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***Abstract*—The scattering of a plane electromagnetic wave by two inverted perfectly electrically conducting (PEC) half-planes loaded by a dielectric layer is studied by using the Wiener-Hopf technique in the spectral domain. The procedure to obtain the solution is based on the reduction of the factorization problem to Fredholm integral equations of second kind. The structure is of interest in antenna technologies, electromagnetic compatibility and electromagnetic shielding. The study of structural singularities allows also the estimation of surface waves and leaky waves for practical engineering applications.**

Keywords—Electromagnetic scattering, Diffraction, Wiener–Hopf method, Integral equations, Circuital modelling, Stratified regions, Wedge, Step, Analytical–numerical methods, Antenna technologies, Electromagnetic compatibility, Electromagnetic shielding, Surface Waves and Leaky Waves.

I. INTRODUCTION

In this paper we consider the classical canonical scattering problem constituted of two inverted staggered Perfectly Electrically Conducting (PEC) half planes separated by a dielectric layer.

Cartesian coordinates as well as polar coordinates will be used to describe the problem. Two origins are considered, see Fig. 1: the edge of the upper half-plane is chosen as origin O for coordinates (x,y,z) and (ρ, φ,z), while the edge of the lower half-plane is chosen as origin O' for coordinates (x_2,y_2,z_2) and (ρ_2, φ_2,z_2). The two reference systems are related through the following relations x_2=x-s, y_2=y+d.

Fig.1 shows the two PEC half planes with zero thickness respectively defined by $x<0, y=0, -\infty<z<\infty$ and $x>s, y=-d, -\infty<z<\infty$; the problem is with translational symmetry along the z axis.

In the following we will consider s as the staggered parameters along x and d as the distance along y between the two half-planes. While d can assume only positive values, s can be either positive as in Fig. 1 or negative. In the last case we have that the two half-planes partially overlap.

Three regions are identified: region 1 is the half space region delimited by y>0 filled by free-space, region 2 is the rectangular finite region with $-d<y<0$ and characterized by the dielectric with relative dielectric permittivity ε_r and region 3 is the half space region delimited by y<-d filled by free-space.

For the sake of simplicity, the structure is illuminated by an E-polarized plane wave from region 1 with azimuthal direction $\varphi=\varphi_o$ ($0<\varphi_o<\pi$) and with propagation constant k:

$$E_z^i = E_o e^{jk\rho\cos(\varphi-\varphi_o)} \tag{1}$$

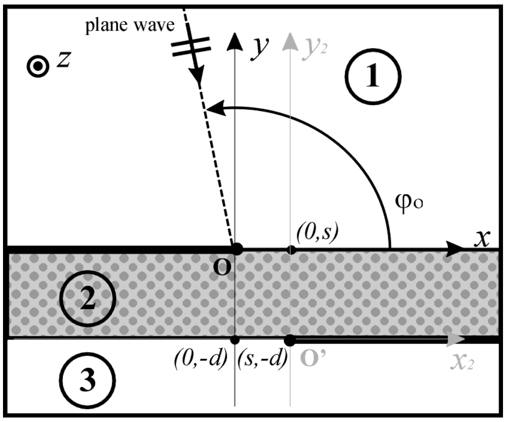

Fig. 1. Scattering by Two Inverted Staggered PEC Half-Planes Loaded by a Dielectric Layer

The literature shows several works that are related to this problem. In particular [1-2] study the problem in absence of dielectric layer via matrix Wiener-Hopf equations. These formulations highlight the presence of problematic exponential behavior of the spectral unknowns depending on the staggered parameter *s*. Other similar problems are described in [3-6].

With the reference to the present structure under investigation, in this work, we formulate the problem in terms matrix Wiener-Hopf equations in the spectral domain.

The procedure to obtain the solution is based on the reduction of the factorization problem to Fredholm integral equations of second kind. This technique is effectively applied in several scattering problems and described in [7,8].

The spectral formulation of the problem reported in the following section will take into consideration the interaction at near field of the edges of the two edges located at (x,y)=(0,0) and (x,y)=(s,-d) and the presence of the dielectric layer by using a unique entire model.

The solution steps are based on: 1) formulation via the Wiener-Hopf (WH) technique [8] with the help of network modelling [9-11], 2) the Fredholm factorization [7,8] and 3) asymptotic estimations of electromagnetic field [12].

II. Wiener-Hopf Formulation and Steps for the Solution

In region 1 the following WH equation (2) holds [8,13-14] in terms WH unknowns (3) defined at y=0.

$$I_{a-}(\eta)+I_{1+}(\eta)=Y_\infty(\eta)V_{1+}(\eta) \tag{2}$$

$$\begin{aligned}
&V_{1+}(\eta)=\int_0^\infty E_z(x,0)e^{j\eta x}dx,\\
&I_{1+}(\eta)=\int_0^\infty H_x(x,0)e^{j\eta x}dx\\
&I_{a-}(\eta)=\int_{-\infty}^0 H_x(x,0_+)e^{j\eta x}dx\\
&V_{1\pi+}(\eta)=\int_{-\infty}^0 E_z(x,0_\pm)e^{-j\eta x}dx=0\\
&I_{1\pi+}(\eta)=-\int_{-\infty}^0 H_x(x,0_-)e^{-j\eta x}dx
\end{aligned} \tag{3}$$

with $\xi(\eta)=\sqrt{k^2-\eta^2}, Y_\infty(\eta)=\xi(\eta)/k\,Z_o$ and the free space impedance Z_o. Note the PEC boundary condition at x<0, y=0.

The dielectric region 2 with PEC boundary conditions at x<0, y=0 and x>s, y=-d is modelled through the following spectral equation (4) [8,11,15] in terms Fourier transforms (5) according to the transmission line modelling of plane stratified media

$$\begin{aligned}
&i(\eta,0_-)=-Y_{11}(\eta)v(\eta,0)-Y_{12}(\eta)v(\eta,-d)\\
&i(\eta,-d_+)=Y_{21}(\eta)v(\eta,0)+Y_{22}(\eta)v(\eta,-d\;)
\end{aligned} \tag{4}$$

$$\begin{aligned}
&v\;(\eta,y)=\int_{-\infty}^\infty E_z(x,y)e^{j\eta x}dx\\
&i\;(\eta,y)=\int_{-\infty}^\infty H_x(x,y)e^{j\eta x}dx
\end{aligned} \tag{5}$$

Note that $Y_{11}(\eta)=Y_{22}(\eta)=-jY_d(\eta)\cot[\xi_d(\eta)d]=Y_l(\eta)$ and $Y_{12}(\eta)=Y_{21}(\eta)=Y_m(\eta)=j\dfrac{Y_d(\eta)}{\sin[\xi_d(\eta)d]}$, $\xi_d(\eta)=\sqrt{\varepsilon_r k^2-\eta^2}, Y_d(\eta)=\xi_d(\eta)/k\,Z_o$.

By defining the following WH unknowns (6) at y=-d we rewrite (4) as (7).

$$\begin{aligned}
&V_{2+}(\eta)=\int_0^\infty \dot{E}_z(x_2,0)e^{j\eta x_2}dx_2=e^{-j\eta s}\int_s^\infty E_z(x,-d)e^{j\eta x}dx=0\\
&I_{2+}(\eta)=\int_0^\infty \dot{H}_x(x_2,0_+)e^{j\eta x_2}dx_2=e^{-j\eta s}\int_s^\infty H_x(x,-d_+)e^{j\eta x}dx\\
&I_{d+}(\eta)=\int_0^\infty \dot{H}_x(x_2,0_-)e^{j\eta x_2}dx_2=e^{-j\eta s}\int_s^\infty H_x(x,-d_-)e^{j\eta x}dx\\
&V_{2\pi+}(\eta)=\int_{-\infty}^0 \dot{E}_z(x_2,0)e^{-j\eta x_2}dx_2=e^{j\eta s}\int_{-\infty}^s E_z(x,-d)e^{-j\eta x}dx,\\
&I_{2\pi+}(\eta)=-\int_{-\infty}^0 \dot{H}_x(x_2,0_+)e^{-j\eta x_2}dx_2=-e^{j\eta s}\int_{-\infty}^s H_x(x,-d_+)e^{-j\eta x}dx
\end{aligned} \tag{6}$$

$$\begin{aligned}
&-I_{1+}(\eta)+I_{1\pi+}(-\eta)=Y_{11}(\eta)V_{1+}(\eta)+Y_{12}(\eta)\exp(j\eta s)V_{2\pi+}(-\eta)\\
&I_{2+}(-\eta)-I_{2\pi+}(\eta)=Y_{21}(\eta)\exp(j\eta s)V_{1+}(-\eta)+Y_{22}(\eta)V_{2\pi+}(\eta)
\end{aligned} \tag{7}$$

Finally region 3 is modelled similarly to region 1 with the WH equation

$$-I_{d+}(-\eta)+I_{2\pi+}(\eta)=Y_\infty(\eta)V_{2\pi+}(\eta) \tag{8}$$

As described previously the factorization problem of the matrix WH equations (2), (7), (8) is reduced to Fredholm integral equations by eliminating the current unknowns via contour integrations. The unknowns in the system of Fredholm integral equations are the voltages $V_{1+}(\eta), V_{2\pi+}(\eta)$. Solution is performed by analytical-numerical techniques in terms of approximate spectra of the voltages.

Asymptotic estimation via UTD [12] of the total field in region 1 an 3 is performed via the voltages. The study of the structural singularities of the spectra [10] allows also the estimation of surface waves and leaky waves for practical engineering applications. Numerical validations and results will be shown during the presentation.

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