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# Correction to: "Material description of fluxes in terms of differential forms"

## "Dedicated to Prof. David Steigmann in recognition of his contributions"

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Although the final result presented in Equation (34) of our work [**?**] is correct, the proof of Equation (34) contains an error. The text starting immediately after Equation (33) with "When a metric tensor $\boldsymbol{g}$..." and ending immediately before Equation (36) with "... in the alternative notation" should be replaced with the text below.

## Correction to the Proof of Equation (34)

Let us assume that the space $\mathcal{S}$ is equipped with a metric tensor $\boldsymbol{g}$, i.e., a symmetric and positive-definite tensor field valued in $[T\mathcal{S}]^0_2$, defining the scalar product of two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ as $\boldsymbol{u}.\boldsymbol{v} = \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{v})$. The metric $\boldsymbol{g}$ induces the

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*musical isomorphisms* $\flat : T\mathcal{S} \to T^\star\mathcal{S} : \boldsymbol{v} \mapsto \flat(\boldsymbol{v}) \equiv \boldsymbol{v}^\flat$, which maps a vector $\boldsymbol{v}$ with components $v^c$ to a covector $\boldsymbol{v}^\flat$ with components $g_{ac}v^c$, and its inverse $\sharp : T^\star\mathcal{S} \to T\mathcal{S} : \boldsymbol{\alpha} \mapsto \sharp(\boldsymbol{\alpha}) \equiv \boldsymbol{\alpha}^\sharp$, which maps a covector $\boldsymbol{\alpha}$ with components $\alpha_c$ to a vector $\boldsymbol{\alpha}^\sharp$ with components $g^{ac}\alpha_c$, where $g^{ac}$ are the components of the inverse of the matrix $[\![g_{ab}]\!]$ of $\boldsymbol{g}$. The isomorphism $\sharp$ and the metric tensor $\boldsymbol{g}$ induce the scalar product of covectors $\boldsymbol{\alpha}.\boldsymbol{\beta} = \boldsymbol{g}(\boldsymbol{\alpha}^\sharp, \boldsymbol{\beta}^\sharp) = \boldsymbol{\alpha}(\boldsymbol{\beta}^\sharp)$.

The $(n-1)$-dimensional tangent bundle $Ts$ of the hypersurface $s$ determines a 1-dimensional sub-bundle of $T^\star\mathcal{S}$ containing the annihilators of $Ts$, i.e., the covectors $\boldsymbol{\nu}$ such that $\boldsymbol{\nu u} \equiv \boldsymbol{\nu}(\boldsymbol{u}) = 0$, for every $\boldsymbol{u} \in Ts$. Moreover, using the scalar product of covectors, we can define the *unit normal covector* $\boldsymbol{n}$ to the hypersurface $s$ as the annihilating covector such that $\|\boldsymbol{n}\|^2 = \boldsymbol{n}.\boldsymbol{n} = 1$.

The integral (33) of an $(n-1)$-form $\boldsymbol{\omega}$ on the hypersurface $s$ can be expressed in terms of the axial vector field $\boldsymbol{w}$ of $\boldsymbol{\omega}$ with respect to the volume form $\boldsymbol{\mu}$, i.e., $\boldsymbol{w}$ is such that $\boldsymbol{\iota_w \mu} = \boldsymbol{\omega}$. If we introduce the *axial projector* $\boldsymbol{a} = \boldsymbol{n}^\sharp \otimes \boldsymbol{n}$ (in components, $a^a{}_b = n^a n_b$) and the *transverse projector* $\boldsymbol{t} = \boldsymbol{i} - \boldsymbol{n}^\sharp \otimes \boldsymbol{n}$ (in components, $t^a{}_b = \delta^a{}_b - n^a n_b$, where $\boldsymbol{i}$ is the spatial identity tensor, it holds that $\boldsymbol{i} = \boldsymbol{a} + \boldsymbol{t}$ and that any vector field $\boldsymbol{w}$ can be decomposed as

$$\boldsymbol{w} = \boldsymbol{iw} = (\boldsymbol{a} + \boldsymbol{t})\boldsymbol{w} = \boldsymbol{aw} + \boldsymbol{tw} = \boldsymbol{w}_a + \boldsymbol{w}_t, \tag{C1}$$

where $\boldsymbol{w}_a = \boldsymbol{aw} = (\boldsymbol{nw})\boldsymbol{n}^\sharp$ and $\boldsymbol{w}_t = \boldsymbol{tw} = \boldsymbol{w} - (\boldsymbol{nw})\boldsymbol{n}^\sharp$ are the axial and the transverse component of $\boldsymbol{w}$, respectively. By construction, $\boldsymbol{w}_t$ is an element of the tangent bundle of the $(n-1)$-dimensional manisold $s \subset \mathcal{S}$. Hence, due to linearity, the $(n-1)$-form $\boldsymbol{\omega} = \boldsymbol{\iota_w \mu}$ can be written as

$$\boldsymbol{\omega} = \boldsymbol{\iota_w \mu} = \boldsymbol{\iota}_{(\boldsymbol{w}_a + \boldsymbol{w}_t)}\boldsymbol{\mu} = \boldsymbol{\iota}_{\boldsymbol{w}_a}\boldsymbol{\mu} + \boldsymbol{\iota}_{\boldsymbol{w}_t}\boldsymbol{\mu}. \tag{C2}$$

Let now $\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{n-1}\} \subset Ts$ be a set of linearly indepdendent vectors spanning $T$s. Since $\boldsymbol{w}_t$ can be expressed as a linear combination of $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{n-1}$, we obtain

$$(\boldsymbol{\iota}_{\boldsymbol{w}_t}\boldsymbol{\mu})(\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{n-1}) = \boldsymbol{\mu}(\boldsymbol{w}_t, \boldsymbol{u}_1, \ldots, \boldsymbol{u}_{n-1}) = 0. \tag{C3}$$

Comparing Eq. (C3) with the definition of $\boldsymbol{\omega}$ in Eq. (C2), we find

$$\boldsymbol{\omega}(\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{n-1}) = (\boldsymbol{\iota_w \mu})(\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{n-1}) = (\boldsymbol{\iota}_{\boldsymbol{w}_a}\boldsymbol{\mu})(\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{n-1}), \tag{C4}$$

and, since (C4) must hold true for all $(n-1)$-tuples $\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{n-1}\} \subset Ts$, we can write

$$\boldsymbol{\omega} = \boldsymbol{\iota_w \mu} \equiv \boldsymbol{\iota}_{\boldsymbol{w}_a}\boldsymbol{\mu}, \tag{C5}$$

i.e., only the axial component of $\boldsymbol{w}$, which is the componet parallel to the vector $\boldsymbol{n}^\sharp$ associated with the normal covector $\boldsymbol{n}$ to the hypersurface $s$, contributes to $\boldsymbol{\omega}$. Finally, by exploiting the result $\boldsymbol{w}_a = (\boldsymbol{nw})\boldsymbol{n}^\sharp = (\boldsymbol{wn})\boldsymbol{n}^\sharp$ and the linearity of the interior product, Eq. (C5) becomes

$$\boldsymbol{\omega} = \boldsymbol{\iota_w \mu} \equiv \boldsymbol{\iota}_{\boldsymbol{w}_a}\boldsymbol{\mu} = (\boldsymbol{wn})\boldsymbol{\iota}_{\boldsymbol{n}^\sharp}\boldsymbol{\mu} = (\boldsymbol{wn})\boldsymbol{\alpha}, \tag{34 corr.}$$

where

$$\boldsymbol{\alpha} = \boldsymbol{\iota}_{\boldsymbol{n}^\sharp}\boldsymbol{\mu} \tag{1}$$

is the $(n-1)$-form induced on the hypersurface $s$ by the volume form $\boldsymbol{\mu}$ and the metric $\boldsymbol{g}$. Therefore, on the basis of these results, the flux of an extensive quantity $q$ across the hypersurface $s$ can be expressed in the alternative notation [...]

## References

1. S. Federico, A. Grillo, R. Segev, Material description of fluxes in terms of differential forms, Continuum Mechanics and Thermodynamics, **28**(1-2), 379-390 (2016) https://doi.org/10.1007/s00161-015-0437-2