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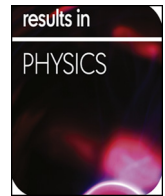
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Time & clocks: A thermodynamic approach

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ABSTRACT

Recently, the definition of time has been introduced by introducing the irreversible thermodynamics into the analysis of the atomic irreversibility. In this way, time is defined by means of the entropy generation and the entropy rate. But, which clock can be used to measure this time? In this paper a thermodynamic clock is designed by using the properties of a black body. So, a relation between the definition of time by entropy and a measure of temperature of a black body has been obtained. The result obtained agree with the Relativity and the thermal time hypothesis, obtaining a link between quantum mechanics, relativity and thermodynamics, by the irreversible thermodynamic clock operational approach.

Introduction

Time is a fundamental quantity in physical sciences and in our everyday life [1–4]. But, in this paper, we will focus our analysis only on the *physical time* of our Universe, based on some analytical results [5] obtained in relation to atomic irreversibility, by considering the continuous interaction between the environmental electromagnetic waves and the matter, due to the continuous non-equilibrium state of reality. In particular, from the analysis of the absorption–emission of a photon by an atomic electron, we have introduced a definition of time. The approach used was the introduction of irreversible thermodynamics into quantum mechanics [6]. Here, we will define the related clock, just by considering the irreversible thermodynamics [7–17].

From a physical point of view, the cosmic time is usually defined as the result of a measurement of a comoving standard clock [18]. In Newton approach, time is a mathematical variable, not necessary a real quantity, useful to evaluate the variation of physical quantities. For Newton simultaneity and time interval are absolute, even if time interval is an abstract property only related to the instants measured by a universal clock [19]. On the contrary, Einstein Relativity introduces a new concept of time related to the *postulate of invariance of the speed of light*: in Relativity, two inertial observers, in different states of motion, measure instants related to a same event with clocks synchronized with respects any single observer; consequently, durations depend on the observer immersed in a mathematical space, the space–time [20,21,19]. Moreover, following the results of Bell [22] and Pauli [23], Brown [24] highlighted that clocks measure some quantity related to the space–time, but not directly the time [19]. Last, recently, Rovelli pointed out that General Relativity describes the evolution of observable quantities

relative to one another, without an objective existence of time as a physical quantity [25,26,19]. Consequently, it has been pointed out the need of a meaning of time: a definition of time related to some physical phenomenon [18,20,21,19].

It is clear that a definition of time is difficult in terms of quantities or phenomena independent of time itself [27]. Consequently, everyone can only agree with a general definition of time as the result of the measurement of a clock. In relation to this definition, some questions arise:

- What is a clock?
- What kind of measurements?

Up today, any attempt to respond to these fundamental questions have always been referred to time itself, because, from a macroscopic point of view, time is a fundamental and basic concept.

Einstein [28] pointed out that the definition of time must be based on the clock measure, but it has been pointed out the need of physical meaning for time coordinates [25]. In the analysis of clocks, a time–clock relation has been introduced [18]; it states that there is a conceptually necessary relation between time and a physical process which functions as the core of a clock. This time–clock relation implies that a physical process must exist as the basis of a clock. Time and the physical process cannot be defined independently. The time–clock relation involves also a reference to a physical process in conformity with physical laws. Consequently, a well-defined use of time requires that time has a physical basis.

Moreover, at least one of the following requirements is necessary to time definition, in order to have a physical basis [18]; some physical

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processes:

- must have a duration shorter or equal to the time interval considered;
- can take place in the same order of the events considered.

In all the critical analyses of physical foundation of the time, the concept of duration has always been highlighted as a fundamental quantity [29,28,30–34]. A fundamental requirement is the time-clock relation, and we can improve it by considering that a global time coordinate t is the time measured by clocks of these observers; so, t is the proper time measured by an observer at rest with respect to the local matter distribution [35]. But, this concept requires a physical basis for the definition of time [18].

In this paper, we wish to suggest a completely new approach, based on the recent results in quantum mechanics and non-equilibrium thermodynamics, obtained by introducing the irreversibility in our analyses.

The thermodynamic approach

Recently, we have proven that time is the footprint of irreversibility of any subset of the Universe, due to the fluxes of energy and mass among them [36,5], introducing the definition of the time interval in relation to the local entropy and the local entropy rate, as follows [5,36]:

$$t = \frac{S}{\dot{S}} \quad (1)$$

We consider that the Universe is in a continuous non-equilibrium state, but we introduce the hypothesis that in any position it is possible to realise a local [37–40] thermodynamic equilibrium, such that we can introduce the definition of temperature T [K] in a statistical way as [37,38,40]:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_V \quad (2)$$

where S is the entropy [J K⁻¹] and U is the internal energy [J].

Now, we must obtain the value of the entropy generation and the entropy rate independently to time. In relation to the entropy rate we can maintain consider that [41]:

$$\frac{T_0}{A} \dot{S} = \frac{1}{2} \epsilon_0 c E_{el}^2 + \frac{1}{2\mu_0} c B_m^2 \quad (3)$$

where E_{el} is the electric field, B_m is the magnetic field, c is the velocity of light, ϵ_0 is the electric permittivity in vacuum and μ_0 is magnetic permeability in vacuum, and A is the area of the border of the thermodynamic control volume, i.e. the surface of the system considered. This relation refers to the electromagnetic wave related to the photon inflowing to the atom. So, we can try to obtain a macroscopic relation also for the photon after the interaction with the atom. To do so, we consider a black body, defined by Kirchhoff as physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence [42]. For a such body Planck introduced the concept of *resonator* for which he evaluated the irreversibility as introduced by Maxwell-Hertz and taking into account the Boltzmann hypothesis of molecular disorder, expressed by the Planck *natural radiation* concept [43]. He obtained the following relation for the entropy in the black body:

$$S = k_B \left[\left(1 + \frac{\bar{U}}{h\nu} \right) \ln \left(1 + \frac{\bar{U}}{h\nu} \right) - \frac{\bar{U}}{h\nu} \ln \left(\frac{\bar{U}}{h\nu} \right) \right] \quad (4)$$

where \bar{U} is the mean vibrational energy of the resonator. In this way, the relation (1) can be written as:

$$t = \frac{\left(1 + \frac{\bar{U}}{\epsilon} \right) \ln \left(1 + \frac{\bar{U}}{\epsilon} \right) - \frac{\bar{U}}{\epsilon} \ln \left(\frac{\bar{U}}{\epsilon} \right) k_B T_0}{\frac{1}{2} \epsilon_0 c E_{el}^2 + \frac{1}{2\mu_0} c B_m^2} \frac{c}{A} = \quad (5)$$

where T_0 is a reference temperature and $\epsilon = (A/2)\epsilon_0 E_{el}^2 + (A/2\mu_0)B_m^2$ is the energy of the electromagnetic radiation.

Now, we must find a relation for the measurement of the time interval. In accordance with the black-body radiation relation the energy density ρ_E of the radiation is related to the absolute temperature T [31]:

$$\rho_E = \frac{8\pi^5 k_B^4}{15h^3 c^3} T^4 f_1 \left(\frac{m_e}{k_B T} \right) \quad (6)$$

where $\pi = 3.141593$, $k_B = 1.380649 \times 10^{-23}$ J K⁻¹ is the Boltzmann constant, $h = 6.626070 \times 10^{-34}$ J s is the Planck constant, $c = 299792458$ m s⁻¹ is the velocity of light, T is the temperature, and [31]

$$f_1(x) = 1 + \frac{7}{4} \left(\frac{4}{11} \right)^{4/3} f_2^{4/3}(x) + \frac{30}{\pi^4} \int_0^\infty \frac{\sqrt{x^2 + y^2}}{1 + \exp(\sqrt{x^2 + y^2})} y^2 dy \quad (7)$$

and

$$f_2(x) = 1 + \frac{45}{2\pi^4} \int_0^\infty \frac{y^2}{1 + \exp(\sqrt{x^2 + y^2})} \left[\sqrt{x^2 + y^2} + \frac{y^2}{3\sqrt{x^2 + y^2}} \right] dy \quad (8)$$

At present $T \approx 2.7$ K microwave background, so [31] $\rho_E \approx 3.97 \times 10^{-14}$ J m⁻³.

Now, we consider the dynamical equation [31]:

$$\dot{R}^2 = \frac{8\pi G}{3} \rho_E R^2 \quad (9)$$

where R is the radius of the Universe and $G = 6.6732 \times 10^{-11}$ N m²kg⁻¹ is the gravitational constant. Moreover, the entropy density can be written as [31]:

$$s = \frac{4}{3} \frac{8\pi^5 k_B^4}{15h^3 c^3} (RT)^3 f_2 \left(\frac{m_e}{k_B T} \right) \quad (10)$$

Now, we introduce by using the relations (5)–(10), the relation between temperature at time in the Universe results [31]:

$$t = - \int \left[\frac{8\pi G}{3} \frac{8\pi^5 k_B^4}{15h^3 c^3} T^4 f_1 \left(\frac{m_e}{k_B T} \right) \right]^{-1/2} \cdot \left[d \ln T + \frac{1}{3} d \ln \left(f_2 \left(\frac{m_e}{k_B T} \right) \right) \right] \quad (11)$$

where $m_e = 9.1093826 \times 10^{-31}$ kg is the mass of the electron. Temperature has been highlighted to be connected to the energy scale.

In relation to the Eqs. (9)–(11) we must highlight that

- The Eq. (9) depends on initial conditions considered; indeed, following the Standard Model of General Relativity [31], we can assume that in the early universe, matter was very nearly uniformly distributed and very nearly in thermal equilibrium at uniform temperature;
- The Eq. (9) is believed to be true only for the early universe [31]; however, this issue might not matter in terms of a local time because locally the universe is flat,

consequently, the results obtained in this paper maintain their validity.

In thermodynamic equilibrium, following the Planck and Einstein results on the analysis of black body radiation, a well known distribution of the photons among the various quantum states with definite values of the oscillation frequency ν corresponds to a defined temperature T . The average frequency grows proportionally to the temperature as $\nu = k_B T/h$. But, the periodic time of oscillation τ is inversely proportional to the frequency, such that:

$$\tau = \frac{1}{\nu} = \frac{h}{k_B} \frac{1}{T} = \frac{4.799243 \times 10^{-11} [\text{Ks}]}{T} \quad (12)$$

which allows us to obtain a relation between a clock (time interval measurement device) and the temperature.

So, it follows:

$$t = n\tau = n \cdot \frac{4.799243 \times 10^{-11} [\text{Ks}]}{T} \approx n \cdot \frac{4.80 \times 10^{-11} [\text{Ks}]}{T} \quad (13)$$

where $n \in \mathcal{N}$. From this relation, we can highlight that:

- Time results a discrete quantity, in accordance with the approach of discrete time of Riek [44];
- Time is the result of the irreversibility in the Universe, consequently it isn't possible to realise reversible clocks;
- Even if the entropy locally decreases, the entropy generation (the entropy variation due to irreversibility) always increases; consequently, time continues to increase, in accordance with the arrow of time;
- The thermodynamic approach holds to a result in accordance with the fundamental results of Briggs, who has shown that only macroscopic classic clocks can be realised [45].

The clock designed allows us to obtain the time interval in relation to the temperature measurements; some values are summarised in Table 1.

Discussion

Here, in relation to the problems related to time definition, we can point out our results:

- Our definition of time is related to the entropy, of which variation is extended to any epoch and domain, since the Universe formation, because any process generates entropy variation;
- Our definition of time is linked to entropy variation and fluxes, so, our definition satisfies that the physical basis for the time scale might break down in the very early universe due to phase transitions;
- How present physical scales might be extrapolated into the past? This extrapolation can be introduced just by considering the entropy and the temperature of any epoch of the Universe, its formation included.

Moreover, local time flow rate is different in relation to global Universe time flow rate, because global Universe flow rate is the global effect of the entropy rate generation, while the local time follows the distribution of the local entropy variation and rate. But, this consideration is in accordance with the Theory of Relativity.

The fundamental problem consist in designing a clock able to measure time in relation to our theoretical results. In this context, the clock introduced can be classified as follows:

- Gravitational clocks: the measurement of a duration is obtained in two different ways:
 - Indirectly: the time scale is obtained by using the positions of particular celestial bodies;
 - Directly: the time scale is obtained by a pendulum;
- Balance-wheel clocks: the time scale is obtained by the periodic rotation of arms around an axis, movement generated by a spiral spring that develops an elastic force;
- Quartz clocks: the time scale is obtained by the oscillations of a quartz resonator;
- Atomic clocks: the time scale is obtained by the energy difference between two fixed quantum states in an atom;
- Radioactive clocks: the time scale is obtained by the decay which produces a quantity of daughter substance from a well known amount of initial parent substance.

Here we have introduced a clock which link time to temperature by introducing the behaviour of a black body. Our approach is based on the concepts of entropy, energy and interaction (fluxes): these quantities are well defined in any position of the Universe. Moreover, we have also introduced the hypothesis of local equilibrium in order to define temperature. But, it has been pointed out the impossibility of the conversion of a number of oscillations into a time standard when the oscillation frequency isn't linked to some physical process setting a scale [18]. Just to avoid this difficulty, we have introduced the definition of time in relation to the entropy generation and entropy production rate.

Moreover, we have considered the photons with a well-defined frequency emitted by a black body in a given rest mass and referred to a temperature scale. In this way, the oscillation frequency is completely linked to the temperature by the analytical relation of dissipation in quantum system.

Our results agree with the thermal time hypothesis [25,26,19] which states that a preferential time doesn't exist in nature because there are no equilibrium states preferred *a priori*. The system is in an arbitrary state to which a *thermal* time is linked. But, we have also introduced the definition of time by using the irreversibility, expressed by the non-negative entropy generation, so that the arrow of time is part of the time physical definition itself.

Our approach, based on the irreversible thermodynamics (the analysis of fluxes and irreversibility at atomic level) hold to a link between quantum, relativistic, and thermodynamic approaches.

Last, we wish to highlight that our results agree with Lorentz transformation; indeed [23,31]:

- Entropy and entropy generation are invariant, so time presents a general definition in relation to the Eq. (1)
- Temperature satisfies the Lorentz transformation:

$$T = \frac{T_0}{\gamma} \Rightarrow \tau = \gamma \tau_0 \quad (14)$$

Table 1

Some values of time intervals in relation to temperature measured by using the Eq. (13).

T	t [s]						
[K]	n = 1	n = 10	n = 10 ²	n = 10 ³	n = 10 ⁴	n = 10 ⁵	n = 10 ⁶
1	4.80 × 10 ⁻¹¹	4.80 × 10 ⁻¹⁰	4.80 × 10 ⁻⁹	4.80 × 10 ⁻⁸	4.80 × 10 ⁻⁷	4.80 × 10 ⁻⁵	4.80 × 10 ⁻⁴
10	4.80 × 10 ⁻¹²	4.80 × 10 ⁻¹¹	4.80 × 10 ⁻¹⁰	4.80 × 10 ⁻⁹	4.80 × 10 ⁻⁸	4.80 × 10 ⁻⁶	4.80 × 10 ⁻⁵
10 ²	4.80 × 10 ⁻¹³	4.80 × 10 ⁻¹²	4.80 × 10 ⁻¹¹	4.80 × 10 ⁻¹⁰	4.80 × 10 ⁻⁹	4.80 × 10 ⁻⁷	4.80 × 10 ⁻⁶
10 ³	4.80 × 10 ⁻¹⁴	4.80 × 10 ⁻¹³	4.80 × 10 ⁻¹²	4.80 × 10 ⁻¹¹	4.80 × 10 ⁻¹⁰	4.80 × 10 ⁻⁸	4.80 × 10 ⁻⁷
10 ⁴	4.80 × 10 ⁻¹⁵	4.80 × 10 ⁻¹⁴	4.80 × 10 ⁻¹³	4.80 × 10 ⁻¹²	4.80 × 10 ⁻¹¹	4.80 × 10 ⁻⁹	4.80 × 10 ⁻⁸
10 ⁵	4.80 × 10 ⁻¹⁶	4.80 × 10 ⁻¹⁵	4.80 × 10 ⁻¹⁴	4.80 × 10 ⁻¹³	4.80 × 10 ⁻¹²	4.80 × 10 ⁻¹⁰	4.80 × 10 ⁻⁹
10 ⁶	4.80 × 10 ⁻¹⁷	4.80 × 10 ⁻¹⁶	4.80 × 10 ⁻¹⁵	4.80 × 10 ⁻¹⁴	4.80 × 10 ⁻¹³	4.80 × 10 ⁻¹¹	4.80 × 10 ⁻¹⁰

where $\gamma = 1/\sqrt{1 - (v/c)^2}$, with v relative speed of the system considered in relation to a rest frame of reference and c speed of light, with the result of obtaining the same time dilatation of Relativity [31].

In relation to the last bullet, we must introduce some comments. At present, there is a huge debate about the Lorentz invariance of temperature. Any deep discussion of this problem is outside of the aim of this paper, but we wish to consider some recent results on this topic [46]. In his paper, Mareš and colleagues has suggested a new result in relation to the question: “Is the body moving with the velocity v relatively to the rest system of coordinates in its own coordinate system colder or hotter than if it were measured in the rest system of coordinates?” [46]. Mareš and colleagues have shown that:

- The Kelvin temperature has to be Lorentz invariant;
- Entropy can no more be Lorentz invariant.

In relation to our results on the time, we have introduced a definition of temperature in relation to the entropy and the internal energy. Consequently, the different results on the debate on Lorentz invariance of temperature allow our results to be however confirmed.

Results and conclusions

In this paper we have obtained the clock which can measure time in relation to temperature. This result represents the required step to prove the definition of time as obtained by the irreversible thermodynamic analysis of the quantum systems.

Indeed, following Einstein, any definition of time must be related to a clock.

In this paper, we have just obtained the physical bases for the designing of a clock, able to link time to temperature, by introducing the results of Planck on the black body. We have named this kind of clock as Constructural clock, because our definition of time has been obtained by considering the photon inflow (and outflow) into the atom and the related irreversibility due to the interaction photon-atomic electron, and in Thermodynamics the Constructural law is the approach related to fluxes.

Authors contributions

U.L. developed the quantum thermodynamic model. U.L. and G.G. designed the thermodynamic clock.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.rinp.2020.102977>.

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