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Efficient numerical evaluation of transmission loss in homogenized acoustic metamaterials for aeronautical application

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Abstract

This work wants to investigate the soundproofing level of passive acoustic metamaterials made of Melamine Foam and cylindrical Aluminum inclusions. Latest research shows promising acoustical possibilities on controlling certain frequencies, varying their geometry or material configuration. Typically, acoustic metamaterials are plates with inclusions that resonate in low frequencies and block basses. Investigation is carried to use them for the improvement of aircraft cabin quietness. An homogenization method is adopted to study the metamaterial: this is modelled as viscoelastic material with inclusions, using a frequency-dependent approach, and the effective properties of the homogenized metamaterial are derived using a micromechanics-based method that allow us to prescribe the simplest mesh for periodical geometries (in this case, cylindrical) by reducing drastically the computation time. MSC Actran is used for vibro-acoustic simulations, in particular for the evaluation of Sound Transmission Loss in metamaterial panels with different volume fractions and in a sandwich panel with metamaterial core. This last has been compared with a classical sandwich panel used in aeronautics and it has been demonstrated that metamaterial gets higher transmission loss over all the frequency range.

1 Introduction

This paper aims to investigate innovative materials, namely metamaterials, suitable for the fuselage of aircraft in order to reduce the problem of noise and vibrations in the cabin [1, 2]. Aeronautical regulations, like FAR and EASA, mainly dictate safety standards, although a part of them is also devoted to environmental noise aspects [3]. Hence, interior noise requirements in civil aviation mainly derive from airline requests [4], which are based on passengers and cabin crew subjective response [5]. Since new products are always expected to have improved technical characteristics in order to compete on the market, the noise problem is nowadays attacking also small aircraft with classical configurations because of their lower technological level compared to big airplanes and the stringency of the aeronautical rules which become with the time more sensitive to noise aspects [6, 7].

Acoustic treatments, that are all the technical solutions installed on board to increase the noise reduction through the fuselage wall or to control the internal noise sources, have a fundamental importance to define the internal noise requirements [8]. Some technologies proposed in the past are resumed in the work by Dobrzynski [9]: thermo-acoustic blankets, skin damping, furnishing panels, mufflers and active noise control systems may be regarded as noise treatments. These need to be optimized taking into account different parameters, particularly the weight, the footprint and the cost; an example is given by the honeycomb acoustic metamaterial proposed by Sui et al. [10], which possesses lightweight and yet sound-proof properties. For these reasons, acoustic metamaterials are analyzed in this paper.

In the last decade, a new research field has emerged to study Metamaterials [11]. This term refers to materials whose properties are "beyond" those of conventional materials. They are made from assemblies of multiple elements fashioned from composite materials such as metals, foams or plastics. The core concept of metamaterial is to replace the molecules with man-made structures called unit cell. They can be viewed as "artificial atoms", usually arranged in repeating patterns on a scale much smaller than the relevant wavelength of the phenomena they influence. Several types of metamaterials can be found: electromagnetic metamaterials [12, 13, 14, 15], mechanical metamaterials [16, 17, 18]. Some are artificial three-dimensional structures which, despite being a solid, ideally behave like a fluid. Thus, they have a finite bulk modulus but vanishing shear modulus, ie. they are hard to compress yet easy to deform. And finally, acoustic metamaterials [19, 20, 21], whose effective properties like compressibility or density can be negative. Negative density or compressibility can only be achieved dynamically. For instance, Helmholz resonators driven just above their frequency of resonance lead to negative dynamic compressibility [22]. According to the same principles of wave propagation in periodic structures [23, 24, 25], acoustic metamaterials are tuned to the acoustic wavelength and can be categorized into non-resonant and resonant materials. Resonant metamaterials are generally heterogeneous materials containing a periodic arrangement of elements smaller than the acoustic wavelength: by selectively tuning the material properties of the metamaterial, the elastic or acoustic behavior can be significantly altered from conventional material properties. Resonant metamaterials can be conveniently applied to aircraft interior, airframe noise in naval vessels, and controlling noise in automobiles as the order of magnitude of the wavelength is 1 m and this is much greater than the reasonable thickness of classical damping materials. Work in studying the properties of a heterogeneous material has been carried out at Virginia Tech in the past two decades [26], and has evolved into what is now termed a heterogeneous (HG) metamaterial that will be described better in the following section.

These metamaterials derive their properties not from the properties of the materials they are composed of, but from their newly designed structures with repeating patterns, hence they need to be 'homogenized'. Indeed, if it is possible to treat them like homogeneous materials with effective properties, their analysis becomes faster and more convenient if the finite element softwares are used.
Finite Element Method (FEM) is well-established and yields accurate results for the structural analysis of any geometrical shape. However, it requires a mesh of all the details of the constituent material. Therefore, when dealing with plates having great numbers of inclusions - such as metamaterials, this method becomes very costly in calculations and time, especially when the macroscopic dimensions of the plate need to be much greater than the characteristic size of the inclusions. Thus, some homogenization methods have been investigated on the last two decades [27, 28]. In particular, Langlet et al. [29] have studied homogenization of passive periodic materials such as a plate periodically perforated across its thickness. In this work, a homogenization method based on the Carrera Unified Formulation (CUF) [30] and Mechanics of Structure Genome (MSG) [31] is investigated. CUF is used to solve the governing equations of MSG for periodically heterogeneous materials. The MSG provides a tool to obtain the complete effective stiffness matrix in a straightforward manner without relying on ad-hoc assumptions and minimizing the loss of information between the original heterogeneous cell and the equivalent homogeneous body. This CUF-MSG based homogenization method has been successfully used to find the homogenized mechanical properties of composites [32] and periodic structures [33], in which the unit cell is a cube containing a cylindrical inclusion and the matrix surrounding it.

This research seeks to improve upon the scientific literature by further investigating a heterogeneous material and developing an efficient finite element model to evaluate the acoustic performances of metamaterials at large scale. This concept is unique from previous studies of HG material in which a periodic arrangement of masses within a poro-elastic material is investigated. Furthermore, these numerical finite element models will allow for an advanced understanding of the physics behind the material functionality and they will be used for conducting parametric studies in order to develop more advanced designs and efficient manufacturing of acoustic metamaterials for specific applications.
In particular, this paper presents the results obtained by numerical simulations performed with Actran, an MSC Software based on the finite element method. This is a powerful tool for the acoustic and vibroacoustic analysis of complex structures, accounting for various geometries, load conditions and materials. Moreover, this software allows different types of analyses to be performed, which have been validated through many applications presented in Workshop Series for Actran 17, Acoustics and Vibroacustics Training [34]. A specific model has been created to evaluate the transmission loss properties of the metamaterial. Transmission Loss is one of the key metrics in evaluating acoustic performances of a material and it quantifies the material's ability to reflect or block sound energy. If transmission loss of a material is high, the sounds emitted on one side of the material tend to stay on this side, and not be heard on the other one, thus the material acts as a barrier for sound. An extensive overview on models for the evaluation of sound transmission loss of panels and comparisons with experimental results is provided in the paper [35] by De Rosa etal.. In this work and other companion articles

[36, 37, 38, 39], authors highlight the importance to build good predictive numerical models for virtual simulation of sound transmission loss test in panels with complex geometry or material configurations, such as laminated composites and sandwich panels.
By using Actran model, the effective properties of metamaterial are validated by comparing the transmission loss of the homogenized plate with the results of the heterogeneous one. Then, some studies are performed to investigate the effect of some design parameters on the acoustic performances of HG metamaterial. Finally, the acoustic performances of a composite sandwich panel with metamaterial core will be evaluated with respect to classical sandwich solutions with Nomex core, by keeping the same weight, in order to demonstrate the potentiality of this material as soundproofing insulator for aeronautical applications. Since the characteristic wavelength of noise sources in aeronautics is usually very high, the response of metamaterial is analyzed in low-frequency range (0-500 Hz).

2 Heterogeneous metamaterial

A heterogeneous (HG) metamaterial is a new class of acoustic metamaterial. It is defined as a composite system consisting of multiple small masses embedded within a passive poro-elastic matrix material. The embedded masses create an array of resonant mass-spring-damper systems within the material that operate at low frequencies where the passive poro-elastic material is no longer effective. By employing the poro-elastic material to provide the stiffness for the embedded masses, the HG metamaterial utilizes two passive control schemes: damping at high frequencies, and dynamic absorption at low frequencies, into a single device for broadband noise reduction. The displacement of the masses against the foam stiffness at their low frequency resonance leads to an increase in mechanical damping losses and absorption. An increased effect of the embedded mass on the poro-elastic material is due to a mismatch in the impedance between the two materials. For optimum absorption a larger impedance mismatch is desired [40].
HG metamaterials can be used for controlling low frequency sound radiation, improving low frequency transmission loss when attached to vibrating structures, and is a lighter and thinner replacement to conventional materials [41, 42]. These materials have shown to significantly reduce interior noise with only a marginal increase in the overall mass of the structure. It has been demonstrated that HG metamaterials can be used as lightweight blanket treatments for effectively controlling low frequency sound radiating from structures [43, 44]. Kidner et al. concluded that HG metamaterial is more efficient when placing the masses to target certain modes by varying the depth, weight, or shape [45]. Proper tuning will result in a mode split of the targeted resonance into two damped peaks above and below the original peak [46]. It was also demonstrated that porous materials having porous inclusions, called composite porous materials, show increased performance in sound absorption and sound insulation [47].
Figure 1 illustrates the arrangement of periodically distributed masses acting as a series of mass-spring-damper systems embedded in a poro-elastic material. There is a wide range of applications available for poro-elastic HG metamaterials. One broad application includes the placement of these materials on aircraft for the damping of sound and vibration. This application is largely dictated by the choice of the poro-elastic matrix material and the material of inclusions.

2.1 Identification of material constituents

The acoustic metamaterials here investigated are made of a frame of poro-elastic material with good acoustic properties in the high-frequency domain and a periodic array of cylindrical inclusions integrated inside the foam as shown in Figure 2.
An investigation is carried out to find materials that obey several criteria according to aviation standards, namely:

- Excellent sound-transmission loss properties in the widest frequency range

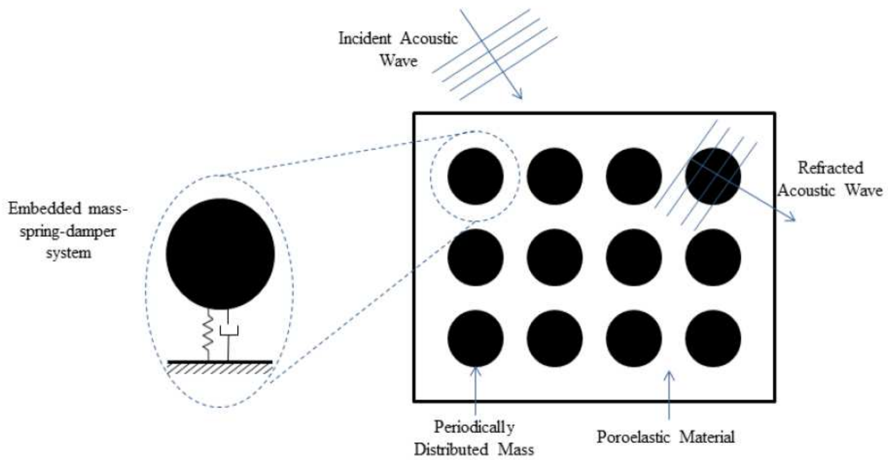


Figure 1: Heterogeneous metamaterial panel with cylindrical through-the-thickness inclusions.

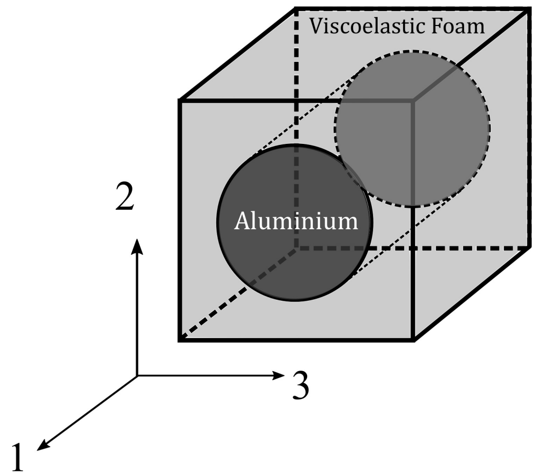


Figure 2: Periodic Unit Cell of the Metamaterial with matrix of viscoelastic foam and cylindrical inclusions of Aluminium.

- Light
- Fire-repellent
- Good strength
- Easy to manufacture
- Cost-effectiveness

The poro-elastic material chosen for the frame is Melamine foam because of its lightness, sound absorption and fire-repellant properties, while Aluminium has been taken for the inclusions because of its good stiffness and mismatch with the density of the foam. By adding the Aluminium cylinders, we aim at increasing the damping properties of the melamine in the low-frequency range (approximately $0 - 500$ Hz) without increasing the thickness of the materials and adding as little weight as possible. In the following, melamine is modelled as viscoelastic material and the definition of its frequency-dependent properties is provided.

2.2 Frequency-dependent parameters for melamine foam

Viscoelastic problems can be treated both in the time and frequency domains. The frequency representation focuses on the material's response to cyclic loads at various frequencies. It helps condensate the material's response through a parameter called "complex modulus of elasticity" which is similar to Young's modulus in linear elasticity, except that it depends on the excitation frequency applied to the material.

In viscoelastic materials, for low amplitudes of excitation (which is our case in aeronautics), the stress response $\sigma(t)$ to a sinusoidal strain excitation $\epsilon(t)$ will also have a sinusoidal shape, of same frequency but with a phase delay of angle δ. We can therefore write :

$$\epsilon(t) = \epsilon_m \cos(\omega t) = Re(\epsilon_m e^{i\omega t}) = Re(\epsilon^*(t))$$

$$\sigma(t) = \sigma_m \cos(\omega t + \delta) = Re(\sigma_m e^{i(\omega t + \delta)}) = Re(\sigma^*(t))$$

where complex numbers are identified by an asterisk *. Then it is possible to keep the formalism of Hooke's law $\sigma = E\epsilon$ by introducing the complex elastic modulus E^*:

$$E^* = \frac{\sigma^*(t)}{\epsilon^*(t)} = \frac{\sigma_m}{\epsilon_m} e^{i\delta} = E' + iE"$$

where :

$$E' = Re(E^*) = \frac{\sigma_m}{\epsilon_m} \cos(\delta)$$

and

$$E'' = Im(E^*) = \frac{\sigma_m}{\epsilon_m} \sin(\delta)$$

Because of the linearity of the behavior law, the complex modulus, defined as above, depends on the amplitude of excitation, and depends also on the excitation frequency: $E^*(\omega)$. One can also rewrite the delayed response $\sigma(t)$ with the real and imaginary part of the modulus of elasticity and trigonometric formulas :

$$\sigma(t) = \sigma_m \cos(\omega t + \delta) = \sigma_m[\cos(\omega t)\cos(\delta) - \sin(\omega t)\sin(\delta)] = \epsilon_m[E' \cos(\omega t) - E" \sin(\omega t)]$$

The real part E' is called Storage Modulus and corresponds to the part of the response which is in phase with the excitation. The imaginary part $E"$ is the loss modulus. It corresponds to the part of the response with a phase of $+90°$, as the derivative of a sinusoidal function. The phase angle δ (in Radian) is the Loss Angle and its tangent :

$$\tan(\delta) = \frac{E"}{E'} = \eta$$

is called the loss factor or damping factor. It gives an indication about the percentage of energy with is dissipated in each cycle by the viscoelastic losses.

Completely analogous definitions apply for the shear modulus. For common structural materials the loss factor is usually constant with frequency, but viscously damped systems or the behavior of viscoelastic materials need to be represented by a frequency-dependent loss factor.

A literature study has been carried to find frequency-dependent complex engineering moduli for melamine foam. Two articles have been found relevant: the first one, Jaouen [48], mainly uses experimentation on a block of melamine foam to obtain real and imaginary parts of E_1, E_2, E_3, ν_{23} and G_{23}. Anyway, some moduli are missing and the frequency-range studied is sometimes too narrow.

Another article, Cuenca [49] provides an easy numerical approach to the frequency dependance of Melamine foam, based on parameters that are identified through experiments on a real cube of melamine foam. Following instructions and data given in this paper, its results have been reproduced in Matlab. This straightforward method lies on the assumption that the stiffness matrix can be seen as the sum of a static term - corresponding to the material's fully relaxed (elastic) state, and a complex, frequency-dependent term related to the (anelastic) relaxation phenomenon. In general, a material can present different types of anisotropy for the elastic and anelastic phenomena, but [49] assumes that the elastic and anelastic parts of the stiffness matrix are colinear, thus simplifying the frequency dependency for the stiffness matrix H_{ij}:

$$H_{ij}(\omega) = C_{ij}\left(1 + \frac{b(i\omega/\beta)^{\alpha}}{1 + (i\omega/\beta)^{\alpha}}\right) \tag{1}$$

with α, β, b and C_{ij} parameters that are given in [49]. The equivalent frequency-dependent moduli are then extracted from the stiffness matrix H_{ij} which is assumed to be in the following symmetric form:

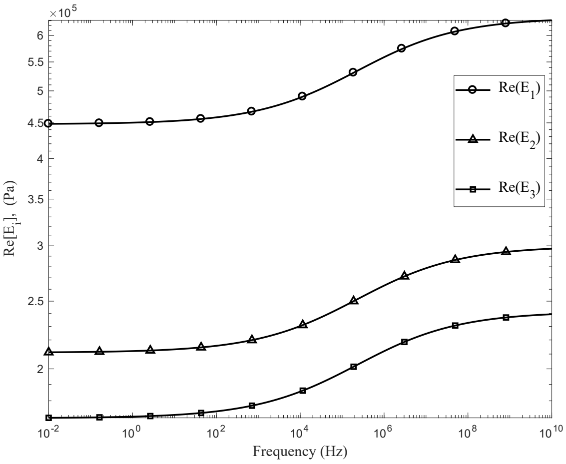
$$H_{ij}(\omega) = \begin{bmatrix} \frac{1}{E_1(\omega)} & -\frac{\nu_{12}(\omega)}{E_2(\omega)} & -\frac{\nu_{13}(\omega)}{E_1(\omega)} & 0 & 0 & 0 \\ & \frac{1}{E_2(\omega)} & -\frac{-\nu_{32}(\omega)}{E_3(\omega)} & 0 & 0 & 0 \\ & & \frac{1}{E_3(\omega)} & 0 & 0 & 0 \\ & & & \frac{1}{G_{23}(\omega)} & 0 & 0 \\ & (sym) & & & \frac{1}{G_{13}(\omega)} & 0 \\ & & & & & \frac{1}{G_{12}(\omega)} \end{bmatrix}^{-1} \tag{2}$$

Results using the method of Cuenca et al. [49] are compared to experimental results of Jaouen et al. [48] in Figure 3. The results do not match exactly because the two articles are not comparing the same Melamine foam cubes. Samples come from different labs so the porosity, density and manufacturing process could be very different. However we can focus on comparing behavior of the mechanical properties for real and imaginary parts, which most of the time are the same. It can also be noticed that the behavior of melamine foam is transversely isotropic. Poisson's Ratio are real and constant with the Cuenca model [49]. This is consistent with Jaouen et al. experiments. Values obtained in Cuenca [49] are $\nu_{12,C} = 0.45, \nu_{13,C} = -0.514, \nu_{23,C} = 0.433$. These values also seem to confirm the transversely isotropy of Melamine Foam according to direction 1. Jaouen et al. find $\nu_{23,J} = 0.44$ through experiments, with direction 2 and 3 in their own coordinate system. Values of $\nu_{23,J}$ and $\nu_{23,C}$ are very close, however direction 2 in Jaouen paper coincide with direction 1 in Cuenca coordinate system. Indeed, melamine seems transversely isotropy according to direction 1 in Cuenca, and transversely isotropic according to direction 2 in Jaouen. A discrepancy still remain between ν_{23} in Jaouen and ν_{13} in Cuenca but it can be reasonably due to the diversity of two samples.

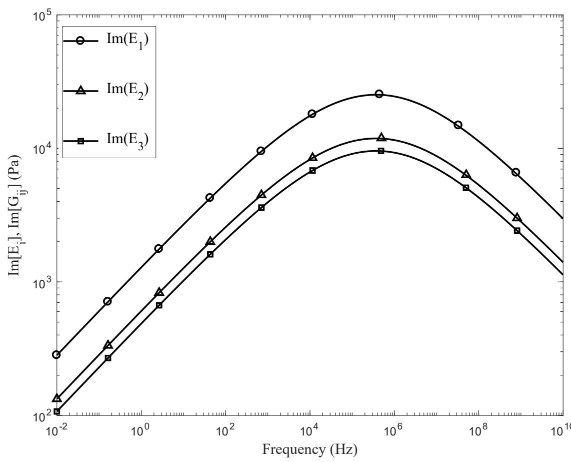
3 Homogenization of metamaterial properties

The homogenization method here adopted is based on a novel higher-order component-wise beam theory in the framework of the Carrera Unified Formulation (CUF) and yields good results for composites with few calculations [32, 33]. To overcome the limitations of classical models and to deal with complex phenomena, such as torsion, warping, or in-plane deformation, the displacement fields of beam theory are enriched with an arbitrary number of higher-order terms.

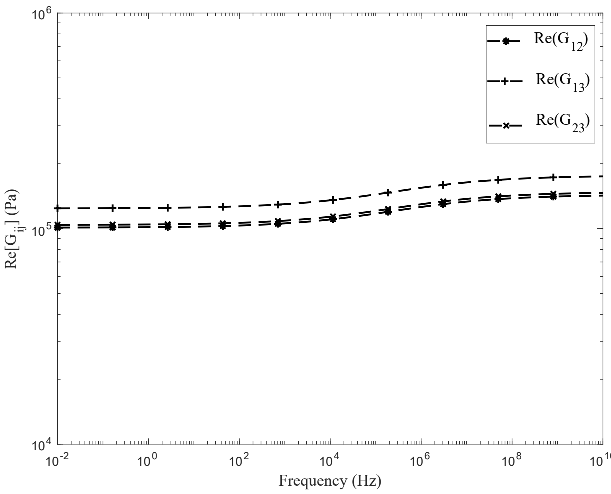
The main feature of this method, leading to study acoustic metamaterials, is the concept of the *Repeating Unit Cell* (hereinafter RUC), just like other homogenization methods for metamaterials (Parallel Transfer Matrix Method, [50]). Moreover it is fast and simple to use, as it is able to homogenize the



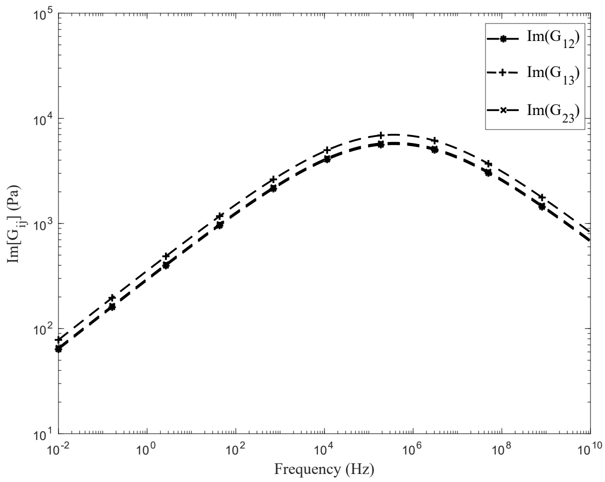
(a) $\mathrm{Re}(E_i)$



(b) $\mathrm{Im}(E_i)$



(c) $\mathrm{Re}(G_{23})$



(d) $\mathrm{Im}(G_{23})$

Figure 3: Equivalent frequency-dependent material moduli Real (a),(c) and Imaginary (b),(d) based on [49] Cuenca model.

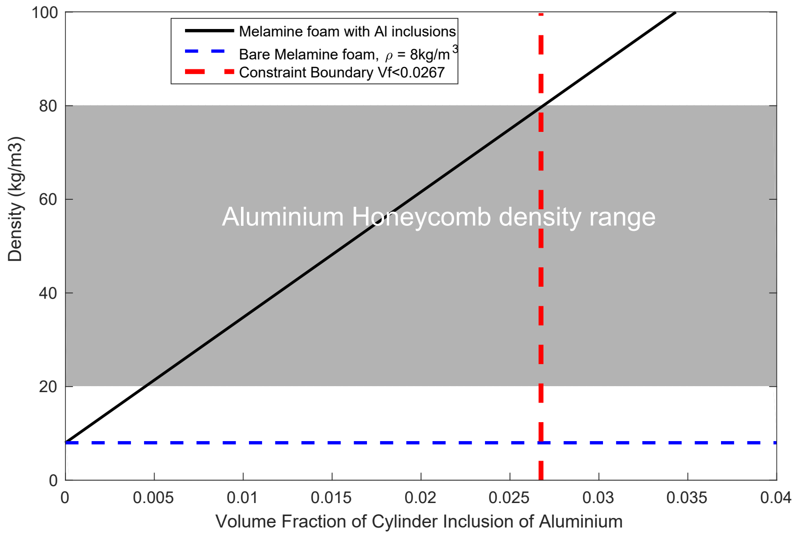


Figure 4: Volume Fraction of Aluminium cylinders within a matrix of Melamine Foam with a density of 8 kg.m^{-3}. In grey, honeycomb Aluminium usual density range (20-80kg.m^{-3}).

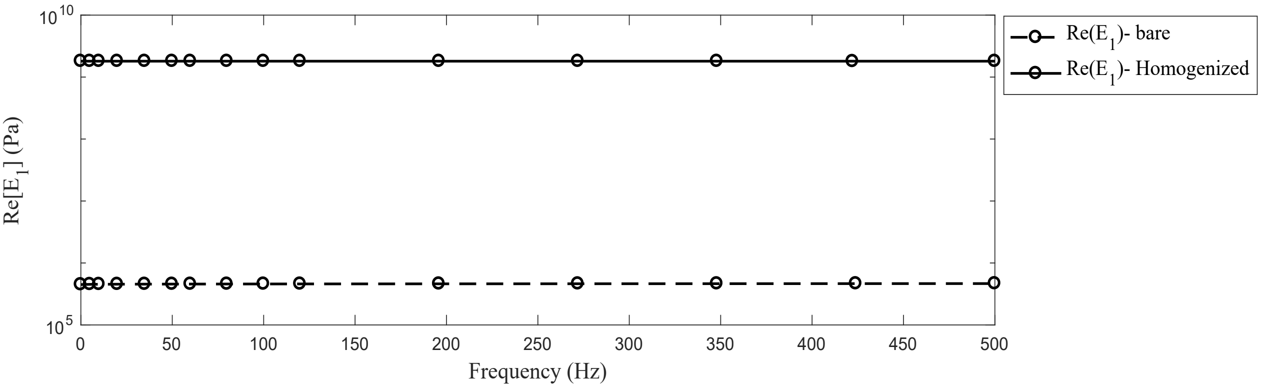
material only by knowing the unit cell geometry and the material properties of its components.
These capabilities are enabled by the use of the Mechanics of Structure Genome (MSG), developed by Yu et al. [31] as a unified theory for the study of multiscale structural problems. MSG is a highly efficient tool to obtain the complete effective stiffness matrix of heterogenous materials in a straightforward manner without relying on ad-hoc assumptions. In particular, the method lays on the concept of Structure Genome (SG), defined as the smallest mathematical building block of the structure, which for the purposes of the present study is equivalent to the Unit Cell.
The mathematical foundation of MSG is the Variational asymptotic method (VAM), introduced by Berdichevskii [51] for the study of periodic systems. VAM can be used for the analysis of stationary value problems in which certain terms are smaller than others. This method is well-known in the mechanics field, where problems featuring heterogeneities at different scales are very common. For instance, in beam structures, the cross-section is usually smaller than the length, and in shell problems the thickness is often negligible in comparison to the global dimensions. In the case of metamaterials and composite materials, VAM is a powerful mathematical tool due to the several geometrical scales that must be accounted in the analysis.

In MSG, the constitutive information is extracted from the RUC by minimizing the difference in terms of elastic energy between the original heterogeneous cell and the equivalent homogeneous material, which can be written as the following functional:

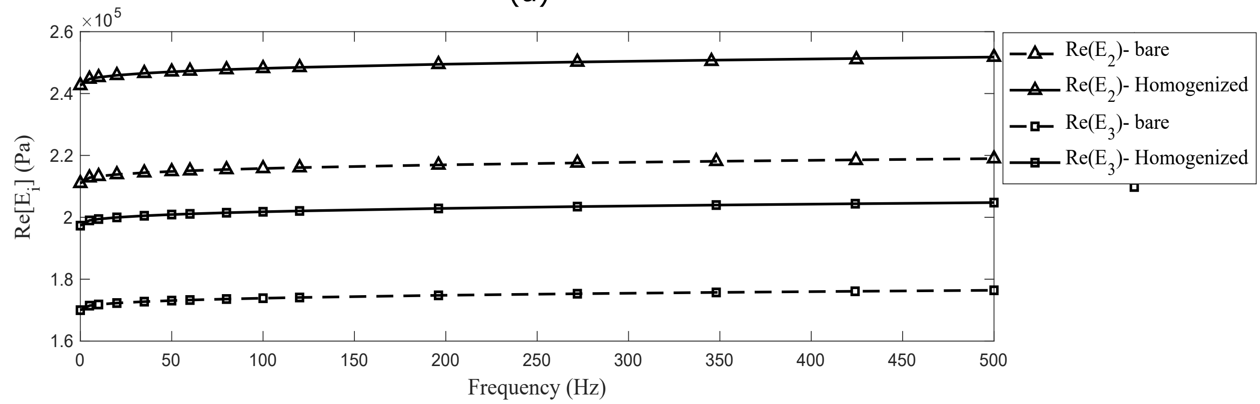
$$\Pi = \left\langle \frac{1}{2} C_{ijkl}\, \varepsilon_{ij}\, \varepsilon_{kl} \right\rangle - \frac{1}{2} C^{*}_{ijkl}\, \bar{\varepsilon}_{ij}\, \bar{\varepsilon}_{kl} \tag{3}$$

where $\langle \bullet \rangle$ denotes the volume average $\frac{1}{V}\int_V \bullet\, dV$. The first term of the functional Π is the strain energy of the heterogeneous body, whereas the second term is the strain energy of the homogeneous material. C_{ijkl} is the fourth-order elastic tensor and ε_{ij} is the second-order strain tensor. C^{*}_{ijkl} and $\bar{\varepsilon}_{ij}$ are those of the homogenized body.

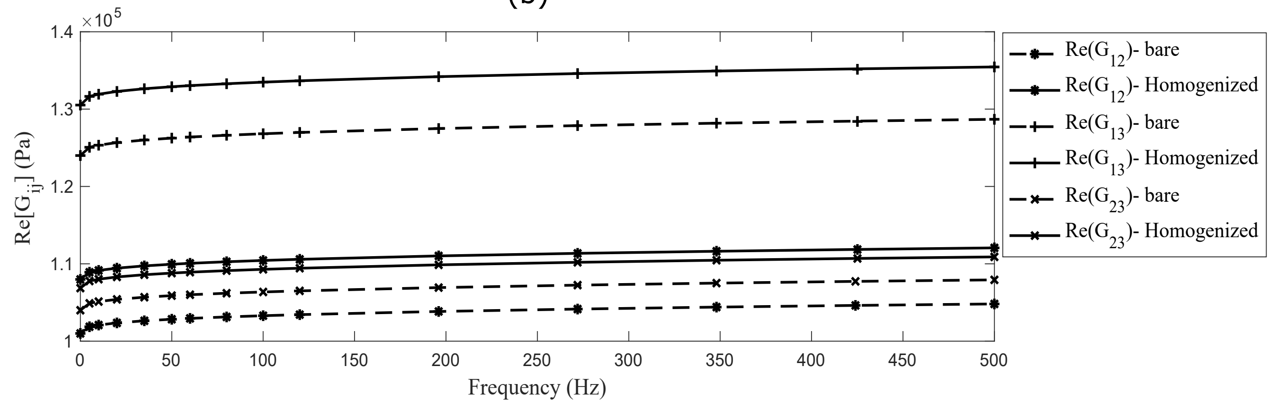
The periodic unit-cell to be homogenized is geometrically similar to the one in the previous section, as shown in Figure 2. The cylindrical inclusion is made of Aluminium with isotropic properties $E = 6.75 \times 10^{10}$ Pa and $\nu = 0.34$. The density of Aluminium is $\rho_{Al} = 2700$ kg.m^{-3}.
Since this material has been conceived to be used as core of the sandwich lining panels of aircrafts and



(a)



(b)



(c)

Figure 5: Real part of frequency-dependent moduli for bare melamine foam (dashed lines) based on [49] results vs. melamine foam with Aluminium cylinders homogenized with CUF-MSG based Code: (a) transversal Young modulus; (b) in-plane Young moduli; (c) shear moduli.

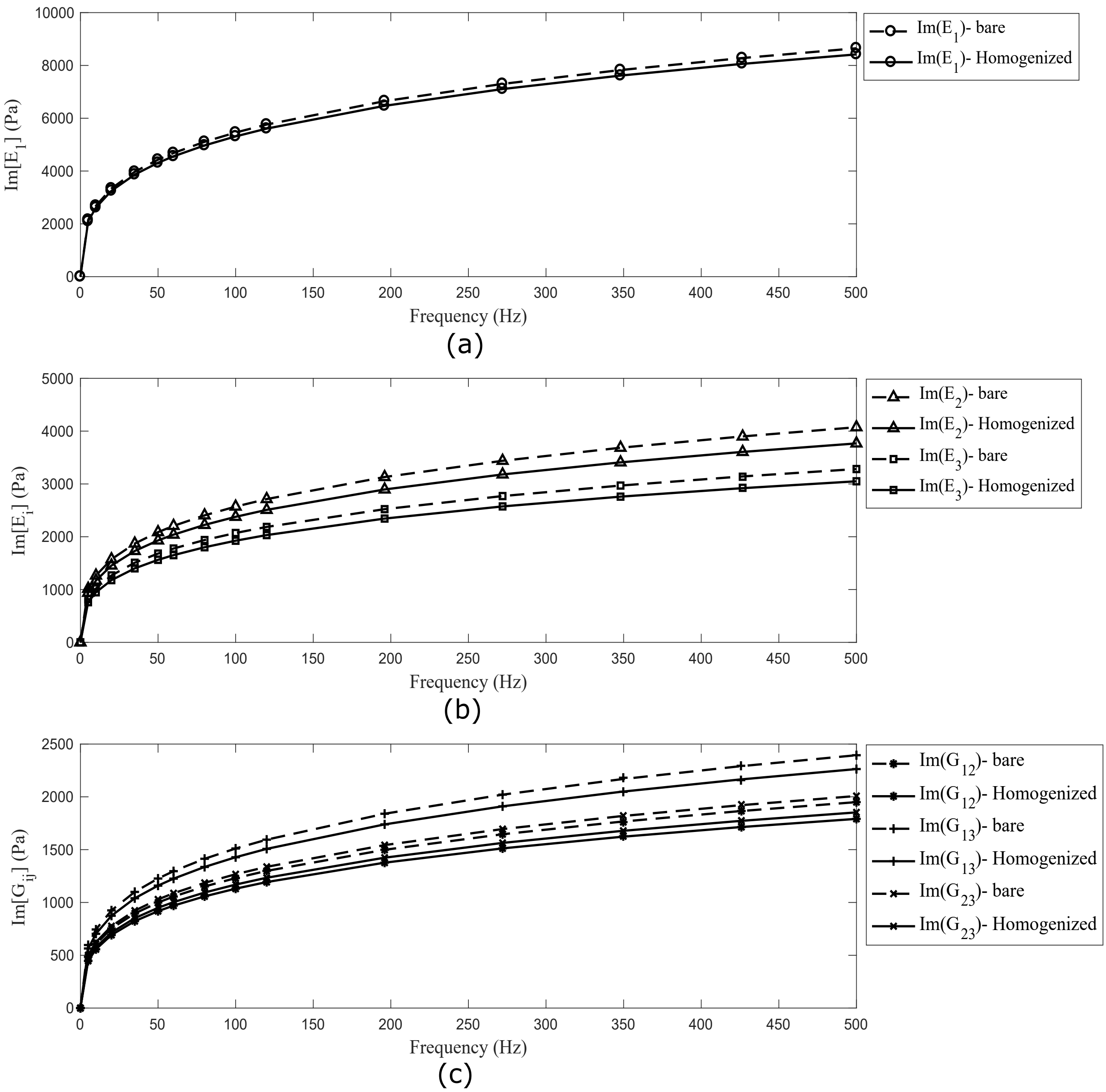


Figure 6: Imaginary part of frequency-dependent moduli for bare melamine foam (dashed lines) based on [49] results vs. melamine foam with Aluminium cylinders homogenized with CUF-MSG based Code: (a) transversal Young modulus; (b) in-plane Young moduli; (c) shear moduli.

its lightness is an aeronautical requirement, the equivalent density of the metamaterial is here imposed to have a maximum value of 80 kg.m^{-3}, that is the maximum density of Aluminium Honeycomb commonly used in aeronautics.
In order to be able to calculate the maximum volume fraction of Aluminium to be considered in the metamaterial, the density of melamine foam is required. However, the density of the foam that was used in the experimental part of the Cuenca method [49] is not mentioned in the paper. Thus, the melamine density is approximated to be $\rho_M = 8$ kg/m^3 which appears to be the low average for this material. The effective density ρ_{eff} of the homogenized material is given by $\rho_{eff} = \rho_M(1-V_f)+\rho_{Al}V_f$. This assumption yields a volume fraction $V_f = 0.0267$ for $\rho_{eff} = 80$ kg/m^3 as the maximal possible value. This information is gathered in Figure 4. For a RUC of side $a = 0.05$ m, this volume fraction represents a cylinder of diameter 9.2$\cdot 10^{-3}$ m according to the formula : $d = \sqrt{\frac{V_f}{\pi}} \times 2a$. This value of volume fraction of Aluminium is used in the homogenization process explained below.

Knowing that viscoelastic materials can be characterized with frequency-dependent complex parameters, our main assumption in applying the homogenization method is that the behavior of the material related to the real part of the complex moduli can be decoupled from the behavior of the material related to the imaginary part of the complex moduli, as conducted in [52].
The complex frequency-dependent properties of melamine calculated in Matlab with the Cuenca model are studied in the frequency range 0-500 Hz. 15 frequencies are chosen within this range and the 15 sets of mechanical complex properties are extracted. These frequencies are highlighted on the curves of Figure 5 and Figure 6 by markers. Each marker corresponds to one measurement set of properties. For each set among the 15 chosen frequencies, the homogenization method was applied twice: one for the real part of the properties and one other for their imaginary part.

The melamine was oriented in such a way that the transversely isotropic behaviour is taken in direction 1 (Figure 2).

	Real	Imaginary
ν_{12}	0.44	10^{-6}
ν_{13}	-0.48	10^{-6}
ν_{23}	0.36	0.0253

Table 1: Homogenized Poisson Ratio.

The values of Poisson's ratio obtained with this homogenization method are shown in Table 1. They are constant and complex even though the imaginary parts of Poisson's ratio in the code are all set as 10^{-7}, to be negligible, as it is assumed in Cuenca that all the Poisson's ratio of Melamine foam are real.

The curves in Figure 5 and Figure 6 gather the results of the homogenization compared to bare-foam for Real and imaginary parts of the material, according to frequency. The real parts of the properties all increase, especially for direction 1. Stiffness is added in this direction because of the cylinder shape but not so much in the others.
However, it can be observed that the imaginary part, which gathers the information on the damping, decreases with Aluminium cylinders, which can be expected as the Aluminium has no imaginary part. In reality, Aluminium moving against the foam would allow more energy to be dissipated, providing additional damping. This additional damping is not captured in the homogenized properties; anyway, the following results in terms of transmission loss can only be conservative.

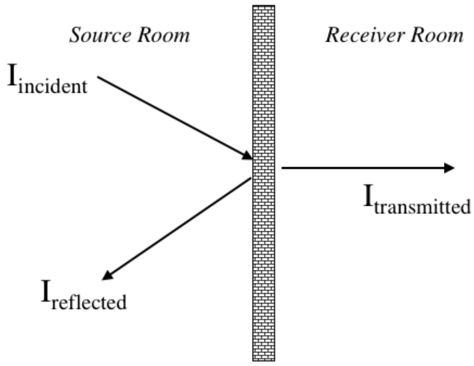


Figure 7: Acoustic behavior of materials for soundproofing: incident soundwaves are partly reflected, partly transmitted when meeting an obstacle such as a wall.

4 Transmission Loss

Transmission Loss (TL) is one of the key metrics in evaluating acoustic performances of metamaterials. TL quantifies the amount of energy that is not transmitted through the material and, using HG metamaterial, it can be increased or tuned at specific frequencies while having a mass or volume smaller than conventional acoustic materials. An extensive study of homogenized metamaterial is performed by studying the effects of the volume fraction of the embedded masses on the transmission loss.
In formulas, the transmission loss (TL) is the reciprocal of the transmission coefficient defined as the ratio of sound energy transmitted through a material to sound energy incident on the material, expressed in decibels (dB).
On Figure 7, incident intensity on the wall is split into reflected and transmitted intensity. The transmission coefficient τ accounts for the percentage of sound intensity that has been transmitted through the obstacle. τ is a frequency-dependent physical property of the material. Sound Transmission Loss is the log ratio of the incident energy to the transmitted energy:

$$TL = 10 \log \frac{I_{incident}}{I_{transmitted}} = 10 \log \frac{1}{\tau} \tag{4}$$

Transmission Loss quantifies the material's ability to reflect or block sound energy. If transmission loss of a material is high, the sounds emitted on one side of the material tend to stay on this side, and not be heard on the other, thus the material acts as a barrier for sound.

4.1 ACTRAN model

Actran MSC is a commercial software created by MSC Software to solve acoustics, vibro-acoustics, and aero-acoustics problems. It is mostly used by automotive manufacturers and suppliers, aerospace and defense companies, and consumer product manufacturers.
This software allowed us to calculate the Transmission Loss of a specific sample with different materials, even those materials having complex frequency-dependent mechanical properties. The samples have been created and discretized with the version 17.1 of Actran with integrated meshing tools (Patran,Apex).
The Incident power is evaluated by the diffuse sound field source: the acoustic source on the right side of the panel in Figure 8 is a 'sample random diffuse field', that is a sound field in which the time average of the mean-square sound pressure is everywhere the same and the flow of acoustic energy

in all directions is equally probable. This source simulates the diffuse field produced experimentally by activating strong acoustic sources in a reverberant chamber with multiple reflections of the waves against the boundary walls.
The Transmitted power is evaluated either by finite fluid volume or Rayleigh surface component. Referring to Figure 8 (left), free field boundary conditions (no reflected waves) are applied to the contour surface of the air volume by means of infinite surface elements, while finite fluid elements are used for the fluid inside the volume. International Standard Air (ISA) is considered: speed of sound $c = 340$ m/s and density $\rho = 1.225$ Kg/m^3.

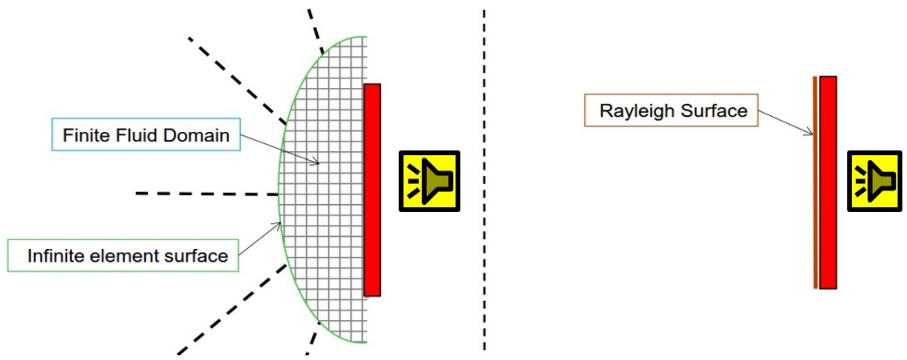


Figure 8: Example of radiating power surfaces.

Another method to simulate the radiating power surface of the panel in Actran is Rayleigh Surface component that represents an interface between the structure and a semi infinite acoustic fluid (see Fig. 8, right). This has been used for the most of cases because of its simple implementation, but unfortunately it uses more RAM because of the high density of the impedance matrix and shows its limitations at higher frequencies. For this reason, the finite fluid volume interfaced with structural elements is employed for the last analyses about sandwich plates.
The computation of TL needs to comply the minimum requirement of 8 elements for structural or acoustic wavelength, depending on the component to which the elements are applied. For an acoustic fluid the wavelength depends on its speed of sound c. In order to have a sufficient level of accuracy, if no flow condition is assumed it needs 8 to 10 linear elements per wavelength, or 4 to 6 for quadratic element interpolation.
If $f_{max} = 1000Hz$ and choosing 8 linear elements/wavelength, the minimum length of the acoustic mesh elements has to be:

$$h = \frac{\lambda_{min}}{4} = \frac{c}{4f_{max}} \tag{5}$$

h=0.0425m

For an orthotropic material, the minimum wavelength over the 3 directions of shear waves is calculated considering the following formula for the speed of sound:

$$c = \sqrt{\frac{G_{ij}}{2\rho}} \tag{6}$$

where

- c is the speed of sound in the considered medium
- G_{ij} is the Shear Modulus over one of the 3 directions

Then Eq.(5) is used to compute the minimum length of the structural mesh elements.

4.2 Validation of effective properties

The plates shown in Figure 9 have been studied to validate the homogenized properties of metamaterial. Clamped boundary conditions are applied to the lateral edges. The volume fraction of the plates with related number of inclusions are provided in Table 2. The analyses have been performed by considering both the heterogeneous plates and the homogenized ones and the results in terms of transmission loss have been compared in Figure 10 for 300 inclusions and Figure 11 for 600 inclusions.

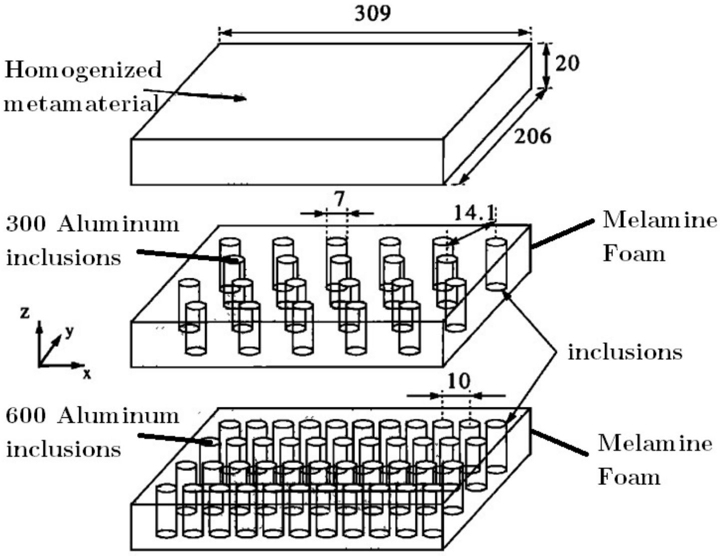


Figure 9: 0.309×0.206×0.020 m plate made of Melamine Foam with 300 or 600 Aluminum inclusions.

Number of inclusions	300	600
V_f	0.192	0.385
a (m)	0.0141	0.010

Table 2: Parameters of metamaterial plates (0.206 × 0.309 m) for different number of inclusions.

Details of the FEM analyses are provided in Table 3. One issue that was encountered is the computational time needed for the modeling of the heterogeneous plates by increasing the number of inclusions. Indeed, each cylinder adds more degrees of freedom than if the plate was completely homogeneous. This fact justifies the use of effective homogenized properties.

	300 inclusions	600 inclusions	Homogenized
Nodes	33310	57065	24336
3D Elements (plate)	20232	33240	19635
3D Elements (inclusions)	12000	24000	-
Time/Frequency step [s]	139	486	50
RAM consumption [GB]	3.9	7.5	1.8

Table 3: Computational time and resources comparison for the analysis of heterogenous and homogenized metamaterial plates. Workstation info: Microsoft Windows® 10, Intel® Core™i5-2500 CPU @3.3 GHz, 8 GB RAM. Only 1 CPU core is used for the analysis.

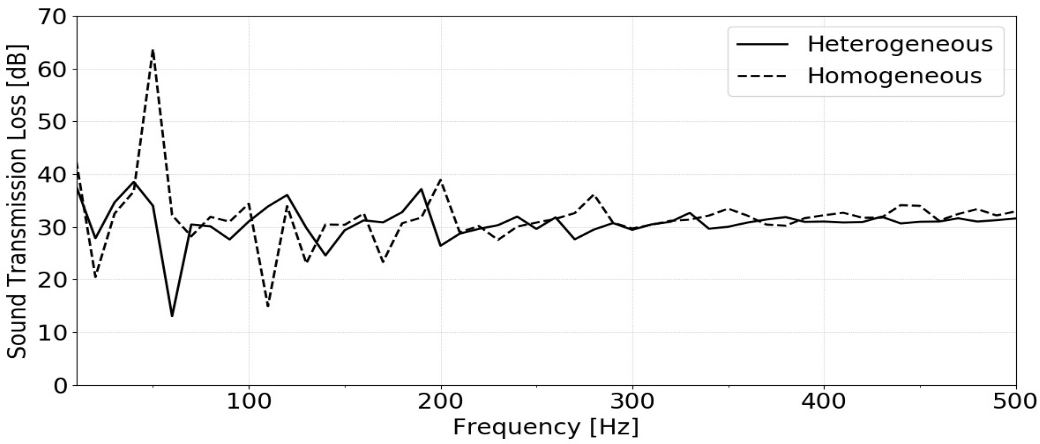


Figure 10: STL of a simply supported plate, Melamine Foam and 300 Aluminum inclusions (19.2% volume fraction): comparison with equivalent homogenized material.

STL curves of heterogeneous plates show some differences with respect to full plates with equivalent homogenized properties at very low frequencies and better agreement over 100 Hz.
The results in Figure 10 shows a good agreement between 10 to 500 Hz, except for 50 Hz, which shows a 30 dB difference, while frequencies 60 and 110 Hz show a 20 dB difference. The plate with 600 inclusions shows an average good agreement along the frequencies; at 40 Hz we have the same resonance peak, even though a 20 dB difference is shown. Differences are lower at higher frequencies. It is fair to say that this work focuses more on the average acoustic performances of the material over all the frequency range; to capture detailed information about specific frequencies, a dynamic homogenization of the metamaterial is required.

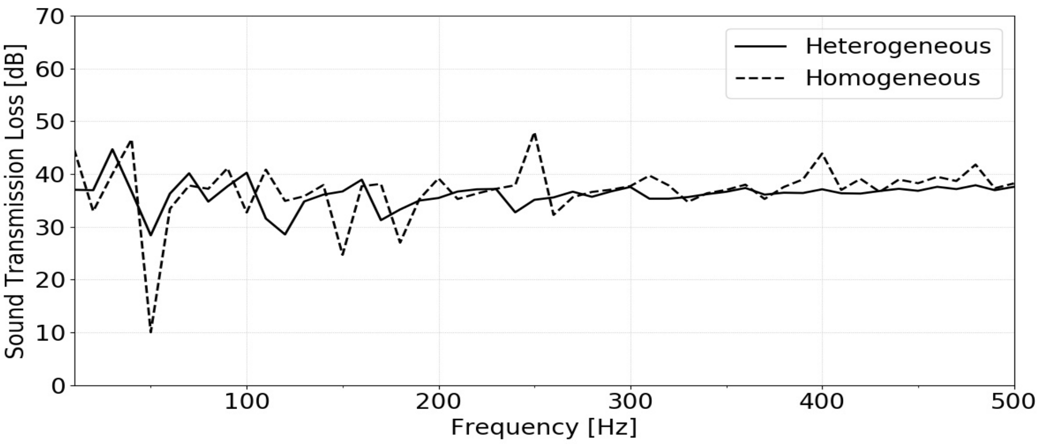


Figure 11: STL of a simply supported plate, Melamine Foam and 600 Aluminum inclusions (38.5% volume fraction): comparison with equivalent homogenized material.

5 Parametric study

A numerical parametric study is performed to explore different configurations of a HG metamaterial in terms of volume fraction.
The homogenized plate considered has the same dimensions of those ones in Figure 9 and different values of volume fractions are considered. Simply supported boundary conditions are here applied to the lateral edges of the plate. For comparison of acoustical performances and to evaluate the effect of inclusions, the Sound Transmission Loss of a full Melamine Foam plate with mass $10.18 \cdot 10^{-3}$ Kg is analyzed too.

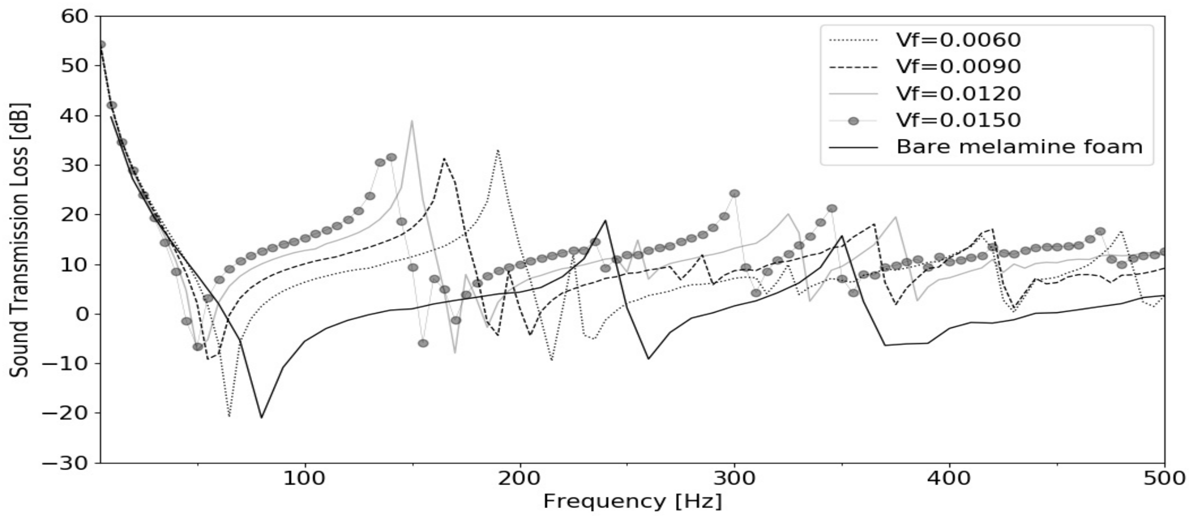


Figure 12: Sound Transmission Loss of metamaterial plates with different volume fractions compared with bare melamine plate.

From Figure 12 it can be observed that, by increasing the volume fraction of inclusions, STL increases (coherently with the mass-law) and its maximum values move toward lower frequencies. Anyway, higher volume fractions lead to increased mass of the panel and taking into account the lightness requirement in aeronautics a compromise should be found between acoustic performances and weight of metamaterial for its application in aircrafts.

6 Sound Transmission Loss of Sandwich Plates

A sandwich plate with dimensions 1×0.6×0.007 m and clamped lateral edges is here considered . The lamination scheme is shown in Figure 13. The skins are composed by two plies oriented at 0°/90° and the material properties are provided in Table 4.

Composite skins	
E_x	20·10^9 Pa
E_y	20·10^9 Pa
E_z	3.6·10^9 Pa
G_{xy}	4·10^9 Pa
G_{xz}	4.3·10^9 Pa
G_{yz}	4.3·10^9 Pa
ν_{xy}	0.13
ν_{xz}	0.27
ν_{yz}	0.27
Specific Weight $[Kg/m^3]$	1950

Table 4: Glass Fabric Pre-impregnated Epoxy Resin.

Two materials are considered for the core: Nomex (see Table 5), that is a typical aeronautical material used for sandwich cores, and metamaterial with a volume fraction of 0.015, that gives an equivalent density equal to the Nomex one ρ = 48 Kg/m^3. The aim is to compare the acoustical performances of the metamaterial core with respect to the classical configuration with Nomex core by maintaining the same global weight. Details of the FEM analyses are given in Table 6.

Nomex core	
E_x	0.1·10^6 Pa
E_y	0.1·10^6 Pa
E_z	90·10^6 Pa
G_{xy}	0.1·10^6 Pa
G_{xz}	15·10^6 Pa
G_{yz}	31·10^6 Pa
ν_{xy}	0.99
ν_{xz}	2·10^{-4}
ν_{yz}	2·10^{-4}
Specific Weight $[Kg/m^3]$	48

Table 5: Core - Nomex Aramid honeycomb.

Sandwich panel	
Nodes (air volume)	348075
Nodes (plate)	26499
2D Elements (top+bottom plate skin)	17280
3D Elements (air volume)	332800
3D Elements (plate)	17280
Time/Frequency step [s]	1151
RAM consumption [GB]	5.5

Table 6: Computational time and resources for the analysis of sandwich panel. Workstation info: Microsoft Windows® 10, Intel® Core™i5-2500 CPU @3.3 GHz, 8 GB RAM. Only 1 CPU core is used for the analysis.

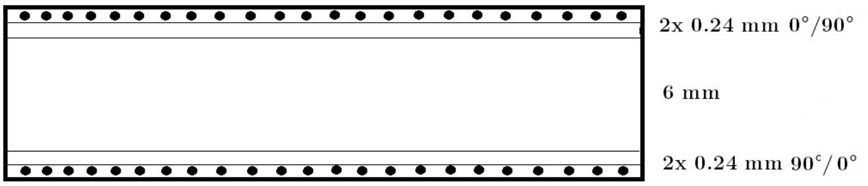


Figure 13: Lamination scheme.

To satisfy the minimum factor of 8 elements/wavelength for both the configurations studied, the maximum length of the element has been taken equal to 0.004 m. Then, the model is composed of 250×150×2 elements.
The effect of the skins is also evaluated: metamaterial with volume fraction 0.0150 and Nomex are analyzed with and without skins. Results in Figure 14 show an average gain of 10 dB in STL with respect to the configuration without skins in both cases of materials. So, it is confirmed that the presence of skins gives a positive effect in term of noise reduction. In particular, metamaterial shows an higher STL than Nomex after 70 Hz with an average gain of 3-4 dB over all the frequencies considered.

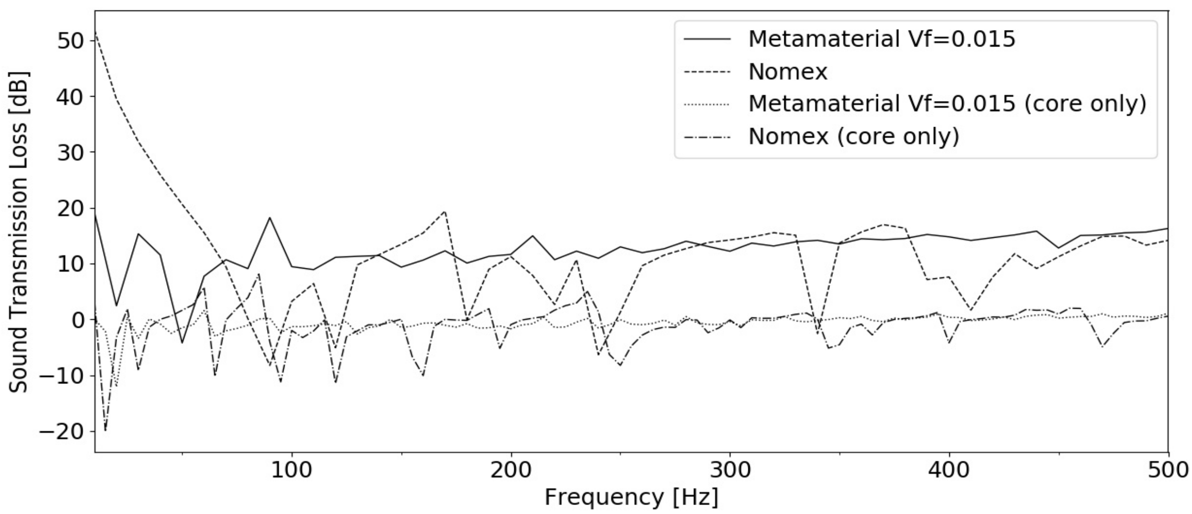


Figure 14: Comparison between Nomex and Metamaterial V_f=0.015, both in single and sandwich configurations.

7 Conclusions

Sound transmission loss in passive acoustic metamaterials for aeronautical applications has been here studied. The homogenized effective properties of plates made of melamine foam with cylindrical Aluminium inclusions have been evaluated. Results have shown that the homogenization method based on CUF and MSG method permits to efficiently predict the acoustic performances of periodic heterogeneous materials. Then, a parametric study has been performed by varying the volume fraction of inclusions in the melamine. This has demonstrated that adding inclusions to the melamine foam the transmission loss increases and its peaks move to lower frequencies. Finally, the metamaterial has been compared to classical materials used for inner lining of cabin in aircrafts. In particular, a sandwich configuration with composite skins has been analyzed by considering two materials for the core: Nomex and metamaterial. Taking into account the lightness aeronautical requirement, the volume fraction of metamaterial has been fixed in order to have a global weight equivalent to the Nomex core case. Results confirm the positive effect of skins on sound transmission loss and demonstrate the high potentiality of metamaterial in soundproofing of aircraft with respect to classical materials.

A companion work will be devoted to experimental validation of these results. First of all, the metamaterial will be manufactured by embedding, without adhesive, Aluminum cylinders in a panel made of melamine foam (for example, Melamine Basotech G+), in which periodic holes have been previously drilled. Afterwards, composite skins will be attached to the metamaterial panel, but this requires to develop an ad-hoc bonding process compatible with both melamine foam and composite. This aspect is actually an open issue and is worthy of another paper work. Finally, different metamaterial configurations will be explored, for example employing other foams with good acoustic and fire-resistant properties, such as polymide, or using spherical inclusions. However, the use of spheres risk to complicate the manufacturing process of large panels without providing any significant acoustic improvement for the application considered that requires small thickness; anyway, this point will be assessed numerically. Absorption coefficient will be evaluated too. From a modelling point of view, a future work will present the dynamic homogenization of metamaterial and the detailed analysis of its dispersion behavior by means of advanced finite elements based on CUF.

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