Analysis of the Surface Impedance of a Sinusoidally Modulated Metasurface

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***Abstract*—Metasurfaces have been extensively exploited in recent years for mantle cloaking applications. In this type of problems it is of fundamental importance to determine the connection between the metasurface geometrical parameters and the realised value of surface impedance, in order to properly design the metasurface. In this paper the surface impedance of a non homogeneous metasurface, based on a sinusoidally modulated metallic pattern is analysed.**

***Index Terms*—Cloaking, metamaterials, periodic structures, surface impedance**

I. Introduction

Matamaterials and metasurfaces have been widely used in last years for a vast range of applications such as leaky wave antennas [1], lenses [2], or cloaking [3]. Thanks to the periodic arrangement and the sub-wavelength dimensions of the unit cell, a metasurface can be described by an equivalent value of surface impedance. This approach is widely used for example in mantle cloaking problems [4]. In particular, in such analysis, the first step is usually to compute the appropriate value of the surface impedance depending on the geometry, material and dimensions of the object to conceal and on the frequency of operation. As a second step, a suitable metasurface is designed to match the surface impedance requirements.

A fundamental information is therefore the connection between the geometrical parameters of the chosen metasurface and the realised value of surface impedance. For some canonical unit cells as for example patches, grids or stripes, analytical models have been developed [5], [6] usually in the limit of a low frequency regime.

In this framework, in this paper the impedance of a metasurface based on a sinusoidal pattern is numerically analysed in order to provide a relation between the different design parameters and its surface impedance value.

II. Problem Description

Consider the mantle-cloaking problem of an infinitely long metallic cylinder with radius a covered by a dielectric layer of thickness $t = b - a$ (the same considerations can be carried out for a dielectric cylinder). The cylinder is illuminated by a linearly polarized plane wave with the electric filed component parallel to the cylinder axis (i.e. TM polarized).

The tangential component of the total electric field in the region outside the cylinder, i.e., in the background medium, can be expressed in polar coordinates (ρ, φ) as an infinite sum of harmonics in the form:

$$E_z(\rho, \varphi) = \sum_{n=-\infty}^{\infty} j^{-n} g_n(k_0\rho) e^{jn\phi}, \qquad \rho > b,$$

where

$$g_n(k_0\rho) = J_n(k_0\rho) + c_n H_n^{(2)}(k_0\rho)$$

and J_n and $H_n^{(2)}$ are the Bessel and Hankel functions, which describe the incident (plane wave) and the scattered field, respectively, c_n are the scattering coefficients, and k_0 is the wavenumber in the background medium.

Mantle cloaking consists of reducing the scattering field so that the total field is equal to the incident one in all directions around the object.

A general approach to solve the problem of mantle cloaking may be as follows. In order to cancel a major portion of the scattered field, the coefficients c_n, $n = 0, \pm 1, \pm 2, \dots, \pm N$, that provide the greatest contribution should be set equal to zero, which is generally impossible. Since $\{c_n\}$ depend on all the problem parameters the cancellation can be achieved by finding an appropriate set of parameters such that a certain criterion of cloaking be fulfilled To this end, by setting an infinitely thin impedance boundary surface on the external boundary dielectric-vacuum (the object interface) [4], the complex impedance of this surface can be applied as a control parameter. Introduction of such a cloaking surface results in a boundary value problem with the corresponding impedance boundary condition imposed at the object interface which can be solved in terms of generalized Watson series in cylindrical functions. Therefore, as an appropriate cloaking criterion it can be chosen the minimization with respect to complex surface impedance Z_S of the quantity:

$$\mathcal{C}_N^D(Z_S) = \sum_{n=-N}^{N} |c_n|^2 \,, \tag{1}$$

which is approximately the total scattering width (SW)

$$\sigma^D = \sum_{n=-\infty}^{\infty} |c_n|^2 \tag{2}$$

of the scattering object (a covered cylinder).

With this problem formulation, it is possible to determine a homogeneous value Z_S of the surface impedance which cancels only one, principal, harmonic of the scattered field. Such a cancellation may be possible because one can determine the required value $Z_S = Z_S^*$ by solving the equation $c_0(Z_S) = 0$.

This result is effective if the cylinder is electrically small with respect to the wavelength of the incident plane wave. In fact, in this case, the scattered field is primarily dominated by the first harmonic (with modal index $n = 0$). Therefore, cancelling the c_0 coefficient leads to a significant reduction of the scattered field.

However, when the cylinder has a diameter comparable to the wavelength of interest several different harmonics contribute to the scattered field; thus a non homogeneous value of surface impedance, or multiple impedance layers, is required to achieve a cloaking effect.

In [7] a homogeneous surface impedance cloaking layer is considered, showing the effective cancellation of one scattering harmonic and the consequent reduction of the scattered field, while in [8] the cloaking of non-small conductive cylinder is discussed showing how a minimum of the scattered field can be achieved with both inductive and capacitive surface impedances depending on the cylinder and the dielectric radius.

In this framework, here a modulated metasurface is proposed. The profile of the metallization within the unit cell, represented in Fig. 1, is described by:

$$W(x) = W_{min} + (W_{max} - W_{min}) \left(\sin \frac{\pi x}{D_u} \right)^{\alpha} \qquad (3)$$

where W_{max} and W_{min} are the maximum and the minimum line width, respectively, D_u is the cell length, and α is the constant which determines the cell modulation. Due to the 1D

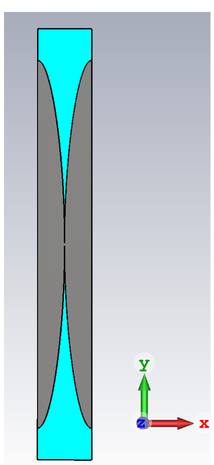

Fig. 1. Geometry of unit cell with a modulated profile.

modulation, the metasurface has a non-homogeneous structure; therefore, it will give rise to a non homogeneous local value of surface impedance.

The entries of the surface impedance tensor matrix for the planar case, used to characterize the non-homogeneous nature of the surface, are obtained by a numerical analysis with a commercial numerical tool and they will be used as first guess for the non-linear analysis for the conformal case.

This analysis is a starting point for an analytical formulation of the surface impedance tensor with respect to the unit cell geometrical parameters, and therefore to the metasurface design both in the planar and in the conformal cases, similarly to the investigation formulated for example in [9].

III. Surface Impedance Results

In the following, the input impedance analysis of a planar infinite metasurface based on the unit cell described in previous section is presented. Both TM and TE excitations have been considered.

The fixed parameters of the unit cell are: D_u = 4 mm, W_{min} = 0.2 mm, while different values for W_{max} and α are considered. Therefore, only the first two floquet modes have been considered.

The equivalent circuit of this structure, represented in Fig. 2, consists of a short-circuited transmission line in parallel with the unknown metasurface impedance Z_S.

The input impedance Z_{in} is computed with the simulation software CST Microwave Studio [10], while the short-circuited line impedance is $Z_{sc}' = Z_d j \tan(k_d z)$, where $z = 2.9$ mm represents the dielectric thickness, $Z_d = \frac{Z_0}{\sqrt{\varepsilon_r}}$ and $k_d = k_0\sqrt{\varepsilon_r}$ are the characteristic impedance and wavemumber of the dielectric layer, respectively, and ε_r = 2.3 is the dielectric relative permittivity.

Considering the problem in terms of admittances:

$$Y_{in} = Y_S + Y_{sc}' \qquad \longrightarrow \qquad Y_S = Y_{in} - Y_{sc}' \qquad (4)$$

Therefore,

$$Z_S = \frac{Z_{in} Z_{sc}'}{Z_{sc}' - Z_{in}} \qquad (5)$$

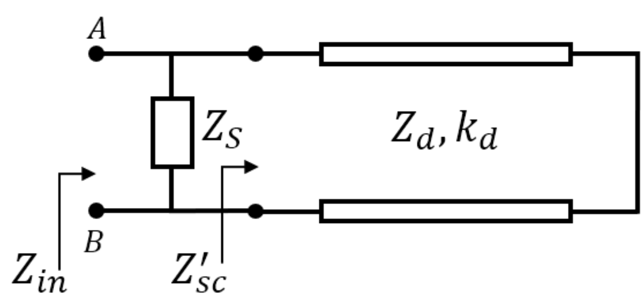

Fig. 2. Equivalent circuit of the planar metasurface structure.

In Fig. 3 the surface impedance tensor $\underline{\underline{Z_s}}$ for TM and TE excitation is shown for α = 0.5 and W_{max} = 27.3 mm. Note that the surface impedance tensor is diagonal because impedances for TE and TM excitations are uncoupled.

$$\underline{\underline{Z_S}} = \begin{bmatrix} Z_{TM} & Z_{TMTE} \\ Z_{TETM} & Z_{TE} \end{bmatrix} \qquad (6)$$

The surface impedances are compared for different values of α and W_{max} while the other design parameters are D_u = 4 mm, W_{min} = 0.2 mm and the transverse dimension of the unit cell is set to D_v = 32 mm.

In Fig. 4 the impedance for α = 0.5 and α = 3 are compared (W_{max} = 27.3 mm). It can be noticed that the variation of α especially influences the impedance seen in the case of TE excitation.

In Fig. 5 the results for W_{max} = 27.3 mm and W_{max} = 15.6 mm (α = 0.5) are reported. A decrease of W_{max} results into a lower impedance Z_S. This information can be exploited for example in mantle cloaking problmes of metallic cylinders, in which for a fixed dimension of the cylinder radius, lower surface impedance Z_S is required with the increasing of frequency.

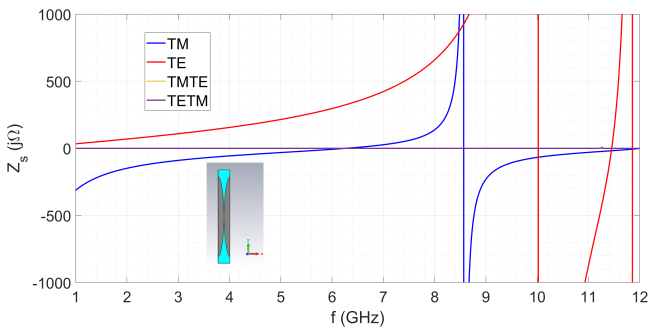

Fig. 3. Metasurface impedance for TM and TE incidence, W_{max} = 27.3 mm and α = 0.5.

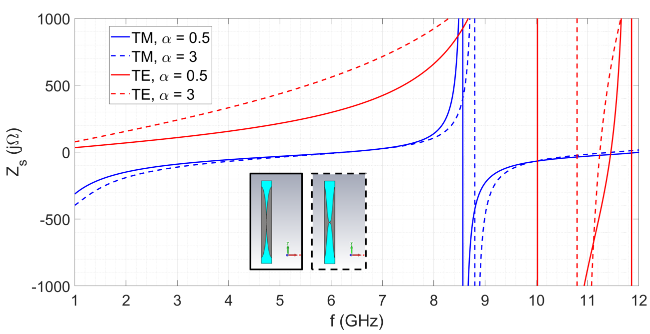

Fig. 4. Comparison of the diagonal terms of the metasurface impedance tensor for α = 0.5 and α = 3, W_{max} = 27.3 mm.

CONCLUSIONS

In this paper the surface impedance of a metasurface characterised by a sinusoidally modulated cell has been investigated.

The possibility to separately change the longitudinal (by W_{max}) and azimuthal (by α) components of the surface impedance tensor simplifies its analytic analysis.

If the surface impedance is characterized by several ($M = 2l, l \geq 1$) complex quantities forming an impedance parameter vector $\mathbf{Z}_S = (Z_{S1}, \ldots, Z_{SM})$, then one can determine (mathematically correct) an "optimal" parameter set $\mathbf{Z}_S = \mathbf{Z}_S^*$ by solving the equation system $\{c_j(\mathbf{h}) = 0\}_{j=-l}^{l}$.

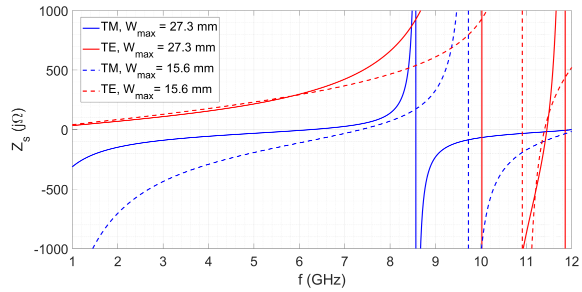

Fig. 5. Comparison of the diagonal terms of the metasurface impedance tensor for W_{max} = 27.3 mm and W_{max} = 15.6 mm, α = 0.5.

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