Summary

The purpose of this PhD thesis is to investigate the miscibility properties of bosonic binary mixtures in ring-lattice and few-site potentials. To this end, a number of different research topics are explored: on one hand, the study of the low energy-excitations of a mixed and uniformly distributed mixture, whose dynamical properties are investigated within a group-theoretic approach based on the application of the dynamical-algebra method; on the other hand, the systematic analysis of the mixing-demixing quantum phase transitions that can occur in these systems, and the introduction of suitable quantum indicators to characterize them. The latter are employed, among other things, to quantify the degree of entanglement between different bosonic modes, but also the degree of mixing and localization of the two quantum fluids. This analysis results in rather rich mixing-demixing phase diagrams, which capture the impact of *spatial fragmentation* onto the miscibility properties of the components. Our analysis indeed takes into account the possible asymmetry between the atomic species and, going also beyond the pointlike approximation of potential wells (by means of a suitable Gross-Pitaevskii treatment), constitutes an effective springboard towards an actual experimental realization. In this regard, we remark that a part of our study has been developed in strict collaboration with the experimental group from Hannover University, with which we have demonstrated that the investigated mixing-demixing quantum phase transitions are within the reach of modern experimental setups. Our analysis has touched, also, on excited states, as it has highlighted several *dynamical* properties of phase-separated states, evidencing, among various aspects, that demixing can be robust and persistent even in the presence of chaos. Eventually, we have have focused on *attractive* interspecies interactions and on the emergence of a rather general mechanism of formation of supermixed solitons.