constant and given by eq. (15). Indicating with $\hat{c}$ this concentration, the outflow volumes can be rewritten as:

$$\mathcal{V}_2^l=(1-\hat{c})\left[\int_{t_k^{ini}}^{t_k^{end}}uh\,dt\right]_{X_k^{end}}\quad;\quad \mathcal{V}_2^s=\hat{c}\left[\int_{t_k^{ini}}^{t_k^{end}}uh\,dt\right]_{X_k^{end}}.$$

We can express these volumes as a function of the outflow volume of the mixture

$$\mathcal{V}_2^{mix}=\mathcal{V}_2^l+\mathcal{V}_2^s=\left[\int_{t_k^{ini}}^{t_k^{end}}(uh)\,dt\right]_{X_k^{end}}$$

in the following way:

$$\mathcal{V}_2^l = (1-\hat{c})\mathcal{V}_2^{mix} \tag{22a}$$

$$\mathcal{V}_2^s = \hat{c}\mathcal{V}_2^{mix} \tag{22b}$$

Finally eq. (21) can be reformulated in the following form that will be used further on:

$$(1-\hat{c})\mathcal{V}_2^{mix} - \mathcal{V}_1^l = (1-c_b)V_{ero} \tag{23a}$$

$$\hat{c}\mathcal{V}_2^{mix} = c_b V_{ero} \tag{23b}$$

Transept $k = 3$: here, V_{dep} denotes the bed volume variation connected to deposition, namely $V_{dep} = V_3^{bed}$ (with $V_{dep} \geq 0$). Moreover, since all the sediments stop in this reach according to the assumptions, $\mathcal{V}_3^s$ must be null. Therefore, the resulting equations are:

$$\mathcal{V}_3^l - \mathcal{V}_2^l = -(1-c_b)V_{dep} \tag{24a}$$

$$-\mathcal{V}_2^s = -c_b V_{dep} \tag{24b}$$

Combining the equations written for each transept, it is now possible to obtain the following useful relations.

BDA relation: substituting eq. (21b) in eq. (24b) it follows:

$$V_{ero} = V_{dep} \tag{25}$$

Moreover, we can derive $\mathcal{V}_2^{mix}$ from eq. (23b)

$$\mathcal{V}_2^{mix} = \frac{c_b}{\hat{c}} V_{ero} \tag{26}$$

and substitute it in eq. (23a):

$$(1-\hat{c})\frac{c_b}{c} V_{ero} - \mathcal{V}_1^l = (1-c_b)V_{ero}$$

By using eq. (25) and (20a) it follows:

$$\left[(1-\hat{c})\frac{c_b}{\hat{c}} - (1-c_b)\right] V_{dep} = V_{rain}$$

that can be simplified to the final form:

$$V_{rain} = \frac{c_b - \hat{c}}{\hat{c}} V_{dep} \tag{27}$$

This relation is the explicit expression of eq. (10) and, despite its simplicity, it represents a key element in our approach.

Volume of erosion: by using the previous relation and eq. (25), we obtain:

$$V_{ero} = \frac{\hat{c}}{c_b - \hat{c}} V_{rain} \tag{28}$$

which links the eroded volume reaching the deposition fan to the relevant volume of rain.

Amplification of the debris-flow volume respect the rain volume: in eq. (23a) we can substitute the expression of V_{ero} obtained from eq. (23b)

$$V_{ero} = \frac{\hat{c}}{c_b} \mathcal{V}_2^{mix} \tag{29}$$

and $\mathcal{V}_1^l$ from eq. (20a), obtaining:

$$(1-\hat{c})\mathcal{V}_2^{mix} - V_{rain} = (1-c_b)\frac{\hat{c}}{c_b}\mathcal{V}_2^{mix}$$

It is then possible to derive the relation

$$\mathcal{V}_2^{mix} = \frac{c_b}{c_b - \hat{c}} V_{rain} \tag{30}$$

that is nothing but the well-known volumetric amplification relation obtained by Takahashi (2007) starting from a quite different context: it expresses the volume of debris flow as a function of the volume of rain and of the equilibrium concentration in a reach with a given slope. It is worth noting that the present derivation allows to appreciate the assumptions underlying this expression.

The liquid volume leaving the domain: by using eq. (20a) and eq. (21a) we obtain:

$$\mathcal{V}_3^l = V_{rain} + (1-c_b)V_{ero} - (1-c_b)V_{dep} \tag{31}$$

and using relation (25), it becomes:

$$\mathcal{V}_3^l = V_{rain} \tag{32}$$

This expresses the somewhat (a posteriori) obvious statement that the liquid volume leaving the domain is equal to the liquid volume entering upstream.

3.4. The role of the rain volume in a debris-flow

A by-product of the approach just presented is the explanation of the role of the rain volume in the simplified dynamics of a debris flow. In a Lagrangian framework, the rain volume firstly generates the hydrological flow, then induces a volume of erosion given by eq. (28), conveys downstream the sediments as a mixture whose volume is given by eq. (30) and finally, after having deposited the eroded volume, eq. (25), leaves the domain, eq. (32).

4. Probability threshold for intensity-duration data: the frequentist method

It is widely accepted (see e.g. Brunetti et al., 2010; Peruccacci et al., 2012; Nikolopoulos et al., 2014) that the relation between rainfall intensity I and duration D is a power law of type:

$$I(D) = \alpha D^{-\beta} \tag{33}$$

where α and β are two constant parameters which are commonly estimated by using least square method (or other statistical approaches) starting from a significant set of couples (I_k, D_k). In this work, we assume that the same type of relation is valid also for the intensities and durations defined in the previous section, i.e.

$$\hat{I}(\hat{D}) = \hat{\alpha}\hat{D}^{-\hat{\beta}} \tag{34}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are two constant parameters similar to α and β, and estimated by using the BDA couple set $(\hat{I}_k, \hat{D}_k)$.

The approach we have chosen in this work to estimate the probability threshold is the frequentist method (see e.g. Peruccacci et al., 2012, among others), nevertheless other methods could be applied as well (see e.g. Berti et al., 2012; Peres and Cancelliere, 2014). For sake of completeness, we present here a summary of the methodology applied to the $(\hat{I}_k, \hat{D}_k)$ couples, but the procedure applies to (I_k, D_k) as well. We address the reader to the mentioned paper for more details.

According to this approach, the threshold is a curve that, in a log-log plot, is a straight line parallel to eq. (34) but with a lower value of the intercept such that the probability that the measured real event data exceeds the threshold value is a given value.

In order to obtain this threshold line, firstly the following difference set must be evaluated:

$$\delta(\hat{D}_k) = \log \hat{I}_k - \log \hat{I}(\hat{D}_k), \quad k = 1, \ldots, N \tag{35}$$

where $\hat{I}_k$ is the value of the k-th registered datum associated to the k-th duration $\hat{D}_k$, $\hat{I}(\hat{D}_k)$ is computed by means of eq. (34) and N is the total number of registered events. Then, the probability density function of this set is approximated by using a Kernel Density Estimation as proposed by Silverman (1986). Afterwards, this last function is sampled at regular intervals and the resulting set of values are used to estimate the parameters of a normal distribution:

$$f(\delta) = \frac{1}{2\pi\sigma} \exp\left(-\frac{(\delta-\mu)^2}{2\sigma^2}\right) \tag{36}$$

Fig. 4. Location of Trentino-Alto Adige/Südtirol region (Italy), the sample study area.

namely the mean μ and the variance σ. Finally, the previous distribution is integrated from $-\infty$ up to a threshold value δ_x such that the relevant probability of non-exceedance is equal to x. Commonly, this value is set equal to 5%. Therefore, the threshold curve can be written as:

$$\hat{I}_{5\%} = \hat{\alpha}_{5\%} \hat{D}^{-\hat{\beta}} \tag{37}$$

where $\hat{\alpha}_{5\%} = \hat{\alpha} - |\delta_{5\%}|$.

5. The rainfall thresholds for a sample study area

In order to verify the feasibility of the proposed method and to highlight the peculiarities of BDA approach respect to the classical one considered in this work, we have calculated the rainfall thresholds according to the procedures above described. The study area is Trentino-Alto Adige/Südtirol, located in the Alps, in the north of Italy (Fig. 4).

5.1. Available data

Between 2006 and 2016, 161 debris flows were recorded by the regional agency. The average area of the catchments affected by the events is nearly $1.5\,\mathrm{km}^2$. Almost none of them has a precise indication about the triggering area while for 139 cases, a quantitative estimate of the deposited volume is available. It ranges from $100\,\mathrm{m}^3$ of the smallest events up to $50{,}000\,\mathrm{m}^3$ of the largest ones. In Fig. 5, the spatial distribution of the debris-flow events is reported on a geological map of the study area. It must be emphasized that all the events, even if occurring in geologically different areas, were characterized by a loose stony nature, and devoid of any significant presence of cohesive mud.

A network of 195 rain gauges is available with an average spatial density of approximately $1/70\,\mathrm{km}^{-2}$ and an average altitude of $1{,}400\,\mathrm{m}$ asl. Nearly each rain gauge used in this study has a record frequency between 5 min and 10 min.

For the same period, records of a C-band Doppler weather radar are available. The radar is located in a central position of the region on the top of Macaion peak at $1{,}866\,\mathrm{m}$ asl (see Fig. 6), and it is effectively used to monitor an area within the range of 120 km. Precipitation is estimated converting the reflectivity Z into intensity of precipitation I (see e.g. Uijlenhoet, 2001). The radar output data are available over a square grid cell with resolution of 500 m, while the temporal resolution is between 5 min and 6 min.

Since generally, the cumulated rainfall depends on the altitude and rapidly decreases with the increase of the distance from the event area (Marra et al., 2016), in order to get reliable rain data relevant to a debris flow, a careful choice among the available rain gauges has been performed. For each debris flow event, the choice of the representative station was obtained by using an automatic procedure. From the knowledge of the coordinates of a debris-flow basin closure point, assumed located where the deposition starts, the horizontal distance between the closure point and each rain gauge was evaluated. Among all the available rain gauges with a distance less or equal to 5 km, the representative station was assumed the one with the smallest altitude difference respect the closure point. If no instrument matched the distance criteria, the debris-flow event has been considered unserviceable for the subsequent analysis.

Rain gauge data has been used only for the CD estimation, while radar data has been used to estimate the (I, D) and $(\hat{I}, \hat{D})$ couples. Nevertheless, because of the complex topography of the region, mountain beam shielding causes, in certain areas, a lack of measurement associated with a reported debris flow event (see Fig. 6). Therefore we have

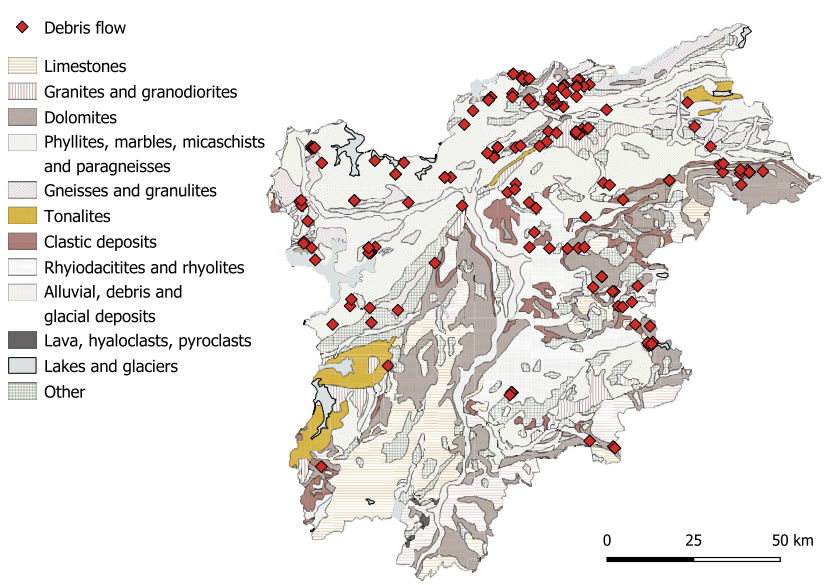

Fig. 5. Simplified geological map of the study area with indication of the locations of debris-flow events.

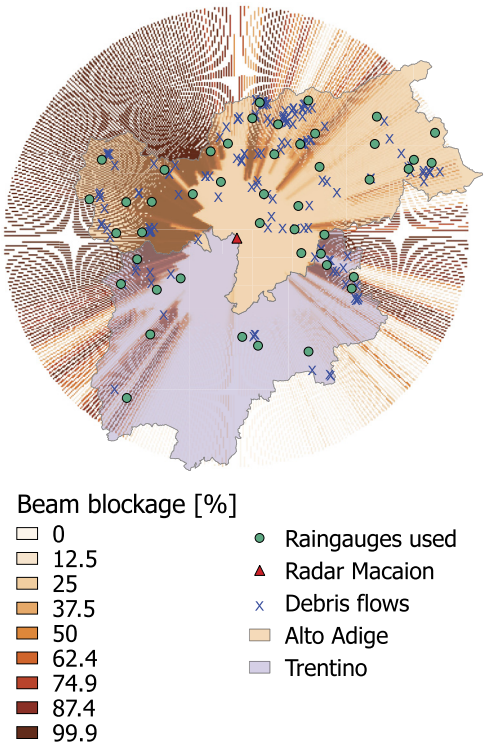

Fig. 6. Percentage of signal attenuation due to beam blocking effect on the debris flows considered with radar beam elevation at 1°.

been preliminary filtered out all the events located in places where the radar signal is weakened more than 90%. Other sources of errors, such as signal attenuation in heavy rain or wet radome attenuation (see e.g. Marra et al., 2014), have not been taken into account.

5.2. The CDM-based threshold

For this site, 45 rain gauges, linked to the 161 debris-flow available events, have been used to determine the monthly CD reported in Table 1 and plotted in Fig. 7.

The CDM-based threshold has been obtained by using only 109 registered events (out of the 161 registered) matching the 5 km distance and the radar signal criteria. The resulting threshold equation is:

$$I_{5\%} = 4.91 D^{-0.7} \tag{38}$$

and is plotted in Fig. 8 along with the relevant (I, D) event couples.

5.3. The BDA-based threshold

The BDA methodology requires the knowledge of the volume of deposited material (as described in Section 3). Due to this constrain, only 84 debris flows of the 109 registered events with unshielded radar data are serviceable. For each debris-flow case, the couple $(\hat{I}, \hat{D})$ was obtained using the following procedure:

1. the relevant basin was extracted from a DTM, setting the closure at the beginning of the deposition zone;
2. a representative bed slope i_f was considered as the average slope of the last 50 m of the basin network;
3. the debris-flow reference concentration $\hat{c}$ was evaluated by means of eq. (15), where we set $\Delta = 1.65$ and $\psi = 35°$;
4. the volume of rain that has caused the deposition was estimated by using eq. (27)
5. the duration $\hat{D}$ should have been evaluated by solving eq. (12); nevertheless, since intensity is actually a piece-wise constant function whose constancy interval is equal to the radar sampling interval δt, the equation cannot be solved exactly. Therefore, the procedure used to obtain $\hat{D}$ was the following:
   - consider the discrete hydrograph pertaining a debris flow event; place the origin of a discrete reference system in the interval where the intensity is maximum and label this interval as i_0;
   - consider the following sequence of discrete integrals of the hydrograph:

$$a_0 = i_0 \delta t$$

$$a_n = a_{n-1} + \begin{cases} i_{n/2+1/2}\,\delta t & if \quad n \text{ is odd} \\ i_{-n/2}\,\delta t & if \quad n \text{ is even} \end{cases}$$

Table 1
Values of the monthly CD, expressed in hours, for the study case.

Month	1st quartile	Median	3rd quartile
Jan	16.5	29.6	63.5
Feb	20.0	42.7	54.4
Mar	14.5	21.4	38.7
Apr	16.8	30.3	39.4
May	6.8	9.0	11.8
Jun	7.3	9.2	11.7
Jul	7.1	8.7	11.1
Aug	4.6	5.8	7.6
Sep	5.3	7.0	8.9
Oct	7.8	10.3	13.1
Nov	12.0	15.0	26.4
Dec	19.4	30.4	44.3

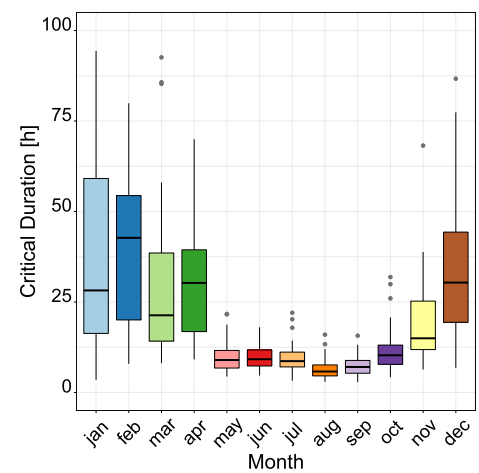

Fig. 7. Box plot of the monthly CD for the study site.

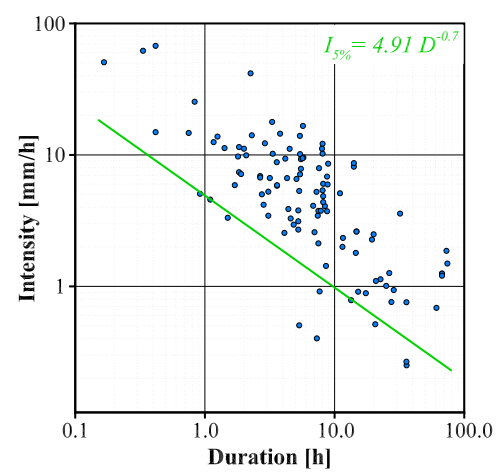

Fig. 8. The CDM-based rainfall threshold for the study area.

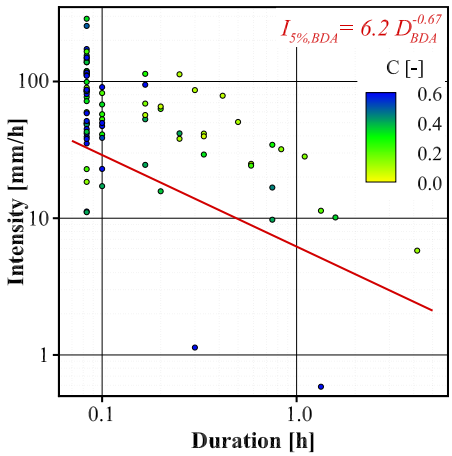

Fig. 9. BDA-based rainfall threshold for the study area. The colour scale indicates the equilibrium concentration, evaluated according to eq. (15), for each event.

where $i_{\pm k}$ is the intensity of the $\pm k$-th interval located on the right and on the left of i_0 respectively;

- consider the integer K as the first integer such that

$$V_{rain} \leq A_b a_K$$

Then, since a_K is nothing but the approximation of the integral term of eq. (12), it follows that:

$$\hat{D} = (K+1)\,\delta t$$

6. the intensity $\hat{I}$ was obtained by using eq. (14)

The resulting BDA-based threshold equation, plotted in Fig. 9 along with the relevant $(\hat{I}, \hat{D})$ event couples, is:

$$\hat{I}_{5\%} = 6.2\hat{D}^{-0.67} \tag{39}$$

6. Discussion

In this Section, we discuss some aspects of the results obtained for the sample study area.

6.1. The CDM-based threshold

The threshold obtained with this method, eq. (38), is similar to the ones obtained by Marra et al. (2014) and by Iadanza et al. (2016) who considered, for areas comparable to the one considered in this paper, rain data associated not only to debris flows but also to landslides. Moreover, it can be noted that in Fig. 8, even if data is distributed over almost three orders of magnitude, the largest number of values are approximately in the interval $[1\,\text{h}, 10\,\text{h}]$.

Regarding the monthly CDs, it is important to notice, Fig. 7, the dispersion of the summer months values is smaller than the dispersion of the other months. This means that for the summer period, the median value of CD is representative of the whole correspondent area. This is quite important because in this period the majority of the debris-flow events occur. Moreover, the summer CDs are shorter than in the rest of the year. This is due to the different structure of the summer storms with respect to the rainy systems of the other months. Similar trend is reported in Iadanza et al. (2016), where an area analogous to the one considered in this paper was investigated.

6.2. The BDA-based threshold

First of all, the assumption that the intensity $\hat{I}$ is a power law function of $\hat{D}$ (see Section 4) seems to be confirmed by results, since data in Fig. 9 shows a clear linear trend in the log-log plot.

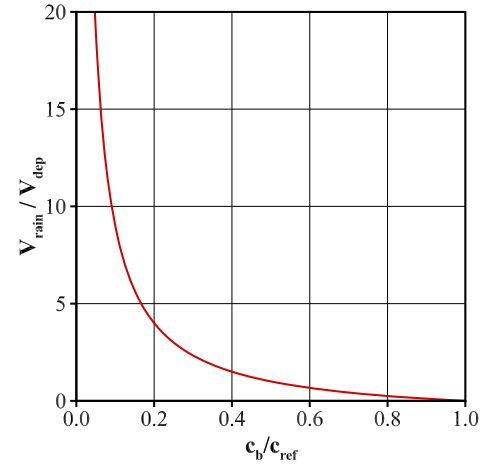

Fig. 10. Trend of the ratio of the volume of rain over the deposited volume as a function of the ratio of the bed concentration over the reference concentration.

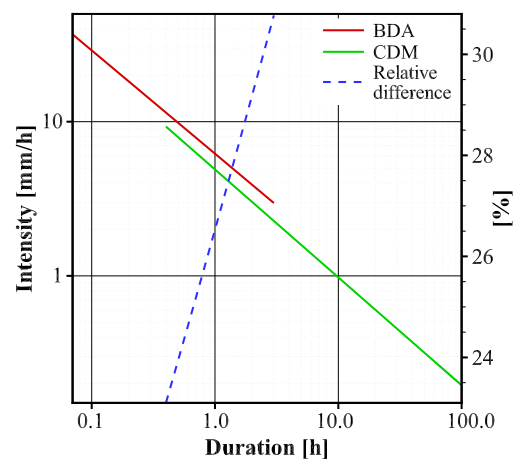

Fig. 11. Comparison between the CDM- and the BDA-based rainfall thresholds for the study area and their relative difference respect the BDA values.

The data is distributed essentially over one scale of magnitude, ranging from 0.1 h to 1 h. This time scale is confirmed by both eyewitness and video testimonies (some of them can be found on the web) even if, a systematic and well documented study is still not available. However, this result allows a first assessment of the BDA relation reliability. On the other hand, the BDA relation is strictly connected to the Takahashi volumetric amplification relation, whose validity is widely accepted and assessed (see e.g. Rosatti et al., 2015). Therefore, the BDA relation appears to be a sufficiently robust estimator, provided that measured data is sufficiently accurate.

An accurate analysis of the result (still Fig. 9) showed several points characterized by a duration equal to the sampling interval of the radar (leftmost points) and this can be due to multiple reasons.

One of these can be the sampling interval is too long with respect to the time necessary to feed a debris-flow.

A second one is an underestimation of V_{dep}, whether for measurement errors or because only part of the debris-flow sediments stopped in the fan, while another significant part is not included in the measurements because it has been carried downstream by the flow. Therefore, the relevant V_{rain} is smaller than the actual value and consequently, the duration is shorter than the real one.

A third possible reason is an overestimation of c_{ref} due to a wrong estimation of a significant slope. This produces an underestimation of V_{rain}, as it can be deduced from eq. (27) and plotted in Fig. 10. It follows an underestimation of the event duration.

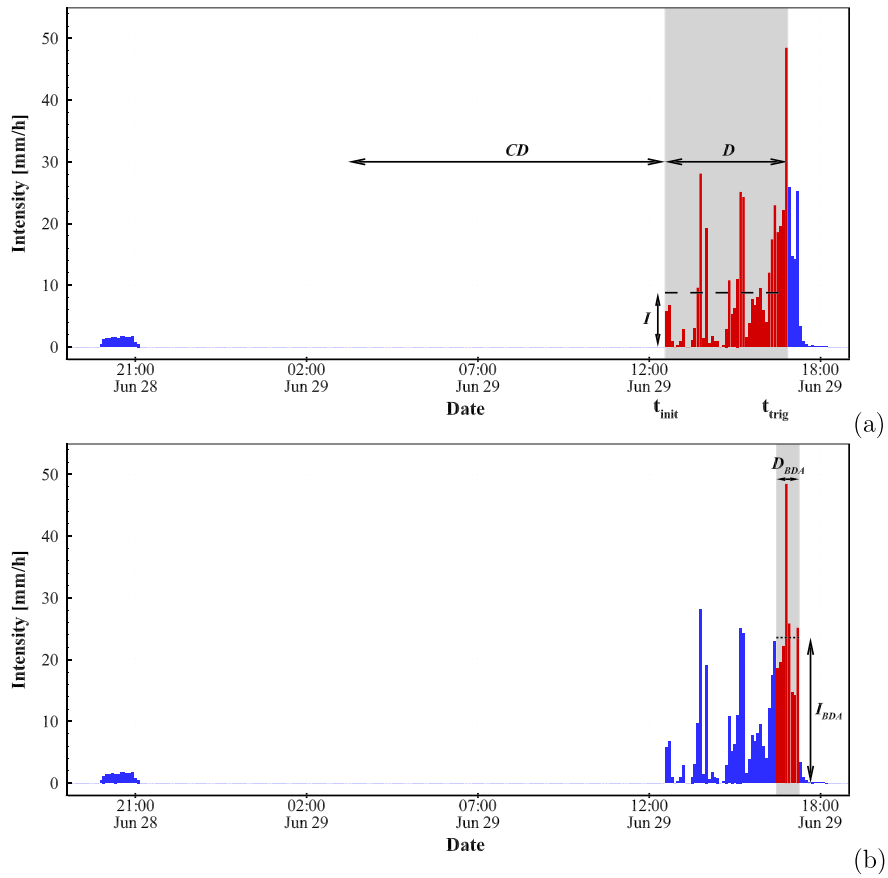

Fig. 12. Portion of hyetograph relevant to the same debris flow event according to (a) CDM and (b) BDA.

Last but not least, despite V_{dep} is essentially correct, the debris flow has not reached the hypothesized equilibrium condition because of lack of sediments, non-erodible zones etc. Therefore, also in this case c_{ref} is overestimated with respect to the actual value and the event duration is underestimated.

We are not able to quantify the errors in our data, or to verify their frequency distribution. Anyhow, we think that the last three reasons are probably the most diffuse and possibly mingled. In any case, we think that our sample is sufficiently reliable. Finally, we are not able even to predict the change in the threshold if more reliable data would be available.

6.3. Comparison between the CDM- and the BDA-based thresholds

A straightforward comparison between the two thresholds, namely eq. (38) and eq. (39), shows that the BDA-based threshold is, as expected, higher than the CDM-based one (see Fig. 11). Nevertheless, we cannot conclude that BDA is a better estimator, since we are comparing quite different approaches and quantities and therefore, some care must be paid in making the comparison.

Since it is not completely right extending the validity of each threshold outside the domain used for the interpolation, there is only a limited overlapping of the two domains. In this range, the relative difference with respect to the CDM data spans from 25% to 31%.

The characteristic time scales of the two approaches are quite different: $[1\,\mathrm{h}, 10\,\mathrm{h}]$ for CDM and $[0.1\,\mathrm{h}, 1\,\mathrm{h}]$ for BDA. Namely, BDA indicates as potential debris-flow inducing rainfalls, short or very short storm durations while CDM suggests quite longer durations. This is not surprising since, as we have already stressed, we are comparing different things, deriving from different points of view. In order to highlight this difference, it is useful to compare graphically (see Fig. 12), how the same hyetograph is considered by the two approaches. It is quite clear how the same event is characterized by different duration and average intensity, and why BDA provides shorter duration and higher intensities with respect to CDM.

6.4. Some issues regarding the forecast use of the BDA-based threshold

The forecast use of the BDA-based thresholds presents some potential issues.

Starting from a forecast hyetograph, while the peak intensity can be easily identified, the potential volume of rain that determines both the duration and the average intensity, is not known. A possible empirical choice, that can be deduced from the available data but that cannot be used in general, is to consider an average duration of $0.5\,\mathrm{h} \div 1.0\,\mathrm{h}$ centred around the peak.

It is not so uncommon in debris-flow events that the relevant hyetograph shows multiple peaks (see e.g. Rosatti et al., 2015). In these cases, it is even more complicated estimating a reference duration because the previous criterion could be largely inaccurate. Moreover, it is quite difficult to forecast if multiple peaks generate multiple events or a single long event.

A validation of the proposed reference duration choice, the development of more sophisticate approaches and the evaluation of the frequency of false positive respect the proposed BDA-based threshold are very interesting topics, but they are beyond the scope of this work and therefore are left for a future widening. Last, but not least, BDA focuses the attention only on one mandatory ingredient for a debris flow: the amount of water necessary to carry downstream the sediments. The other key element, the saturation of the basin, is completely disregarded in this approach and therefore, a more complete methodology considering both the ingredients is desirable.

7. Conclusions

The conclusions that can be drawn from this study are the following:

- The Backward Dynamical Approach presented in this paper appears to be a reasonable theoretical framework able to single out the amount of rain strictly pertaining a debris-flow event and, consequently, the related $(\hat{I}, \hat{D})$ couple. Other less simplified relations can be obtained relaxing one or more assumptions herein considered.
- Every method of extracting a threshold from a (I, D) couple set can be applied to the BDA couples as well.
- Analysis of a sample study area shows that the characteristic duration of the rain generating a debris-flows evaluated with BDA is one order of magnitude shorter than the characteristic duration given by CDM. This result appears to be consistent to several field observations, but a systematic validation is still missing because of lack of reliable data.
  Moreover the BDA-based threshold is, as expected, higher than the CDM-based one, provided that the same methodology is used to obtain the thresholds from the intensity-duration couples. Even if this result seems to overcome a bit the limit of the traditional approaches, it is not yet possible to state that this method performs better than the literature ones because a systematic validation is still not possible, once again, for lack of measured data.
- The use of the BDA-based thresholds for forecast purpose presents some potential issues. Moreover, the approach does not account for the reaching of any "triggering condition"(e.g. saturation of the basin, threshold condition for transport, abatement of possible superficial sediment cohesion, etc.) and for the effect of earthquakes on these conditions (see e.g. Shieh et al., 2009; Tang et al., 2009). A more complete, and presumably more reliable methodology for the rainfall threshold determination should consider, in some way, not only the characteristics of the rain strictly pertaining to the debris flow but also the characteristics of the rain leading to the triggering conditions and at least some features of the sediments.
- To fully validate the proposed method, large samples of data from different areas should be considered. Our forthcoming work aims to achieve this goal. Unfortunately, the number of well-documented debris-flow events are rather few, not just because debris flows are infrequent, but also because so far the detailed survey of the deposited volumes was extremely difficult and not considered so important by the public agencies in charge of debris-flow data collection. Nevertheless, in the last years the application of drone technology to land surveying has gratly simplified the measurement of debris-flow deposits.

We are confident that in some years the number and quality of available data should help to validate both the reliability of the proposed method and its possible future enhancements.

Declarations

Author contribution statement

Giorgio Rosatti: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

Marina Pirulli: Analyzed and interpreted the data; Wrote the paper.

Daniel Zugliani, Marta Martinengo: Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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