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# A methodology for multicopter aircrafts power budget analysis

## Abstract

**Purpose** – The primary objective of this article is to analyse performance of multicopter UAS platforms for passenger transport and compare them with an ordinary helicopter solution. The study aims to define a standard procedure for power budget analysis of unconventional vehicles recently proposed in the aerospace industry, providing guidelines on rotor sizing in terms of required power and total number of rotors. The ultimate purpose of the proposed work is to describe a methodology for power estimation with regard to emerging Electric Vertical Takeoff and Landing (EVTOL) vehicles.

**Design/methodology/approach** – In the context of urban mobility, short range passenger transport between critical hubs in cities is accounted for and compared between innovative aircraft vehicles and traditional helicopters on the same mission profile. The power budget equations used in the helicopter literature are revisited to consider different multi-rotor configurations (up to 20 rotors) and evaluate feasibility of innovative aerospace vehicle design.

**Findings** – The paper includes insights about the maximum rotor numbers which ensure a significant, relative power reduction compared helicopter platforms (the power-to-cruise over power-to-hover ratio appears to be improved). Based on this preliminary analysis, results suggest reducing the installed rotors to avoid excessive power loss in forward flight.

**Practical implications** – The proposed study provides guidelines for further design considerations and the future development of EVTOL multi-rotor aircrafts.

**Originality/value** – This paper fulfils the identified need of a systematic approach on performance analysis for innovative vehicles involved in commercial applications.

**Keywords** Innovative multi-rotor vehicles, air-taxi, urban mobility, power budget analysis, EVTOL aircrafts, performance

**Paper type:** Research paper

## Introduction

Small Unmanned Aircraft Systems (UAS) have become popular for commercial applications owing to advanced autopilot flight modes and high flexibility. Among all vehicle configurations, multi-rotors are the most widespread and commonly employed because of their vertical take-off and landing as well as hover capabilities. The propulsion system of multi-rotor platforms is based on a combination of brushless motor and a suitable propeller design. Compared to helicopter vehicles, the overall actuator system is less complex. For small UAS vehicles, a direct link between the propeller and the motor shaft is exploited so that thrust generated by each rotor is directly controlled by the motor speed. Swashplates and propeller pitch actuation systems are not installed as the manufacturers usually prefer system simplicity and weight reduction instead of vehicle's performance. As a result, small UAS suffer limited endurance, payload and range capabilities. Nevertheless, thanks to their high reconfigurable architecture and potential for EVTOL solutions, they have recently inspired new aircraft design for passenger transport applications. Example of these innovative solutions are reported in Table 1. The limited maximum take-off weight and flight endurance make these vehicles suitable for small range missions such as mobility in urban environments only.

**Table 1** Innovative aircraft design for passenger transport applications based on small UAS vehicles.

Vehicle	Number of rotors	Passengers	Endurance [min]	MTOW [kg]	Max speed [km/h]
Ehang 184	8	1	25	360	100
Fly Astro	16	2	20 - 25	360	70
Boeing PAV	8	2	n/a	n/a	n/a
Volocopter	18	2	27	450	100
City Airbus	8	4	15	n/a	120
Lilium	36	1	n/a	n/a	300
Bell Air Taxi	8	4	n/a	n/a	n/a

A review of the major *Urban Air Mobility* (UAM) solutions is provided in Swartz [1] and Whittle [2]. Industrial and academic communities agree that aviation technologies have reached an advanced level of maturity and innovative architectures will be soon available such as aircraft on-demand mobility (Mueller, [3]). More recently, Polaczky et al. [4] have investigated current technologies for urban air mobility analysing 44 EVTOL projects currently implemented or under testing. According to Ullman et al. [5], innovative vehicle configurations can be classified in three main categories: rotorcrafts (multicopters), VTOL and Ultra Short Take-Off and Landing aircraft (USTOL). A similar subdivision is reported by Rossien et al. [6] based on a 100 aircraft catalogue provided by Electric VTOL News webpage<sup>1</sup>. According to Ullman et

<sup>1</sup> <https://evtol.news/aircraft/>

al. [5], multicopters suffer short range capabilities, higher noise level as well as higher required power compared to USTOL.

In the context of Smart Cities, citizen's request for fast urban mobility can be accomplished by EVTOL air vehicles known as *Air-Taxi*. According to the work by Antcliff et al. [7], EVTOL Air-Taxi allow improvements in travel time up to 3.6 times compared conventional manned ground vehicles. Many research articles deal with conceptual aircraft design for urban mobility purposes. As an example, Johnson [8] proposed three aircraft design based on different passenger, range capabilities as well as propulsion system. Vehicle performance and optimal blade geometries are calculated based on NASA CAMRAD II software, which accounts for complex multirotor aerodynamic interactions (e.g. rotor-to-rotor). The article provides guidelines for NASA research activities on VTOL Air-Taxi vehicles and highlights critical design of light-weight high-power batteries, suitable drivers for highly efficient electric motors, aerodynamic interactions between the airframe and rotors as well as noise reduction. The propulsion system for Urban Air Mobility projects pose technology challenges; the use of batteries for air taxi is problematic and other form of energy should be leveraged [5]. Sustainability of air taxi and delivery UAS is discussed by Rez [9]: the author defines a metric based on onboard power, vehicle mass and geometry to compare the efficiency of conventional helicopter and UAS. Rez [9] shows that the higher the propellers installed the less efficient is the vehicle compared to helicopter with large rotors. As reported in the study performed by Rossien et al. [6], the potential of UAM is still not completely clear: guidelines and best practise suggestions to define the optimal vehicle configuration are not defined. In April 2017 Uber presented its vision for air mobility [10], based on piloted EVTOL architectures. Ullman et al. [5] highlight that Uber does not plan to deploy its own air taxi aircraft but the ultimate objective is to provide the required infrastructure needed for UAM applications. The reason is due to the huge vehicle configurations under development by private and public organizations. According to Kellermann et al. [11], there is a strong need to provide scientific evidence of benefits concerning UAM solutions and their impacts on society. The authors reviewed 111 multidisciplinary publications between 2013 and 2019 and founded that the debate on UAM and delivery with UAS is still open and needs further clarifications. Traffic congestion reduction, travel time savings and environmental sustainability of innovative solutions are important topics that require deeper understanding for industrial applications.

A lack of published data on EVTOL multirotor performance analysis for preliminary design consideration still exists and the purpose of this article is to describe a simplified standard procedure to evaluate the power budget along a predefined mission profile. The methodology proposed by the authors is described providing equations, assumptions and discussing limitations. The power budget analysis is computed considering an urban mission profile for both conventional and innovative aircraft design. The implementation is based on a Robison R-44 helicopter as reference vehicle for comparison with different multi-rotor architectures. Finally, major results and future works are reported to conclude the article.

## Methodology

A methodology for a preliminary performance study of several VTOL architecture is here proposed. All equations listed below are derived from widely accepted lower order mathematical models generally used for performance estimations of a conventional helicopter with a single main rotor and a tail rotor. Thus, all equations are adjusted to take different architectures into account such as multicopters with multiple main rotors and no dedicated anti-torque tail rotor system. For the task at hand, a simplified formulation is used to assess the performance of each selected configuration. Precisely:

- a common mission profile is selected and is meant to be representative of a typical air-taxi mission performed over a largely populated city ([18] and [19]);
- performance is assessed in terms of overall required power for each configuration;
- the overall required power is calculated by summation of rotor power, for each mission phase, in terms of profile power, parasite power, induced power and additional averaged powers (a detailed description of the mathematical models used is reported below for each of the listed terms).

### Hover, Take-Off and Landing Performance

The same formulation is exploited for the hover condition as well as the take-off and landing ones, since in both the last two phases, translational speeds are considered as small. Nevertheless, when not strictly in hover conditions, the overall thrust and figure of merit ( $FM = 1/k$ ) shall be slightly modified according to the analysed case (for example, during take-off the overall thrust will be slightly higher than what needed to counteract the sole weight of the rotorcraft to allow this to gain vertical acceleration and start climbing. Thus, in the Eq. (2),  $W$  shall be replaced with  $T = nW$ , where  $n$  is the current load factor. The rotor required power is obtained by summation of induced and profile power, therefore

$$P_{mr} = P_{i,h} + P_0 \quad (1)$$

The induced power  $P_i$  can be related to the energy involved in the downward momentum generated by the rotor wake and exerted on the air. The parasite power  $P_0$ , instead, is the term accounting for the energy dissipation related to the viscous drag of the blade. Although both terms can be estimated using more accurate theories as well as exploiting experimental results, the aim of this study is indeed to give an overview of how different, unconventional layouts can impact on the overall performance of a rotorcraft, therefore a simplified method as that proposed by Johnson [12] allows to have a rough but still accurate estimation of the required powers. As a result, the induced power can be expressed referring to rotor thrust and applying a correction factor  $k$  (which is typically a value between 1.10 and 1.15) to the momentum theory, therefore

$$P_{i,h} = \frac{k\sqrt{W^3}}{\sqrt{2\rho A}} \quad (2)$$

Accordingly, the parasite power can be expressed as

$$P_0 = \rho A (\Omega R)^3 \frac{\sigma C_{d0}}{8} \quad (3)$$

Both Eqs. (2) and (3) can be computed for each main rotor, in case of a multi-rotor configuration and the tail rotor, in case an anti-torque system is involved.

### Climb and Descend Performance

For the task at hand, a vertical climb is considered, therefore the equations in [12] relative to the vertical flight theory can be exploited. The overall required equation can shall once again refer to the general formulation as in Eq. (1). Nevertheless, while the formulation of profile power given by Eq. (3) is still valid, the induced power shall now be modified. Recalling the Momentum Theory, the rotor induced velocity in hover condition can be written as

$$v_h = \sqrt{\frac{W}{2\rho A}} \quad (4)$$

According to [13], the power required to climb can be obtained from the power to hover as

$$\frac{P_c}{P_{i,h}} = \frac{V_c}{2v_h} + \sqrt{\left(\frac{V_c}{2v_h}\right)^2 + 1} \quad (5)$$

Consequently, Eq. (4) can be introduced in Eq. (5), yielding to

$$P_{i,c} = \frac{k\sqrt{W^3}}{\sqrt{2\rho A}} \left[ \frac{V_c}{2v_h} + \sqrt{\left(\frac{V_c}{2v_h}\right)^2 + 1} \right] = kW \left[ \frac{V_c}{2} + \sqrt{\left(\frac{V_c}{2}\right)^2 + \frac{W^2}{2\rho A}} \right] \quad (6)$$

Eventually, Eq. (6) can be used to account for the induced velocity during climb or descend for small rates of climb and descend: this formulation is physically not strictly valid when considering a climb rate laying within the vortex ring state operating condition of the rotor, precisely when  $-2 \leq V_c/v_h \leq 0$ . In such a condition the momentum theory is no longer valid and the average induced velocity on the rotor can no longer be measured directly. However, the reader shall note that as referred in [13], the non-physical trend follows the experimental data up to  $V_c/v_h \approx -1.5$  so that results are slightly underestimated but still qualitatively consistent (a linear offset can be introduced to better fit experimental results), while for  $-2 \leq V_c/v_h \leq -1.5$  a semi-empirical approximation such that of Johnson [12] and Young [14] shall be used in order to get reasonable results. In short, depending on the descend rate considered, the equation shall be used carefully and addressing the analysed condition properly.

### Cruise Performance

According to what described above in this article, cruise performance is assessed by exploiting the conventional formulation derived by the momentum theory applied to the forward flight case. In on one hand, the induced power in forward flight shall require a numerical solution due to the implicit formulation of the inflow equation, some simplified analytical solution can still be exploited if the angle of attack of equivalent rotor disk is assumed to be zero. Strictly speaking, this angle is generally different from zero, since a slight tilt of the rotor is indeed needed to allow the forward flight, yet this assumption still allows to get some rough, reasonable estimation and make some simple comparisons among different rotorcraft architectures such as the purpose of this work. As further described in both [12] and [13], if the angle of attack relative to the rotor disk ( $\alpha_D$ ) is zero, the induced velocity can be expressed using

$$v_{i,cr}^2 = -\frac{V_{cr}^2}{2} + \sqrt{\frac{V_{cr}^4}{4} + v_{i,h}^4} \quad (7)$$

Subsequently, the induced power can be computed as

$$P_i = kT v_{i,cr} = kT \sqrt{-\frac{V_{cr}^2}{2} + \sqrt{\frac{V_{cr}^4}{4} + v_{i,h}^4}} = kT \sqrt{-\frac{V_{cr}^2}{2} + \sqrt{\frac{V_{cr}^4}{4} + \left(\frac{W}{2\rho A}\right)^2}} \quad (8)$$

Furthermore, the profile drag formulation must be modified as well in order to account for the losses due to the compressibility effects. While the radial flow effect cannot be omitted [15], the effect of the reversed flow on retreating blades can be reasonably neglected up to about  $\mu = 0.4$ . Thus, the following relation is exploited, [13]:

$$P_{0,cr} = \rho A (\Omega R)^3 \left[ \frac{1}{8} \sigma C_{d0} (1 + K\mu^2) \right] \quad (9)$$

Additionally, the power loss resulting from viscous shear effects and flow separation on airframe, rotor hob and so forth shall be considered and addressed as parasitic power. The parasitic power ( $P_p$ ) can be written using the equivalent wetted area ( $f$ ), [13]:

$$P_p = \frac{1}{2} \rho f V^3 \quad (10)$$

Eventually, the overall required power during cruise can be expressed as:

$$P_{cr} = P_{0,cr} + P_{i,cr} + P_p \quad (11)$$

### Modified Formulation for Multicopters

What presented above is generally valid for a conventional helicopter architecture. For the task at hand, instead, several multi-copter layouts are to be evaluated and therefore the mathematical formulation shall be modified: the major modification made is that of equally dividing the weight of the rotorcraft over the number of rotors and then summing the required power for each one of them, so that the overall load is ideally equally distributed depending on the number of rotors of each architecture considered. This assumption is based on current available analytical formulations of multi-

rotor configurations such as tandem and side-by-side rotors: as extensively reported in several widely accepted reference such as [12], [13] and some new publications such as [16], the mutual-interference of the rotors can potentially involve positive effects in terms of required power as long as the relative distance between the rotors is sufficiently small, otherwise it shall be neglected.

$$\chi = \frac{P}{P_{isolated}} - 1 = \frac{2}{\left(1 + \frac{l}{2R}\right)^2} - 1 \quad (12)$$

As a matter of fact, according to Johnson in [12], the equation proposed for the side-by-side effect of the rotors and reported in this paper with Eq. (12), “[...] should only be used out to about  $\frac{l}{R} = 1.75$ , beyond which the interference decreases to zero”. However, for the vehicles listed in Table 1, the ratio  $l/R$  between subsequent rotors is always greater than 1.75 (closed to 2) so that all the rotors have been considered as *isolated rotors* without accounting for mutual aerodynamic interference.

The power budget computation for multicopter analysis is based on helicopter performance equations. To achieve a suitable comparison between conventional and innovative vehicles, the multicopter computation was performed referring to the same helicopter total disk area and introducing circle packing factor  $k$  (Stephenson, [17]), related to the total number of rotors  $N_R$  (Eq. 12).

$$A_{disk_{multicopter}} = k A_{disk_{helicopter}} \quad (13)$$

Given the effective total area, the single rotor radius  $R$  can be computed assuming the same geometry for all the rotors. As an example, Volocopter propeller radius reported in design specification <sup>[2]</sup> is 0.9 m whereas the computed value is 0.94 m, overestimated by 4%.

The angular rate of each rotor ( $\omega$ ) is computed based on the assumption of constant propeller tip speed ( $V_{tip}$ ) for both helicopter and multi-rotor configurations (Eq. 12)

$$\omega = \frac{V_{tip}}{R} \quad (14)$$

Finally, the rotor chord dimension  $c$  is evaluated on the assumption of constant solidity  $\sigma$  for both helicopter and multirotor vehicles (Eq. 12).

$$c = \frac{\sigma \pi R}{N_R} \quad (15)$$

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<sup>2</sup> [https://www.volocopter.com/assets/pdf/2017\\_04\\_Design\\_specifications\\_2X.pdf](https://www.volocopter.com/assets/pdf/2017_04_Design_specifications_2X.pdf)

## Software Architecture

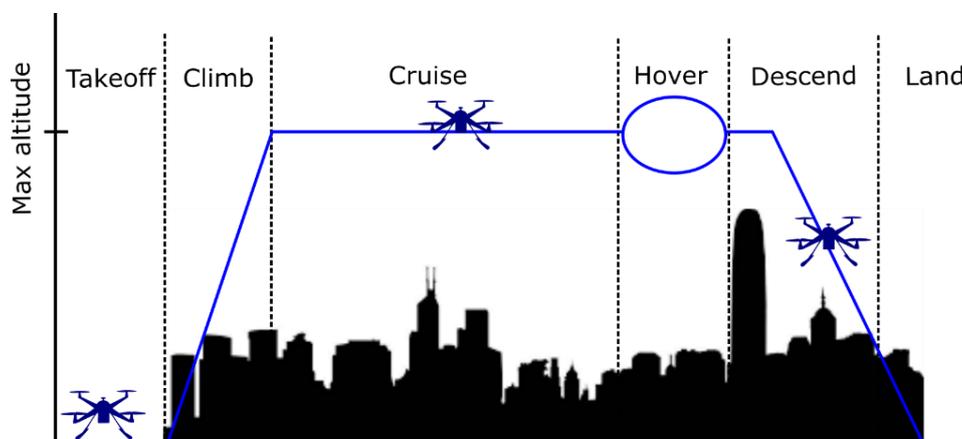
The mathematical model as briefly described above was implemented in a MATLAB® -based, proprietary model: the user can define the geometry of the desired multicopter architecture as well as the mission profile of interest then the routine will automatically compute evaluate the power profile for each mission phase depending on the selected architecture and based on the suitable mathematical model among those described above in this paper. Such a software architecture allows the comparison of different suitable rotorcraft configuration rather easily and a preliminary estimation of a power budget.

## Mission profile and assumptions

The methodology previously described is implemented for passenger transportation in urban environments. The software is highly reconfigurable and different missions can be studied combining the following phases: i) take-off, ii) climb, iii) hover, iv) cruise, v) descend, and vi) land.

A short-range urban mission is considered because representative of typical UAM applications as reported by Patterson et al. [18] and Goyal [19]. Figure 1 shows a schematic of the mission profile, while Table 2 provides details of each phase. The total distance covered is closed to 15 km and the maximum altitude 300 m, according to the minimum flyable altitude in cities. The total time required to accomplish the mission is 27 min which is comparable with endurance of EVTOL vehicles reported in Table 1. Maximum rate of climb, descend, and load factors have been defined as for standard values for helicopter applications. An additional 2 min hover time before descend phase is included to account for traffic management and unpredicted delay before clearance for landing. Three forward speed (10, 20 and 30 m/s respectively) have been considered to envelope the characteristics of vehicles in Table 1. The same available power has been considered for both helicopter and multicopter configurations.

**Figure 1** Urban mission profile



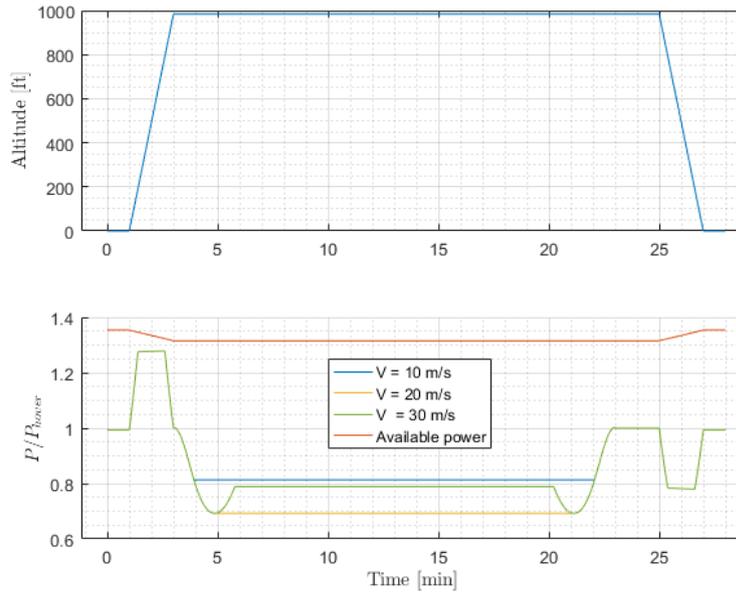
**Table 2** Details of urban mission phases

Mission phase	Duration [min]	Forward speed [m/s]	Vertical speed [m/s]	Altitude [m]	Load factor [n/a]
Take-off	1	0	0	0	1
Climb	2	0	3	300	1.05
Cruise	20	10/20/30	0	300	1.10
Hover	2	0	0	300	1
Descend	2	0	-3	0	0.95
Land	1	0	0	0	1

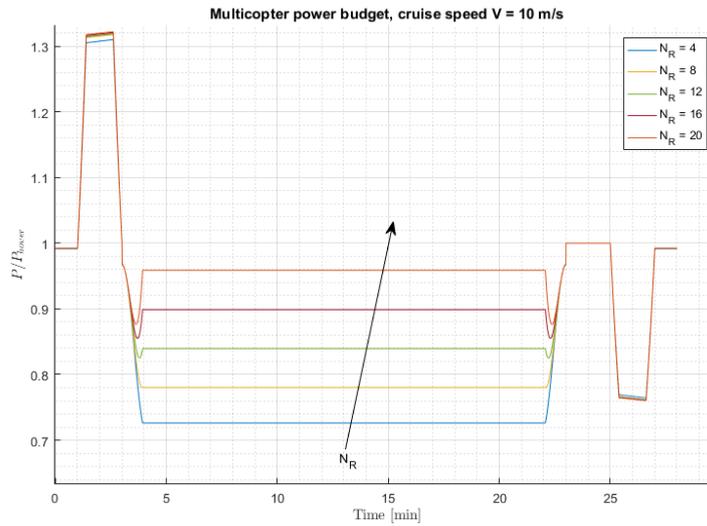
## Power budget results

Results of the analysis for a Robinson R44 and multicopter vehicles are here reported. The power budget is given in terms of  $P/P_{hover}$ , the ratio between computed required power, as explained in the Methodology section, and hover power to allow a comparison between different vehicles. Results for the Robinson R44 are reported in Figure 2: the top diagram is the altitude profile while at the bottom the non-dimensional power ratio is given for different forward flight velocities. The required power is always lower than the maximum value the motor can provide. Climb is the most expensive mission phase; cruise is strongly affected by helicopter speed and we can state the minimum power cruise speed is close to 20 m/s. As expected, a power reduction is highlighted during final descend phase. Figure 3 shows the estimated power budget for different multi-rotor configurations assuming a forward velocity of 10 m/s. Rotor radius, motor angular speed as well as chord dimension as a function of the total number of rotors are reported in Figure 4 based on the assumption of constant total disk area, propeller tip speed and solidity ratio. The higher the number of rotors, the smaller the radius while the higher the motor angular speed. The effect of forward flight speed on required power is in Figure 5 for a 16-rotor multicopter, showing worst performance as the forward speed is increased (for 30 m/s the available power is not sufficient to complete the mission). Considering Figure 3, it can be noticed that the required power during take-off, descend and landing is almost the same for all the architectures while the cruise power is strongly affected by the overall number of rotors. The higher the rotor number, the more the power required during cruise with a negative impact on the overall performance. Recalling that the ratio  $P/P_{hover}$  for the R44 at 10 m/s, is close to 0.81, Figure 3 suggests that for multirotors with more than 12 rotors, a cruise speed of 10 m/s does not ensure a lower power consumption than that required for climb, meaning that the aerodynamic efficiency of the rotor is likely to limit the operational cruise speed or the range of the rotorcraft, compared to solutions with smaller number of rotors. Figure 6 shows the effect of cruise speed on power ratio  $P_{cruise}/P_{hover}$ ; the R44 rotorcraft performance are directly compared to different multicopter configurations.

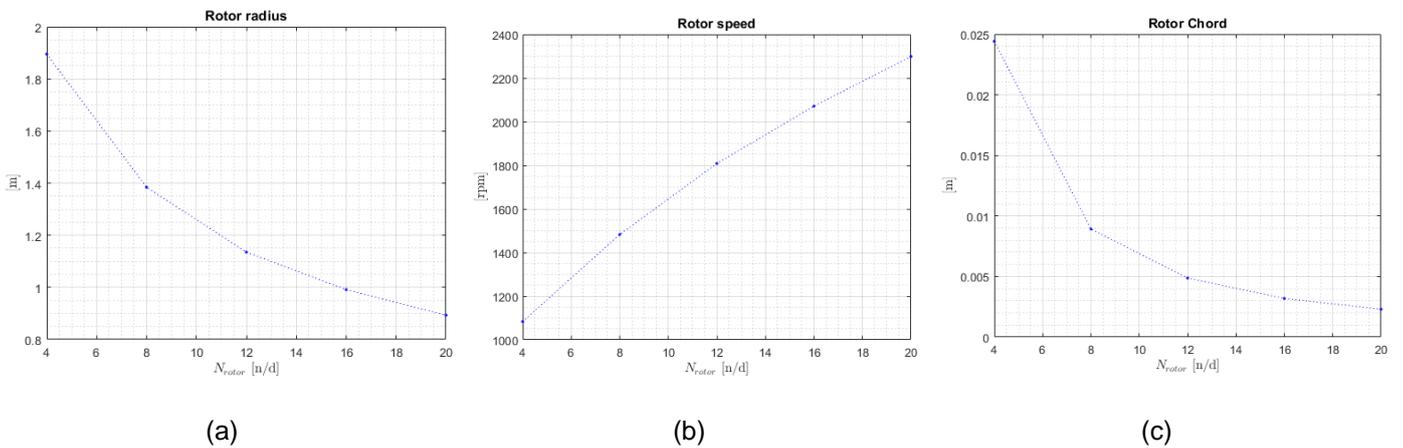
**Figure 2** Robinson R44 power budget estimate



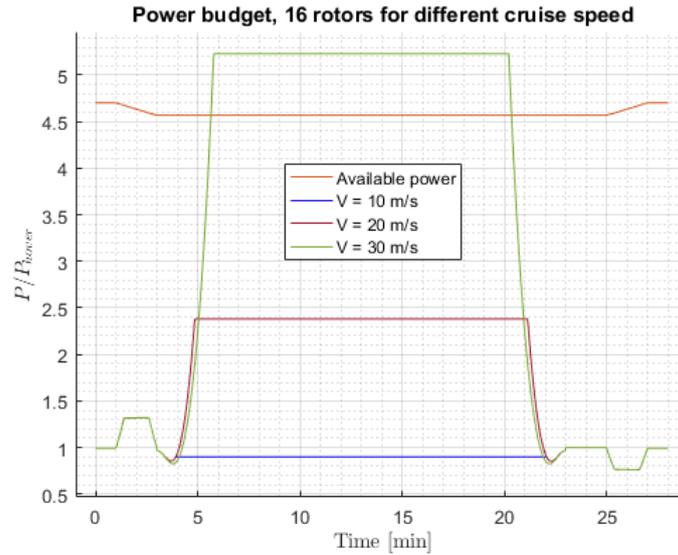
**Figure 3** Multicopter power budget, 10 m/s cruise speed



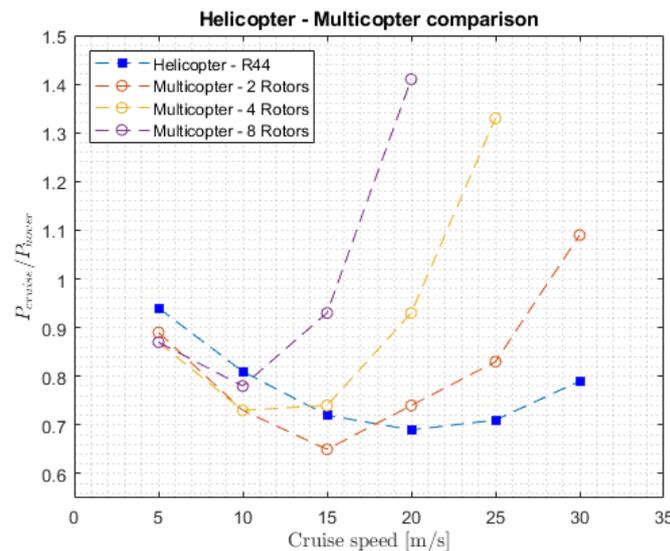
**Figure 4** Multicopter rotor radius, motor speed and chord dimension as a function of total rotors



**Figure 5** Cruise speed effect on power budget for 16 rotors



**Figure 6** Cruise speed effect on helicopter and multicopter cruise power



Considering the R44 helicopter, a cruise speed of 20 m/s minimizes the required power during cruise and, as expected, a parabolic trend is shown. A similar behaviour can be evinced by multicopter architectures; however, important differences are highlighted. Firstly, the minimum cruise power is achieved for lower cruise speed as the total number of rotors is increased. Secondly, the  $P_{cruise}/P_{hover}$  ratio rapidly increases when moving from the optimal cruise speed compared to a conventional helicopter solution. The 2-rotors multicopter shows the lowest  $P_{cruise}/P_{hover}$  ratio even though for cruise speed higher than 20 m/s performances are worse than the R44. Figure 6 highlights that multicopter configuration may achieve lower power consumption than traditional helicopters if limited cruise speed are considered. However, even in the range of cruise speed lower than 15 m/s, the higher the rotor numbers, the worst the cruise performance. Consequently, the study suggests that multicopters implementing a greater number of rotors may well not be beneficial from an aerodynamic point of view. What highlighted in this work and stated here above, is in line with a broadly shared perception among the scientific community (Ullman et al. [5], Rez [9]).

## Conclusion

A methodology to estimate the power budget of innovative multicopter aircrafts is presented. Starting from power budget analysis of conventional helicopters, guidelines are provided to modify all the equations to take novel configurations into account. A power budget over a user-defined mission profile is computed and a comparison between a Robinson R44 and unconventional multirotor vehicles is proposed. Results show that a good balance in terms of overall rotor installed is required to avoid excessive power requirement. Furthermore, multirotor cruise performances are greatly affected by forward flight speed resulting in better performance for limited velocities. Even though the aerospace industry is proposing innovative multi-rotor-based architectures, helicopters remain promising aircraft solutions for short-range missions, especially if a possible conversion to electrical propulsion is considered, a case in which better performance strongly impact on the overall weight of the rotorcraft, due to the limited energy density of state-of-art batteries.

## Further Work

Future works include the analysis of multirotor performance removing the assumption of the same rotor radius. Furthermore, maintenance, reliability and acoustic emission of multicopter platforms for passenger transport will be considered for a more realistic comparison with conventional helicopters.

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