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A proposal of a unique formula for computing compliance in bolted joints

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Abstract

The connection system between mechanical parts with the greatest advantages in terms of production is the threaded connection. This type of connection has considerable stiffness but also high weight. Often the search for the reduction of the masses clashes with the limits dictated by production needs. A considerable effort has been made in making screws with higher performance materials and therefore guaranteeing greater tightening forces with smaller cross sections, but there have not been as many notable developments on the method of determining the compliance of tightened elements. The classical theory identifies three different conditions for calculating deformability, which are sometimes not easy to interpret and implement. The use of numerical techniques such as finite elements allows designers to be very precise, but requires a great deal. To facilitate the work of the designers and provide them with a more manageable tool to better understand the type of threaded connection to be designed, the present work proposes an analytical formulation that allows a quick assessment of the compliance value of the clamped elements, regardless of the geometric relationships and materials. To achieve this, starting from a literature analysis, a parametric finite element model was developed and, based on the results obtained, a formula is proposed that covers all the possible scenarios for determining compliance. The results were compared with the classical theory in order to verify the correctness and applicability of the formulation. At the moment the formulation is valid for screws whereas for the bolts with nut unified formula is under investigation.

Keywords: Bolted joints; connection design; threaded joint

1. Introduction

The aim of this paper is to investigate the possibility to use a unique formula for depicting the stiffness behaviour of different bolted connections. In particular, the focus is on the computation of the compliance of the clamped parts, because, several methods are available on the basis of geometrical features of the joint. In traditional machine design theory, three different conditions are taken into account, based on Rotscher's equivalent cone frustum. Other researchers try to suggest different solution for the computation of clamped elements stiffness. In Brown et al. (2008)

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and Canyurt and Sekercioglu (2015) comparisons of different models are proposed, aside critical discussions of all the published works related to the topic. A different approach is depicted in Routh and Das (2016) where the stiffness of the clamped member is computed by subtracting the stiffness of the hole from the stiffness of the Rotscher's frusta. The authors decide to not report the same literature review of the three cited documents and suggest the reader to refer to those documents of any further information. In the following section, a discussion about literature outcomes is developed. As main reference, authors use the VDI 2230 standard VDI2230 (2003) and in the following all the terms are compliant with that standard. The main difference between the idea depicted in the present paper and the rest of available literature consists in the usage of a parametric FE model made with different clamped element materials and geometry, with the aim to unify the stiffness computation in a unique formula.

Nomenclature

d	diameter of the screw shank
d_h	diameter of the hole in which the screw is inserted
d_w	diameter of the screw head
l_K	thickness of the clamped member
l_i	thichness of one of the clamped parts
D_A	diameter of the clamped member
A_n	Equivalent area of the clamped members
E	Young modulus of the material
E_i	Young modulus of one of the clamped parts
E_p	Young modulus of the clamped members
K_p	Stiffness of the clamped members
φ	angle of the pressure cone
$D_{A,Gr}$	diameter of the pressure cone
l_v	length of the pressure cone
w	identifier for distinguishing tapped threaded joint from bolted joint
A_m	parameter dependent on clamped member materials
B_m	coefficient dependent on clamped member materials

2. Review of literature models

To underline the difference between the proposed methodology and the literature ones, a brief comparison of all the literature models is presented. The oldest model is based on the Rotscher's theory (see Figure 1), that is used as reference in the VDI standard. It is referred to the dimension of a frustum that undergoes from the bolt head to the interface surface between the clamped members and computes the stiffness of the members on the basis of frustum elasticity.

The theory is valid both for tapped thread joint (ESV) and bolted joint (DSV) and VDI standard unifies the formulation as in Equations (1) and (2); in the present paper all the formulas will be referred to the stiffness and not to the compliance of the joints.

If $D_A \geq D_{A,Gr}$

$$K_p = \frac{w \cdot E_p \cdot \pi \cdot d_h \cdot \tan\varphi}{2 \cdot \ln \dfrac{(d_w + d_h) \cdot (d_w + w \cdot l_K \cdot \tan\varphi - d_h)}{(d_w - d_h) \cdot (d_w + w \cdot l_K \cdot \tan\varphi + d_h)}} \tag{1}$$

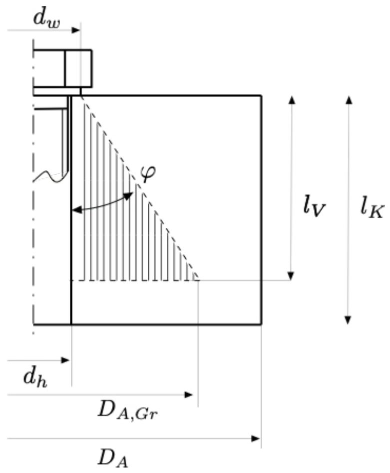

Fig. 1. Rotscher's cone.

if $d_w < D_A < D_{A,Gr}$

$$K_p = \frac{E_p \cdot \pi}{\frac{2}{w \cdot d_h \cdot \tan\varphi} \ln \frac{(d_w + d_h) \cdot (D_A - d_h)}{(d_w - d_h) \cdot (D_A + d_h)} + \frac{4}{D_A^2 - d_h^2} \cdot \left[l_K - \frac{D_A - d_w}{w \cdot \tan\varphi} \right]} \tag{2}$$

By analysing Equations (1) and (2), it is possible to rewrite them in simple form as $K_p = \frac{E_p \cdot A_n}{l_K}$. In particular, if $D_A \leq d_w$ the equivalent area A_n becomes

$$A_n = \frac{\pi}{4} \cdot \left(D_A^2 - d_h^2 \right) \tag{3}$$

if $1 < \frac{D_A}{d_w} \leq 3$

$$A_n = \frac{\pi}{4} \cdot \left(d_w^2 - d_h^2 \right) + \frac{\pi}{8} \cdot \left(\frac{D_A}{d_w} - 1 \right) \cdot \left(0.2 d_w \cdot l_K^* + \left(\frac{l_K^*}{10} \right)^2 \right) \tag{4}$$

if $\frac{D_a}{d_w} > 3$

$$A_n = \frac{\pi}{4} \cdot \left[(d_w + 0.1 \cdot l_K^*)^2 - d_h^2 \right] \tag{5}$$

where l_K^* is defined as the minimum value between the actual length of the clamped members or 8 times the diameter of the bolt or screw head, and w represents a flag for distinguishing the ESV ($w = 2$) from DSV ($w = 1$) conditions.

This kind of approach is not easy to apply, because an analysis of the joint geometry has to be performed in advance and sometimes is not easy to define which one of the previous equation has to be used. In addition, the results are not always in agreement with the experimental data. This theory is based on the presence of a reference geometry (frustum in the Rotscher's work, but other authors used different geometry like cylinders or spheres), that represents the area where the stress is exchanged between clamped members and bolts. This is an approximation that can be overcome by using a Finite Element (FE) model of the joint. The first work is from Wileman et al. (1991), where Ansys was used to estimate the deformation of the clamped member and an equation for the calculation of clamped member stiffness was proposed. The model proposed by Wileman results in an equation where two parameters A_m and B_m are related to the clamped member materials and the main parameters are the diameter of the screw shank and the length of the clamped member, so the stiffness becomes:

$$K_p = E \cdot A_m \cdot e^{B_m \cdot \frac{d}{l_K}} \tag{6}$$

In Lehnhoff et al. (1994) a wide FE analysis was conducted on bolted joints with different geometries and clamped materials and an easy equation was proposed for computation of the member stiffness based on the interpolation of results. The equation has the form of a parabola where coefficients change with respect to materials and geometry. The same approach is proposed in Al-Hiniti (2005).

In Filiz et al. (1996), by means of FE method, the stiffness of the members was studied considering the effects of bolt diameter, connection length and thickness ratio. A practical formula was suggested for the calculation of member stiffness:

$$K_p = \frac{\pi}{2} \cdot \frac{1}{1-\beta_2} \cdot d \cdot E \cdot e^{\left(\frac{\pi}{5}-\beta_1\right)\cdot \frac{d}{l_K}} \tag{7}$$

where $\beta_1 = \left(0.1 \cdot \frac{d}{l_K}\right)$ and $\beta_2 = \left(1 - \frac{l_1}{l_2}\right)^8$.

In Musto et al. (2006), an extension of the Wileman's work was proposed, taking into account different materials for the clamped members. It is proposed to use an effective elastic modulus, that combines the effects of the two materials:

$$E_{eff} = \frac{1}{\frac{1}{E_{ms}} + n \cdot \left(\frac{1}{E_{ls}} - \frac{1}{E_{ms}}\right)} \tag{8}$$

where subscript ms stands for the stiffer material and ls for the lesser one, and $n = \frac{l_{ls}}{l}$ is the ratio between the length of the clamped members.

With this approach, the clamped stiffness is computed as:

$$K_p = E_{eff} \cdot d \cdot \left(m \cdot \left(\frac{d}{l}\right) + b\right) \tag{9}$$

where m and b are based on the materials stiffness ratio.

In Nassar and Abdoud (2009), an analytical approach is developed and compared to FE analysis for better computing the equivalent area A. The authors divided formulation in two possible solutions on the basis of the stress distribution. If the stress envelope is completely inside the joint thickness ($D_A \geq l_K \cdot \tan\varphi + \gamma \cdot d$, with γ equal to the ratio between the contact area under the bolt head and bolt shank diameter) the clamped member stiffness is expressed as:

$$K_p = \frac{E_1 \cdot E_2 \cdot d \cdot \pi \cdot \tan\varphi}{(E_1 + E_2) \cdot \ln \dfrac{\gamma + 3}{\gamma - 1} + E_1 \cdot \ln \dfrac{\gamma \cdot d + 2 \cdot l_2 \cdot \tan\varphi - d}{\gamma \cdot d + 2 \cdot l_2 \cdot \tan\varphi + 3 \cdot d} + E_2 \cdot \ln \dfrac{\gamma \cdot d + 2 \cdot l_1 \cdot \tan\varphi - d}{\gamma \cdot d + 2 \cdot l_1 \cdot \tan\varphi + 3 \cdot d}} \tag{10}$$

If the stress is only partially developed inside the joint ($\gamma \cdot d < D_A < l_K \cdot \tan\varphi + \gamma \cdot d$):

$$K_p = \frac{E_1 \cdot E_2 \cdot \pi \cdot \tan\varphi}{\dfrac{E_1 + E_2}{d} \cdot \ln\left(\dfrac{\gamma + 3}{2 \cdot D_A + 3 \cdot d} \dfrac{D_A - d}{\gamma - 1}\right) + \dfrac{4 \cdot E_1 \cdot (2 \cdot l_2 \cdot \tan\varphi - D_A + \gamma \cdot d) + 4 \cdot E_2 \cdot (2 \cdot l_1 \cdot \tan\varphi - D_A + \gamma \cdot d)}{(D_a + 3d) \cdot (D_a - d)}} \tag{11}$$

Similar approach of Nassar and Abdoud (2009) is proposed in Haidar et al. (2011), where the difference is related to the pressure distribution assumed inside the clamped members. In such a case a third order polynomial distribution is assumed, and the obtained equations are closed to that of Nassar and Abdoud (2009):

$$K_p = \frac{0.5 \cdot E \cdot \pi \cdot \tan\varphi}{\dfrac{l_K}{d} \cdot \ln \dfrac{(3 \cdot \gamma + 7)(D_A - d)}{(3 \cdot D_A + 7d) \cdot (\gamma - 1)} + \dfrac{10 \cdot (\gamma \cdot d - D_A + l_K \cdot \tan\varphi)}{(3 \cdot D_A + 7 \cdot d) \cdot (D_A - d)}} \tag{12}$$

If the stress is only partially developed inside the joint ($\gamma \cdot d < D_A < l_K \cdot \tan\varphi + \gamma \cdot d$):

$$K_p = \frac{0.5 \cdot E \cdot \pi \cdot \tan\varphi}{\ln \dfrac{(3 \cdot \gamma + 7) \cdot (d - D_A + l_K \cdot \tan\varphi)}{(\gamma - 1) \cdot (3\gamma \cdot d + 7 \cdot d + 3 \cdot l_K \cdot \tan\varphi)}} \tag{13}$$

In Yildirim (1988) the equation for computing the clamped member stiffness was derived not considering the support of FEA but analysing experimental data. Without direct access to the publication Yildirim (1988), data to be reported here are collected in Canyurt and Sekercioglu (2015). The stiffness of the clamped members can be computed using:

$$K_p = 0.86 \cdot \frac{\pi}{4} \cdot d \cdot E \cdot \left(\frac{l_1}{l_2}\right)^{0.045 \cdot \frac{l_1}{l_2}} \cdot \left(\frac{l_K}{d}\right)^{-0.0075 \cdot l_K} \tag{14}$$

All the previous models require identification of a validity range or they are not complete in terms of ratio between bolt and clamped member action diameter, or they do not distinguish between ESV or DSV. According to those criticisms, the authors developed the following analysis procedure to settle down a unique formula able to easily compute the stiffness of the clamped members.

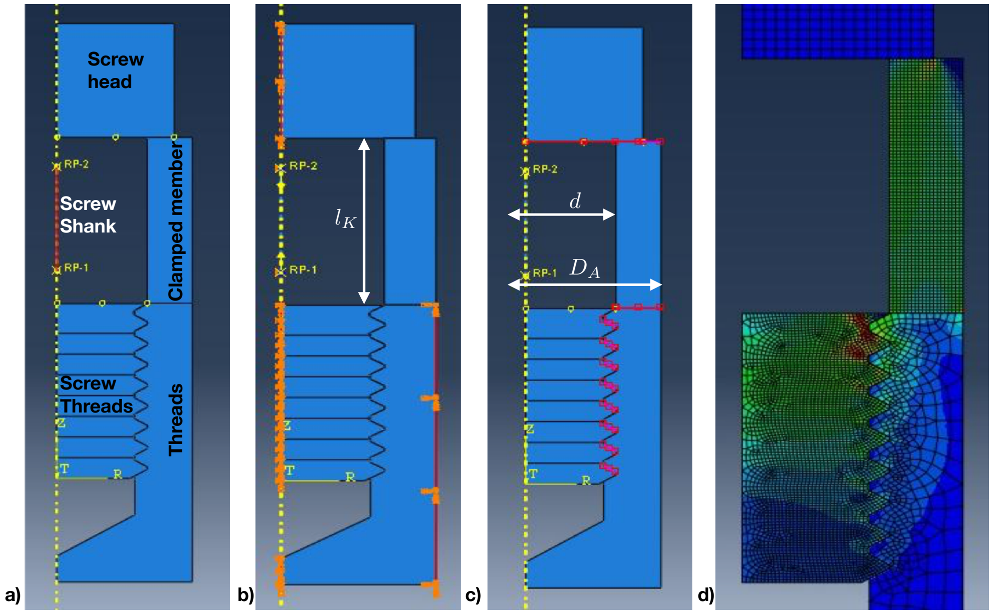

Fig. 2. Model for the tapped thread joint (ESV): (a) simulated elements, (b) constraints, (c) contact conditions and (d) example of stress results.

3. Analysis procedure

By using a commercial FE software, a parametric model of a half of clamped member cross section was developed, both for the ESV and DSV cases. Both the models are axisymmetric. The majority of the elements are modelled using 2D axisymmetric quadrangular element (4 nodes, reduced integration, plain stress), the screw shank is depicted as an uniaxial element (a spring) in order to reduce the number of degree of freedom and to maintain the requested accuracy. This mono-dimensional element is connected to the screw head and to the screw threads by means of two kinematic couplings.

In Figure 2 picture a) it is possible to see the elements that are modelled for the tapped thread joint (ESV); in particular, the screw head, the screw threads, the clamped element and the threads into the clamped element. In this case the separation between the not threaded part and the threaded one is placed at the beginning of the screw threads, as in the majority of the applications. In addition, the stiffness of the screw is not part of the investigation, so the presence of a complete threaded shank or partially threaded is not considered. Having a look of picture b), it is possible to see in orange the boundary conditions, and in particular along the axis of the screw the axisymmetric constraint, on the right side of the image the encastre condition for the threaded clamped part and finally the forces equal to the nominal axial force of the tightened screw applied on the points RP-1 and RP-2 in order to simulate the traction effect on the system. In picture c) the contact conditions are highlighted; they are imposed between engaged threads, between clamped members and between clamped members and screw head. A really fine mesh is used in the not threaded clamped part, whereas a coarser mesh is used for the rest because they are not directly investigated parts. In picture d) it is possible to appreciate the stress distribution into the model.

In Figure 3, the model used for the bolted joint (DSV) analysis is proposed. No great differences can be highlighted between the two models, except for the external dimension of the threaded clamped member, in fact in the DSV model this element is a nut with fixed dimension. All the constraints and boundary conditions are the same.

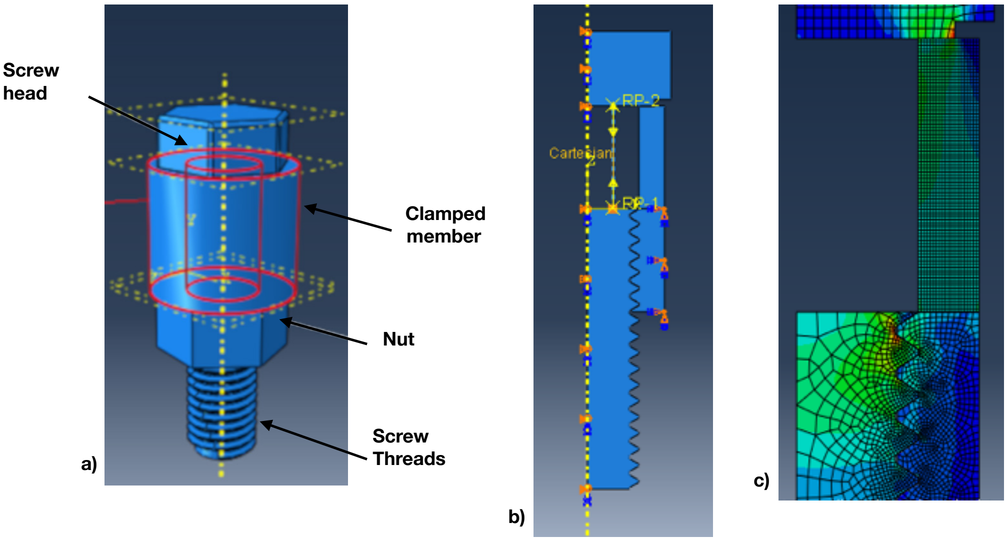

Fig. 3. Model for the bolted joint (DSV): (a) simulated elements, (b) constraints, (c) example of stress results.

Taking as reference the images in Figure 2, the main variables can be highlighted: the screw or bolt shank diameter d, the external reference diameter of the clamped member D_A and the clamped member length l_K. Those will be the main parameters that will be varied in the models in order to evaluate the stiffness of the clamped member. In this paper only the models referred to a screw or bolt shank diameter of 8 mm are taken into account. The tests are limited to the variation of the clamped member length l_K (from 1 to 12 times the bolt shank diameter), the external diameter D_A (from less than screw head diameter d_w to more than three times d_w) and the material of the non threaded clamped members (steel, aluminium, brass, cast iron and magnesium). The matrix of tests is populated with 49 cases for each material, for an overall number of test equal to 245 simulations for each model.

4. Numerical simulation

By running the simulations, a matrix of clamped member stiffness can be built. The variables are the external diameter and the length of clamped elements. In particular the length l_K assumes the values 1, 2, 4 , 6, 8, 10, 12 times the shank diameter, whereas the external diameter assumes values equal to 0.96, 1, 1.15, 1.92, 2.69, 3, 3.46 times the diameter of the screw head. Coefficient 1 and 3 are used to evaluate the transition events between the classical theory applied in Rotscher's theory, as described in Equations (3), (4) and (5). In Table 1 the parameter of the screw are highlighted; in Table 2 the matrix of results is reported.

In Figure 4 the comparison between FE analysis and VDI calculation is reported. As highlighted in literature, the VDI method is confirmed to overestimate the stiffness of the clamped member, in particular for increasing external diameter of the clamped member. A direct comparison with literature is not easy because all the parameters are not declared.

Once the data of the simulations are computed for all the models, by using a curve fitting method, based on regression procedure, it is possible to estimate an equation depending on the geometrical parameters of the joints. As a matter of fact, the dependency is clearly related to the ratio between external diameter of the clamped member and

Table 1. Screw features.

Mechanical feature	Data	Units
Shank diameter d	8	mm
Head diameter d_w	10.4	mm
Pitch	1	mm
Class	8.8	

Table 2. Stiffness results in N/mm.

$\frac{D_A}{d_w} \setminus \frac{l_K}{d}$	1	2	4	6	8	10	12
0.96	740045	371159	185841	123952	92986	74399	62005
1	915950	453591	227476	151802	113909	91155	75978
1.15	1372328	750583	393186	266360	201398	161910	135367
1.92	1871058	1399276	1000760	776462	634309	536151	464303
2.69	1906086	1492744	1216762	1047598	918058	817008	736004
3	1912782	1503268	1247923	1104153	988064	893852	816049
3.46	1909725	1508126	1268636	1154222	1058968	977496	907635

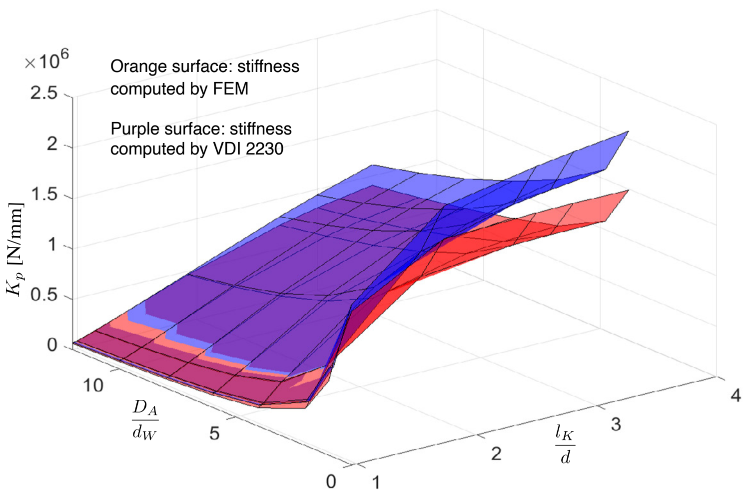

Fig. 4. Comparison between stiffness computed with VDI 2230 and the results of the simulations.

the diameter of the bolt head D_A/d_w and the ratio between thickness of the clamped members and the shank diameter l_K/d, as depicted in Table 2. By considering these two ratios as the variables of a space, the surface depicted in Figure 4 can be interpolated by means of a function of two variables. By slicing the graph of Figure 4 firstly maintaining constant D_A/d_w, assumed as x variable, and then by keeping constant l_K/d, assumed as y variable, it is possible to highlight that the so defined curves are power curves with the generic expression:

$$a = c \cdot x^d + e \tag{15}$$

By combining the two effects, the equation that can interpolate the surface obtained by the simulation is:

$$z = (c \cdot x^d + e) \cdot y^{f \cdot x^g} \quad (16)$$

By using the regression procedure, the coefficient c, d, e, f and g can be estimated. This process was performed for each model and for each material, by interpolating the matrix of results.

5. Results and discussion

By applying the above illustrated procedure, five equations are estimated both for tapped thread joint (ESV) and bolted joint (DSV). In this section the intermediate results are reported for ESV, just to illustrate the procedure for obtaining the final equation. The five equations for the different materials are summed up in Table 3.

Table 3. Equations for clamped member stiffness evaluation in tapped thread joint (ESV).

Material of the clamped member	Equation [N/mm]
Steel	$K_p = 10^6 \cdot \left[1.927 - \left(\frac{d_w}{D_A}\right)^{4.5}\right] \cdot \left(\frac{d}{l_K}\right)^{d_w/D_A}$
Aluminium	$K_p = 10^6 \cdot \left[0.836 - 0.501 \cdot \left(\frac{d_w}{D_A}\right)^{3.5}\right] \cdot \left(\frac{d}{l_K}\right)^{d_w/D_A}$
Brass	$K_p = 10^6 \cdot \left[1.077 - 0.631 \cdot \left(\frac{d_w}{D_A}\right)^{3.7}\right] \cdot \left(\frac{d}{l_K}\right)^{d_w/D_A}$
Cast iron	$K_p = 10^6 \cdot \left[1.39 - 0.8 \cdot \left(\frac{d_w}{D_A}\right)^{3.8}\right] \cdot \left(\frac{d}{l_K}\right)^{d_w/D_A}$
Magnesium	$K_p = 10^6 \cdot \left[0.558 - 0.351 \cdot \left(\frac{d_w}{D_A}\right)^{3.05}\right] \cdot \left(\frac{d}{l_K}\right)^{d_w/D_A}$

It is evident that the five equations have the same form, and the only difference lies in the coefficients. So the next step was identifying a possible unique expression for all the coefficients, just depending on the Young modulus of the material. The generic formula can be expressed as:

$$K_p = 10^6 \cdot \left[A - B \cdot \left(\frac{d_w}{D_A}\right)^C\right] \cdot \left(\frac{d}{l_K}\right)^{d_w/D_A} \quad (17)$$

By plotting the values A, B and C assumed by the five different equations with respect to the Young modulus of the material, it is possible to highlight a relationship. In particular a linear relationship can be recognised for A and B coefficient, whereas a polynomial relationship can be stated for the C coefficient. In Figure 5 the fitting process is highlighted and in Table 4 the results are listed.

Table 4. Coefficients for generalised equation of tapped thread joint (ESV).

Coefficient	Expression	R^2
A	$A = 8.664e^{-6} \cdot E + 0.1786$	0.995
B	$B = 4.147e^{-6} \cdot E + 0.188$	0.990
C	$C = 1.241e^{-15} \cdot E^3 - 4.505e^{-10} \cdot E^2 + 5.64e^{-5} \cdot E + 1.318$	0.999

By considering Equation (17) with the coefficient of Table 4 for the ESV joint and making an evaluation of the errors with respect to the interpolated equation of Table 3, the maximum error is about 6%.

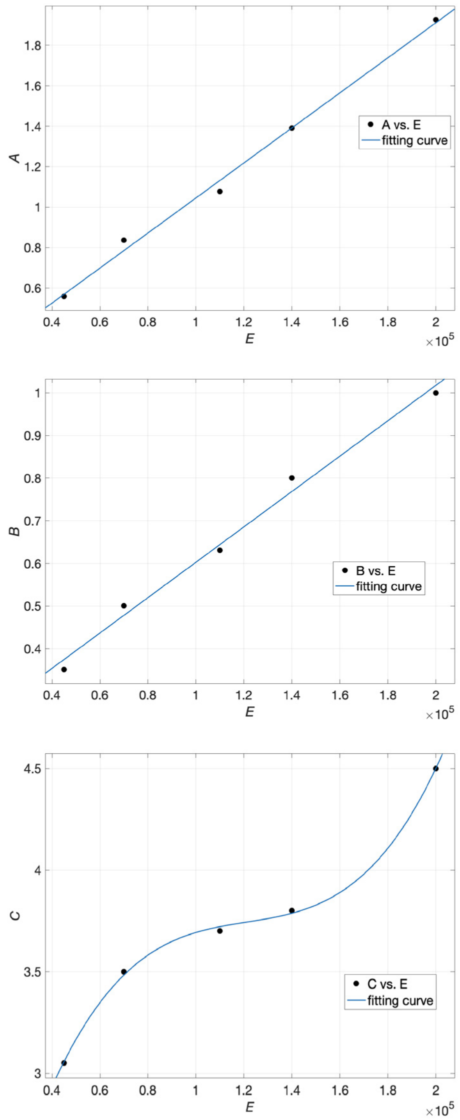

Fig. 5. Interpolation of the coefficients with respect to the Young modulus.

The same procedure was applied to the bolted joint (DSV) and the five equations are listed in Table 5. In this situation, the equations are a little bit different, as expected, from the previous ones of ESV. In addition, some coefficients of the exponent are not equal to 1, so it is not easy to build a unique formula as for the ESV. In the authors opinion this is due to the way the stiffness is evaluated. In both the models the method for defining the stiffness is the same, i.e. considering the maximum difference of displacement between the two faces of the clamped member and the clamping force. This procedure is easy to implement but maybe can provide not so accurate results. The authors are investigating this aspect and are planning to set experimental tests in order to evaluate the proposed methodology.

6. Conclusion

In the present paper the possibility to identify a unique formula for the computation of clamped member stiffness in bolted joints was investigated. By means of several parametric FE models, a matrix of possible geometric proportions

Table 5. Equations for clamped member stiffness evaluation in bolted joint (DSV).

Material of the clamped member	Equation [N/mm]
Steel	$K_p = 10^6 \cdot \left[1.89 - 0.981 \cdot \left(\frac{d_w}{D_A}\right)^{4.75}\right] \cdot \left(\frac{d}{l_K}\right)^{d_w/D_A}$
Aluminium	$K_p = 10^6 \cdot \left[0.804 - 0.471 \cdot \left(\frac{d_w}{D_A}\right)^{3.74}\right] \cdot \left(\frac{d}{l_K}\right)^{d_w/D_A}$
Brass	$K_p = 10^6 \cdot \left[1.097 - 0.634 \cdot \left(\frac{d_w}{D_A}\right)^{3.5}\right] \cdot 0.94 \cdot \left(\frac{d}{l_K}\right)^{0.91 \cdot d_w/D_A}$
Cast iron	$K_p = 10^6 \cdot \left[1.421 - 0.829 \cdot \left(\frac{d_w}{D_A}\right)^{3.74}\right] \cdot \left(\frac{d}{l_K}\right)^{0.96 \cdot d_w/D_A}$
Magnesium	$K_p = 10^6 \cdot \left[0.573 - 0.368 \cdot \left(\frac{d_w}{D_A}\right)^{3.1}\right] \cdot \left(\frac{d}{l_K}\right)^{0.99 \cdot d_w/D_A}$

of the joint both for tapped threaded and bolted joints was analised. The results were processed by means of surface and curve fitting algorithms and a series of equations were derived.

In the case of the tapped threaded joint a unique formula is proposed, capable to consider several materials and geometric ratios of the joints. The same was also investigated for bolted joint but some further analyses have to be performed. The main conclusion of this paper is that a unique formula for the joint clamped member stiffness can be proposed, surely an experimental campaign is needed for the validation of the proposed formula and a more accurate method for the estimation of the stiffness with FE models has to be investigated. In this validation, literature cannot be useful, so a dedicated study has to be defined.

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