The effect of gravity of the plastic zones on the behavior of supports in very deep tunnels excavated in rock masses

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ABSTRACT

The vertical load acting on a support structure is affected by the loss of self-bearing capacity of the rock inside the plastic zone. This load can then be accounted for by analytical calculation methods capable of evaluating the stresses in the tunnel support system to proceed with the tunnel design. Generally, the effect of the rock's own weight in the plastic zone is considered in a simplistic way by evaluating an additional vertical load given by the weight of the rock due to the thickness of the plastic zone. This approach leads to a significant increase in the vertical load with the risk of overdesigning the support structure. In this work, the effect of the rock's own weight in the plastic zone was considered by modifying the numerical solution of the convergence-confinement method for tunnels built in rock. In this way, through the intersection of the characteristic curve of the tunnel and the intersection line of the support structure, it is possible to determine both the vertical loads (with the effect of the weight of the rock) and the horizontal load (without the effect of weight of the rock). The application of the method to a project in the Alps allowed to detect the magnitude of the percentage increase of the vertical load and a significant increase in the thickness of the plastic zone with the consequences that this may have on the designing of the radial bolting length in that zone. Increasing the plastic radius leads to an increase in the length of the bolts. This is interesting because in the area of the crown where the weight of the plasticized rock is considered the bolts are usually installed with a greater length. In the final part of the paper a new procedure is illustrated to define the vertical and horizontal loads acting on the support structures, starting from the convergence-confinement curves, obtained for the crown and for the lateral areas (sides).

KEY WORDS: convergence-confinement method; hyperstatic reaction method; base tunnel; plastic radius; rock

ABBREVATIONS AND NOTATION LIST

a exponent of the failure criterion of Hoek and Brown

A_s area of the lining section

D parameter that varies between 0 and 1, which considers the disturbance of the rock mass due to the excavation operations

E_r elastic modulus of the rock mass in the plastic field

F_n limit strength of the elastic-plastic behavior of the normal springs

f_r component of body forces per unit volume in the radial direction

f_θ component of body forces per unit volume in the tangential directions

I_s inertia moment along the lining section

k stiffness of the lining

K_n stiffness of the normal interaction springs

K_s stiffness of the shear interaction springs

k_0 earth pressure at rest

M bending moments along the support

m_b strength parameter, which depend on the GSI (Geological Strength Index)

m_i strength parameter that refers to intact rock and which depends on the typology of the rock

N normal forces along the support

p pressure inside the tunnel

p_{cr} critical pressure

p_0 natural lithostatic stress at the tunnel depth

r_0 tunnel radius

R_{pl} plastic radius

s strength parameter, which depend on the GSI (Geological Strength Index)

u_{crown} support displacements at the crown of the tunnel

u_R radial displacement

u_{sides} support displacements at the sides of the tunnel

$\varphi_{res,app}$ apparent residual friction angle of the rock mass

σ_{Rpl} radial stress at the plastic radius

σ_r radial stress

$\sigma_{R,crown}$ vertical load applied at the crown of the support

$\sigma_{R,sides}$ horizontal load applied at the side of the support

$\sigma_{r,i}$ incremental radial stress

$\sigma_{\theta,i}$ incremental tangential stress

σ_θ tangential stress

σ_1 principal maximum stress

$\sigma_{1,lim}$ maximum principle stress upon failure of the rock mass

σ_3 principal minimum stress

$\tau_{R\theta}$ shear stress

θ angle representing the evaluation point in the circular support in the tunnel

γ weight of the rock

ψ dilatancy expressed in radians

ν Poisson ratio of the rock mass

CC Characteristic curve

CCM Convergence-confinement method

GSI Geological Strength Index

HRM Hyperstatic reaction method

INTRODUCTION

Determination of the stresses and displacements around circular openings has been one of the most fundamental problems in geotechnical, petroleum, and mining engineering. Design of tunnel liners and the validation of numerical models are among the practical applications of displacement analysis around circular openings. The most common support design techniques are based on analytical approaches, such as the hyperstatic reaction method, HRM (Do et al., 2014a; Oreste et al., 2018a; 2018b) and the Einstein and Schwartz (1979) method. However, these analytical methods require knowledge of the loads acting on the support structures. The loads depend on:

- The dimension and depth of the tunnel;
- The geomechanical characteristics of the ground;
- The stiffness characteristics of the support structure itself and;
- The distance from the excavation face where the structure is to be installed.

Interaction between the rock mass and the support system is generally evaluated with the convergence-confinement method (CCM). CCM describes the relation between the decreasing tunnel internal pressure and the increasing tunnel radial convergence and can be constructed from elasto-plastic analysis of a circular tunnel subjected to hydrostatic far-field stress and uniform internal pressure (see Panet, 1995; Peila and Oreste, 1995; Oreste, 2009; 2014; Spagnoli et al., 2016; 2017). CCM allows to obtain an estimate of the loads acting on the supporting structures, proceeding with the intersection of the ground characteristic curve of the tunnel and the characteristic curve of the supporting structure.

In the base tunnels in rocks, the extension of the plastic zone around the tunnel may be significant, above all when the GSI value is low, the strength of the intact rock is reduced, the tunnel radius is high and the lithostatic pressure is high. The development of an adequate plastic zone is, however, necessary also to be able to contain the loads on the supporting works. Rock with plastic behavior loses a significant part of its resistance and does not generally have the ability to self-sustain itself. For this reason, it is prudent in the design phase, to consider the rock with plastic behavior in the calculation of the applied loads to the support system with its own weight.

A large body of work currently exists on the stress and deformation analysis of tunnels with the consideration of different failure criteria and rock mass behaviors including the elastic-perfectly plastic, elastic-brittle-plastic and elastic-strain-softening models (e.g., Brown et al., 1983; Carranza-Torres and Fairhurst, 1999; Carranza-Torres, 2004; Alonso et al., 2003; Park and Kim, 2006; Lee and Pietruszczak, 2008; Park et al., 2008; Fahimifar and Hedayat, 2008; 2009; Fahimifar et al., 2010; Hedayat, 2016). CCM is also applied to rock masses, presenting a non-linear rupture criterion, as described by Hoek and Brown (1980).

In the present work a procedure to obtain the characteristic curve of the tunnel considering the effect of the rock weight in the plastic zone is presented. An example of a calculation, relating to a tunnel built in the Alps will allow to evaluate the weight of the rock in the plasticized zone on the loads acting on the support. The final analysis with HRM will allow to detect the importance of the additional load on the tension state that develops in the support work and, therefore, on the stability conditions of the support itself.

THE CONVERGENCE-CONFINMENT METHOD CONSIDERING GRAVITY EFFECT

When the internal pressure in tunnels falls below a critical pressure, p_{cr}, a plastic zone develops around the tunnel. The tunnel conditions typically assumed for elasto-plastic analysis include a deep circular tunnel in a continuous, homogeneous, isotropic, and initially elastic rock mass subjected to the hydrostatic far-field stress.

In base tunnels with high overburden pressure, the initial stress state of the rock mass generally approaches the hydrostatic conditions (k_0 is approximately equal to 1). As the internal pressure decreases, the tunnel radial convergence increases. Brown et al. (1983) summarized a large number of solutions obtained for an axisymmetric tunnel problem and presented a closed-form solution for rocks with elastic-brittle plastic behavior as well as a step-wise sequence of calculations for rock with an elastic-strain-softening behavior. Wang (1996) improved the accuracy of the solution in predicting the plastic radius. Carranza-Torres (2004) proposed a rigorous, elasto-plastic solution by rewriting the generalized Hoek-Brown failure criterion in terms of transformed stress quantities.

In the present work a detailed analysis method of CC for rock tunnels will be employed, based on a numerical solution to finite differences (Oreste, 2014). The rock around the tunnel is subdivided into several thin concentric rings, in which the values of apparent cohesion and of the apparent friction angle are continuously determined, linearizing the Hoek-Brown strength criterion according to the value of the radial tension reached. The dilatancy is determined for each concentric ring, as a predefined percentage of the apparent residual friction angle of the specific ring considered. The calculation is repeated for each concentric ring, varying the internal pressure acting on the perimeter of the tunnel. Provided that the inner pressure falls below a critical pressure, p_{cr}, a plastic region of radius R_{pl} develops around the tunnel. Because the dead weight of the broken zone around the tunnel can significantly increase the required support pressure at the roof, the effect of gravity must be considered in the interaction between the ground and the support system. In other words, the dead weight of the broken zone exerts higher pressures to the support system at the crown of the tunnel and needs to be considered in the elasto-plastic analysis of the tunnel. It is suggested that the readers refer to Oreste (2014) for more detailed information. Limited work has been conducted on the effect of the gravitational forces acting on the ground (Hoek and Brown, 1980; Detournay, 1984; Panet 1995; Zareifard and Fahimifar, 2012). To account for the gravity effect, Hoek and Brown (1980) and Panet (1995) suggested an increase in the required support pressure by the amount of $\gamma(R_{pl} - r_0)$, where γ is the unit weight of the rock and r_0 is the tunnel radius. This adjustment assumes that the full weight of the broken zone at the tunnel crown will be transferred to the support system, resulting in ground response curves that are too conservative. Therefore, there is a critical need to study the true interaction between the ground and the supporting system. The weight of the broken zone around the tunnel can be so high that it may affect the tunnel stability. The most critical point is the tunnel crown and the stability assessment needs to be carried out by the construction of the ground response curve at the crown.

Assuming a state of hydrostatic stress field and plane strain condition around a circular tunnel, the equilibrium equations in the radial and tangential directions taking the gravity forces into account are defined by assessing the stress state. Consider a small element ABCD shown in Fig. 1, since the tunnel problem is under plane strain condition, all stress components are functions of radial and tangential directions. Hence, it is possible to use polar coordinate instead of cylindrical one. Let f_r and f_θ be the components of the body forces per unit volume in radial and tangential directions, respectively (see Fig. 1).

Summation of forces parallel to the radial direction through the center of element yields:

$$\left(\sigma_r + \frac{\partial \sigma_r}{\partial r} dr\right)(r + dr)d\theta - \sigma_r r d\theta - \left[\sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta - \sigma_\theta\right] dr \sin\frac{d\theta}{2} + \left[\tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial \theta} d\theta - \tau_{r\theta}\right] dr \cos\frac{d\theta}{2} + f_r r dr d\theta = 0 \quad (1)$$

Since $d\theta$ is infinitesimal, $\sin\frac{d\theta}{2}$ and $\cos\frac{d\theta}{2}$ can be replaced by $\frac{d\theta}{2}$ and unity, respectively. Neglecting the small quantities of higher order and dividing the above equation by $rdrd\theta$, the equation of equilibrium in radial direction can be found as follows:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} + f_r = 0 \quad (2)$$

Similarly, summation of tangential components of forces may result in the equilibrium equation in direction.

$$\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} + f_\theta = 0 \quad (3)$$

As the mentioned tunnel problem has axial symmetry, the radial and tangential stresses in the rock mass will be principal stresses (i.e. $\sigma_r = \sigma_3$ and $\sigma_\theta = \sigma_1$) and consequently the radial equilibrium equations can be reduced to:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + f_r = 0 \quad (4)$$

In above equation, f_r is the component of body forces per unit volume in the radial direction. Since the construction of ground response curve at the roof of the tunnel, which is the most critical point, is of great importance, γ , unit weight of the rock mass, must be substituted for f_r in the equilibrium equation. In fact, at the roof of the tunnel, the gravity forces per unit volume are radially toward the tunnel center and are equal to γ. Thus, the equilibrium equation within the broken zone can be rewritten as:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + \gamma = 0 \quad (5)$$

Fig. 2a demonstrates schematically the body forces acting at the roof of the tunnel according to equation (4). Figure 2b shows the graphical representation of the theoretical ground response curve at the roof with and without consideration of the gravity effect from the plastic zone. The ABC curve represents the ground response curve without consideration of the gravity forces while curve ABMD demonstrates the ground response curve with consideration of gravity forces. The curve ABMD represents comparably larger values of convergence at a given internal pressure. The existence of a minimum M in the ground response curve shows the importance of the dead weight of the broken zone compared with the in situ stresses.

Zareifard and Fahimifar (2012) developed an interesting elasto-plastic, analytical-numerical solution, which considers the curvilinear strength criterion of the previous solution of Hoek and Brown (1988). This solution applies to a strain-softening behavior of the rock and links some fundamental rock parameters in the plastic field to the actual deviatoric plastic strain and to the critical value of the deviatoric plastic strain. In order to represent the rock behavior correctly, sophisticated laboratory tests are necessary in order to define in detail the parameters required by the adopted behavior model. Taking into account that the solution of Zareifard and Fahimifar (2012) does not consider the updated generalized failure criterion (i.e. Hoek et al., 2002), equation 6 has also been used in the numerical procedure formulated by Oreste (2014), in order to obtain the description of the CC of the tunnel considering the effect of the weight of the plastic zone around the tunnel. Besides, regarding the dilatation of the rock mass (difficult to evaluate in the laboratory), this is linked in the proposed procedure to the residual friction angle of the rock mass, as a fixed percentage of the latter.

THE PROPOSED NUMERICAL MODEL

The last update of the failure criterion of Hoek and Brown (Hoek et al., 2002) for rock masses has the following expression:

$$\sigma_{1,lim} = \sigma_3 + \sigma_{ci} \cdot \left(m_b \cdot \frac{\sigma_3}{\sigma_{ci}} + s \right)^a \qquad (6)$$

where: $\sigma_{1,lim}$ is the maximum principle stress upon failure of the rock mass;

σ_3 is the minimum principle stress (confinement);

σ_{ci} is the uniaxial compression strength (UCS) of the intact rock;

m_b and s are the strength parameters, which depend on the GSI (Geological Strength Index) (Marinos and Hoek, 2000; Marinos et al., 2005) and on the parameter D:

$$m_b = m_i \cdot e^{\left(\frac{GSI-100}{28-14 \cdot D}\right)}; \; s = e^{\left(\frac{GSI-100}{9-3 \cdot D}\right)}$$

D is a parameter that varies between 0 and 1, which considers the disturbance of the rock mass due to the excavation operations (D=0 for non-disturbed mass; D=1 for intensely disturbed mass);

m_i is a strength parameter that refers to intact rock and which depends on the typology of the rock;

a is the exponent that is present in eq. 6: $a = 0.5 + \frac{1}{6} \cdot \left(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}} \right)$.

For the good and medium-quality rock masses exhibiting a fragile elasto-plastic behavior it is necessary to describe two failure criteria, one for the peak conditions and another for the residual conditions. The former governs the stress state in the rock mass at the elastic limit, the latter the stress state in the plastic zone. The resistance parameters are duplicated for the two-different peak and residual criteria: m_{bp}, s_p, a_p, m_{bres}, s_{res}, a_{res}. Peak parameters can be determined by using the GSI referring to the initial conditions of the rock mass, whilst the residual ones to a suitably reduced GSI (GSI_{res}) (Oreste, 2014).

In order to deal with the study of the conditions of the rock mass in the plastic zone, the criterion of residual failure of Hoek and Brown can be locally linearized by deriving it from the minimum principle stress σ_3, which is the radial stress σ_r:

$$\frac{d\sigma_{1,lim}}{d\sigma_3} = 1 + a_{res} \cdot m_{bres} \cdot \left(m_{bres} \cdot \frac{\sigma_3}{\sigma_{ci}} + s_{res}\right)^{a_{res}-1} \quad (7)$$

This derivative allows to directly obtain the apparent residual friction angle $\varphi_{res,app}$ of the rock mass, according to the existing minimum stress σ_3:

$$sin\varphi_{res,app} = \frac{a_{res} \cdot m_{bres} \cdot \left(m_{bres} \cdot \frac{\sigma_3}{\sigma_{ci}} + s_{res}\right)^{a_{res}-1}}{a_{res} \cdot m_{bres} \cdot \left(m_{bres} \cdot \frac{\sigma_3}{\sigma_{ci}} + s_{res}\right)^{a_{res}-1} + 2} \quad (8)$$

The numerical procedure for defining the characteristic curve of the tunnel occurs is as follows (Oreste, 2014):

1. Calculation of the radial stress at the plastic radius (σ_{Rpl}): if this value is less than 0, no plastic zone is created around the tunnel; if it is greater than zero, a plastic zone is created for pressures inside the tunnel (p) lower than σ_{Rpl}; the evaluation of σ_{Rpl} occurs by solving the following equation numerically:

$$p_0 - \sigma_{Rpl} = \frac{\sigma_{ci}}{2} \cdot \left(m_{bp} \cdot \frac{\sigma_{Rpl}}{\sigma_{ci}} + s_p\right)^{a_p} \quad (9)$$

2. In the presence of a plastic zone, we proceed with a finite difference method dividing the rock at the outline of the tunnel in concentric rings. Starting from the tunnel outline, where the radial stress is equal to the applied internal pressure (p), the stress is increased radially by a small amount of $\Delta\sigma_r$ value at each ring, until the σ_{Rpl} value is reached. At each value of $\sigma_{r,i}$,

the corresponding value of $\sigma_{\theta,i}$ is used, employing the residual failure criterion of Hoek and Brown;

3. By replacing in Equation 6 the term (σ_θ-σ_r) derived from the criterion of residual failure of Hoek and Brown:

$$\sigma_\theta - \sigma_r = \sigma_1 - \sigma_3 = \sigma_{ci} \cdot \left(m_{bres} \cdot \frac{\sigma_3}{\sigma_{ci}} + s_{res}\right)^{a_{res}} \quad (10)$$

and transforming the equation obtained in incremental terms, the following expression is obtained:

$$\frac{\sigma_{r,i+1} - \sigma_{r,i}}{r_{i+1} - r_i} = \frac{\sigma_{ci} \cdot \left(m_{bres} \cdot \frac{(\sigma_{r,i+1} + \sigma_{r,i})/2}{\sigma_{ci}} + s_{res}\right)^{a_{res}}}{(r_{i+1} + r_i)/2} - \gamma \quad (11)$$

which allows to obtain all the values of r_{i+1} useful for determining the distances of the lateral surfaces of the concentric rings, comprised between the outline of the tunnel and the plastic radius R_{pl}; this procedure also allows to obtain the value of the plastic radius R_{pl} ;

4. Starting then from the plastic radius backwards towards the tunnel contour, the deformation problem is solved. Knowing the stress state in all the concentric rings, the value of the radial displacement u is obtained on each surface of the concentric rings on the basis of the following differential equation valid for the plastic field under axisymmetric conditions:

$$\frac{du}{dr} = \frac{(1-\nu^2)}{E_r} \cdot \left[(\sigma_r - p_0) \cdot \left(1 - N_\psi \cdot \frac{\nu}{1-\nu}\right) + (\sigma_\theta - p_0) \cdot \left(N_\psi - \frac{\nu}{1-\nu}\right)\right] - N_\psi \cdot \frac{u}{r} \quad (12)$$

where: $N_\psi = \frac{1+sen\psi}{1-sen\psi}$

ψ is the dilatancy expressed in radians (dilatancy is an angle that can vary between 0 and the residual friction angle of the material);

ν is the Poisson ratio of the rock mass;

E_r is the elastic modulus of the rock mass in the plastic field (it can be evaluated referring to the value of $\mathrm{GSI}_{\mathbf{res}}$).

5. This differential equation is also transformed into incremental terms in order to be able to use it in the adopted finite difference method;
6. The dilation angle ψ is changed at each concentric ring, determining it as a percentage value of $\varphi_{res,app}$ evaluated through equation 9, where for σ_3 the mean radial stress at the specific ith-ring is considered:

$$\sigma_3 = \frac{(\sigma_{r,i+1} + \sigma_{r,i})}{2} \quad (13)$$

Once the outline of the tunnel is reached, it is possible, therefore, to obtain the radial displacement u_R to be associated with the acting internal pressure p. The pairs of values $p - u_R$ allow to trace the characteristic curve of the tunnel, considering the weight of the rock material present in the plastic zone. A simplified sketch of the plastic zone for the case considered in this paper is shown in Fig. 3.

RESULTS AND DISCUSSION

The proposed solution was adopted in relation to a tunnel built in the Alps in metamorphic rocks such as calcareous schists with a GSI of 35 (GSI_{res} = 35). The general geology of the area comprises zones of contact between units of the **co**ntinental crust and units preserving the characters of oceanic crust. The unit where the tunnel is designed consists of a crystalline basement and limited-strength permo-mesozoic metasediments mainly micascists and calcareous schists. The initial section consists of calcareous schists then a gneiss with massive-to-foliated structure is encountered.

This tunnel was assumed to be circular with an equivalent radius of 5m. The average overburden was assumed as 170m (p_0 = 5MPa). The parameters of the rock mass are assumed as following:

- σ_{ci} of the intact rock is 60MPa;
- m_i = 12;
- disturbance factor D = 1;
- angle of dilatancy ψ equal to the angle of residual friction (associated flow rule assumption);
- Poisson ratio, ν, equal to 0.3.

On the basis of these data, the following parameters have been estimated:

- elastic module E = 1635 MPa;
- parameter of resistance m_b equal to 0.115;
- parameter of resistance s equal to $2x10^{-5}$;
- parameter of resistance a = 0.519;
- the unit weight γ of the rock was evaluated at 30 kN/m^3.

Fig. 4 shows the CCs of the tunnel, in the radial displacement range between 0.05 and 0.25m. The curves referring to three cases are shown:

1. Case 1: Without considering the weight of the rock in the plastic zone (black continues line);
2. Case 2: Considering the weight of the rock in the plastic zone according to the calculation procedure illustrated in this article (grey dashed line);
3. Case 3: Considering the additional load due to the weight of the plasticized rock, according to the conservative approach of Hoek and Brown (1980) and Panet (1995) (black dotted line).

In the same figure, the reaction line of the lining is shown in black, consisting of a 25cm thick fiber-reinforced sprayed concrete lining. Assuming an average elastic modulus during the setting time equal to 10000MPa, a stiffness, *k*, of the lining is obtained equal to 105MPa/m, which provides the slope to the reaction line. The load on the lining is obtained from the intersection of the reaction line with the characteristic curve:

1. Case 1: 1.57MPa
2. Case 2: 1.62MPa
3. Case 3: 1.87MPa

It can be noted that the solution proposed in this article leads to an increase in load on the linings of 3.2% due to the weight of the rock in the plasticized zone, while the simplified solution considers an additional load equal to the weight of the rock multiplied by the thickness of the plastic zone (case 3) leading to a considerable increase of 19.1% of the original load (case 1).

Figure 5 shows the trend of the plastic zones for case 1 (absence of weigh of the rock) and case 2 (proposed solution). In case 3, the trend coincides with case 1. It can be seen that the proposed solution leads to a noticeable increase of the plastic radius until reaching a value of 29.4m for internal pressure p equal to 0, compared to 19.4m of the case 1.

The proposed solution of the convergence-confinement method, therefore, provides vertical loads slightly higher than the original ones, however with a significant increase in the thickness of the plastic zone with respect to the case a) (without considering the weight of the rock inside the plastic zone).

Three different load conditions on the lining have been studied from the three cases examined in Fig. 4:

- condition a) refers to the presence of the same load in the crown and on the sides of the tunnel, without considering the effect of the rock's own weight in the plastic zone (case 1);
- condition b) with the vertical load obtained from the proposed calculation procedure (modified by Oreste, 2014) (case 2) considering the weight of the rock in the plastic zone and the horizontal load without considering the weight of the rock in this zone (case 1);
- condition c) considering the simplified method of Hoek and Brown (1980) to determine the effect of the weight of the rock in the plastic zone at the crown (case 3, simplified method) and the horizontal load without considering the weight of the rock (case 1).

Figure 6 shows the trend of the bending moments along the development of half of the support (starting from the center of the inverted arch up to the center of the crown) by applying the loads as described above:

- Condition a): 1.57MPa in the vertical direction (case 1) and 1.57MPa in the horizontal one (case 1);
- Condition b): 1.62MPa in the vertical direction (case 2) and 1.57MPa in the horizontal one (case 1);
- Condition c): 1.87MPa in the vertical direction (case 3) and 1.57MPa in the horizontal one (case 1);

These results were obtained using the hyperstatic reaction method (see Oreste, 2007; Do et al. 2014a; 2014b; Oreste et al., 2018a and 2018b for more details). This method provides for the subdivision of the support into one-dimensional elements placed in succession, so as to represent its entire development. The connection points between the elements are called nodes and on them the springs simulating the interaction of the support with the rock face, both in the normal direction and in the shear direction, are anchored. The loads acting on the support are represented with nodal forces and the solution of the problem consists in obtaining the displacements of the nodes of a support structure subjected to vertical and horizontal loads. Once the nodal displacements have been obtained along the structure it is then possible to obtain the trend of the bending moments and of the normal forces, useful for verifying the static conditions of the support. The following data were used in the calculation:

- Stiffness of the normal interaction springs K_n: 490.5MN/m;
- Stiffness of the shear interaction springs K_s: 245.2MN/m;
- Limit strength of the elastic-plastic behavior of the normal springs F_n: 0.26 MN;
- Cohesion of the support-rock interface: 0.2MPa;
- Friction angle at the support-rock interface: 25°.

Because of the non-symmetry of the applied load, bending moments appear along the lining, which reaches maximum values in the center of the invert, in the center of the crown and in the middle of the side wall of the tunnel. These maximum bending moments, together with the values of the normal force (Fig. 7) in correspondence with the same points at maximum moment, allow to derive the internal stresses developing in the sprayed concrete linings. By comparing the maximum internal stresses acting in the sprayed concrete with the strength of the sprayed concrete, it will be possible to verify whether the hypotheses on the thickness of the lining and on the quality of the sprayed concrete are compatible with the stability of the tunnel.

The analysis of the results shown in Fig. 6 allows to verify how the load condition c), which is based on the evaluation of the vertical load in a simplified way, would lead to an overestimation of the maximum bending moments of 600% with respect to the load condition b), for which the evaluation of the vertical load was performed using the proposed calculation method. In the load condition a), the bending moments are zero, thanks to the symmetry of load in the two directions (vertical and horizontal). With regard to the normal forces (figure 7), a constant value is detected along the support in the load condition a); in the load condition of b) the maximum normal force is 3.2% higher than the value of the load condition a); finally, in the condition of load c) the maximum normal force is about 19% higher than that found in the load condition a) and about 15% with respect to that obtained under load condition b). It can be noted that the simplified methods, proposed for the determination of vertical load due to the weight of the plastic rock band, lead to a sensible overestimation of the maximum bending moments in the support and a minor overvaluation even of the maximum normal forces.

ANALYSIS OF THE LINING-TUNNEL INTERACTION USING THE OBTAINED CONVERGENCE-CONFINEMENT CURVES OF A BASE TUNNEL

Considering the two convergence-confinement curves at the crown and for the sides of the tunnel that can be obtained from the indicated procedure (Fig. 8A), there is a link load-displacement characterizing the perimeter of the tunnel in the crown point and at the sides, based on the behavior of the rock mass present at the boundary of the tunnel. It is also possible to determine the behavior of a supporting structure in a simplified manner on the basis of its axial and flexural stiffness, referring to the studies by Einstein and Schwartz (1979). By combining the two behaviors (of the tunnel and of the supporting lining) it is possible to reach the exact determination of the actual loads acting in the crown and at the sides of the tunnel, the displacements shown by the lining and finally the bending moments and the normal forces that they develop along the support structure. In this way it is possible to make an initial hypothesis about the type and size of the support structure and then verify whether it is able to resist the bending moments and the normal forces induced inside it. More specifically, referring to the work of Einstein and Schwartz (1979), it is possible for the absence of sliding between the support and the rock wall (no-slip case) to obtain the following equations of the support displacements at the crown (u_{crown}) and at the sides (u_{sides}) of the tunnel:

$$u_{crown} = \frac{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides})}{2 \cdot (d^2 - e^2) \cdot E} \cdot R \cdot (1+\nu) \cdot \left\{ a_0^* \cdot \left[1 + \frac{1}{d} \cdot \left(\frac{\sigma_{R,sides} \cdot (d^2 - e^2)}{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides})} - e \right) \right] - \left[1 - \frac{1}{d} \cdot \left(\frac{\sigma_{R,sides} \cdot (d^2 - e^2)}{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides})} - e \right) \right] \cdot h \right\} \quad (14)$$

$$u_{sides} = \frac{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides})}{2 \cdot (d^2 - e^2) \cdot E} \cdot R \cdot (1+\nu) \cdot \left\{ a_0^* \cdot \left[1 + \frac{1}{d} \cdot \left(\frac{\sigma_{R,sides} \cdot (d^2 - e^2)}{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides})} - e \right) \right] + \left[1 - \frac{1}{d} \cdot \left(\frac{\sigma_{R,sides} \cdot (d^2 - e^2)}{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides})} - e \right) \right] \cdot h \right\} \quad (15)$$

where: $\sigma_{R,crown}$ is the vertical load applied on the support at the crown;

$\sigma_{R,sides}$ is the horizontal load applied on the support at the sides;

R is the tunnel radius;

E and ν are respectively elastic modulus and Poisson modulus of the rock mass;

$e = \frac{1}{2} \cdot (1 - a_0^*) - (1 - 6 \cdot a_2^* + 4 \cdot b_2^*)$;

$d = \frac{1}{2} \cdot (1 - a_0^*) + (1 - 6 \cdot a_2^* + 4 \cdot b_2^*)$;

$h = 4 \cdot (1 - \nu) \cdot b_2^* - 2 \cdot a_2^*$;

$f = \frac{1}{d} \cdot \left[\frac{\sigma_{R,sides} \cdot (d^2 - e^2)}{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides})} - \mathrm{e} \right]$;

$a_2^* = \beta \cdot b_2^*$;

$$\beta = \frac{(6+F^*)\cdot C^*\cdot(1-\nu)+2\cdot F^*\cdot \nu}{3\cdot F^*+3\cdot C^*+2\cdot C^*\cdot F^*\cdot(1-\nu)};$$

$$b_2^* = \frac{C^*\cdot(1-\nu)}{2\cdot[C^*\cdot(1-\nu)+4\cdot\nu-6\cdot\beta-3\cdot\beta\cdot C^*\cdot(1-\nu)]};$$

$$a_0^* = \frac{C^*\cdot F^*\cdot(1-\nu)}{C^*+F^*+C^*\cdot F^*\cdot(1-\nu)};$$

$C^* = \frac{E\cdot R\cdot(1-\nu_s^2)}{E_s\cdot A_s\cdot(1-\nu^2)}$ (where A_s is the area of the lining section);

$F^* = \frac{E\cdot R^3\cdot(1-\nu_s^2)}{E_s\cdot I_s\cdot(1-\nu^2)}$ (where I_s is the inertia moment of the lining section).

Considering the convergence-confinement curve referred to the conditions at the crown (CCC crown, obtained with the presence of the weight of the rock in the plastic zone), for each point belonging to the CCC, the values of $\sigma_{R,crown}$ and u_{crown} are determined and are inserted in equation 14. From this equation $\sigma_{R,sides}$ is obtained and inserted in equation 15. Then, from this u_{sides} is obtained.

We proceed moving on the convergence-confinement curve referred to the crown conditions, until the pair of values $\sigma_{R,sides} - u_{sides}$ obtained from equations 14 and 15 are compatible with the convergence-confinement curve referred to the conditions at the sides (CCC sides, obtained without considering the weight in the plastic zone).

When compatibility is found, i.e. the correspondence between the behavior of the rock to the tunnel contour and the behavior of the support, the procedure stops and the values of the loads assessed on the crown $\sigma_{R,crown}$ and on the sides of the tunnel $\sigma_{R,sides}$ are used to determine bending moments (M) and normal forces along the support (N), by changing the angle θ which is the evaluation point in the circular support in the tunnel (see Fig. 8B):

$$M = \frac{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides}) \cdot R^2}{4 \cdot (d^2 - e^2)} \cdot (1 - f) \cdot (1 - 2 \cdot a_2^* + 2 \cdot b_2^*) \cdot cos(2\theta) \qquad (16)$$

$$N = \frac{(d \cdot \sigma_{R,crown} - e \cdot \sigma_{R,sides}) \cdot R}{2 \cdot (d^2 - e^2)} \cdot [(1 + f) \cdot (1 - a_0^*) + (1 - f) \cdot (1 + 2 \cdot a_2^*) \cdot cos(2\theta)] \qquad (17)$$

CONCLUSIONS

During design of the supporting structures it is often necessary to evaluate the load applied by the rock mass to the outline of the tunnel, above all when using analytical methods of wide application such as the hyperstatic reaction methods. Generally, load estimation is performed using the convergence-confinement method, where the characteristic curve of the tunnel intersects with the support reaction line.

In the vertical direction, due to the loss of self-bearing of the rock present in the plastic zone, it is necessary to consider the weight of the rock in the study of the evolution of stress and deformations. In this work the methodology in which this aspect can be taken into consideration has been presented. The numerical solution of the convergence-confinement method introduced by Oreste (2014) has been modified to take into account the weight of the rock within the plastic zone. In this way it is possible to obtain the modified characteristic curve in order to reach the correct evaluation of the vertical load acting on the supporting work. As for the horizontal load, however, it is possible to still refer to the original convergence-confinement curve, without taking into account the weight of the plasticized rock.

The proposed solution was then applied to a case of a tunnel built in the Alps. Based on the available data, the convergence-confinement curves of the tunnel were drawn and the loads acting on the linings were evaluated. These loads were then used in the hyperstatic reaction method in order to obtain the trend of the bending moments and of the normal forces along the development of the lining.

From the study developed, it was possible to detect how the proposed solution allows to evaluate vertical loads greater than a few percentage units compared to the case of weightlessness of the plastic zone. The simplified solution considering an additional vertical load equal to the weight of the rock by the thickness of the plastic zone, on the other hand, involves significant increases of about 20%, which are not justifiable in practical terms. Moreover, from the example of calculation carried out, it has been possible to identify a non-negligible increase in the value of the plastic radius when considering the weight of the plastic zone. This increase in the plastic radius can be of great interest due to the repercussions it may have in defining the length of the radial bolting in the crown area.

Finally, a specific and quick procedure for the evaluation of the mechanical behavior of the support of a base tunnel has been illustrated, referring to the convergence-confinement method and to the method of analysis of a circular support by Einstein and Schwartz (1979). This procedure starts from determining the two convergence-confinement curves of the tunnel (relative to the crown zone, considering the weight of the plastic zone, and to the lateral areas, without considering the weight of the plastic rock). More specifically, for each point of the convergence-confinement curves of the crown zone (CCC crown) the values of the horizontal load and of the horizontal displacement of the support at the sides of the tunnel are evaluated, compatible with the mechanical behavior of the support. The procedure continues until this pair of values is compatible with the convergence-confinement curves of the lateral zone of the tunnel, i.e. until the stress and deformation analysis of the rock mass is compatible with the stress and deformation analysis of the supporting structure. At that point it is possible to obtain the actual loads (vertical and horizontal) acting on the supporting lining and, therefore, the trend of the bending moments and of the normal forces along the development of the supporting structure.

DISCLAIMER

BASF is not involved in any form with the research presented in this paper and it is only the current affiliation of one of the authors.

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FIGURE CAPTION

Fig 1. Stresses on element ABCD in polar coordinate.

Fig. 2. (a) Schematic representation of the body forces at the tunnel roof. b) Graphical demonstration of the ground response.

Fig. 3. Schematic representation of the tunnel with radius, *R*, of 5m with an overburden pressure, p_0, of 5MPa excavated in a rock with GSI_{peak} of 35 and UCS, σ_{ci}, of 60MPa showing a plastic radius R_{pl} of 7.3m without considering the weight effect (not to scale).

Fig. 4. Convergence-confinement curves of the tunnel analyzed in the three considered cases

Fig. 5. Trend of the plastic radius of the tunnel as the internal pressure changes for case 1 (weightlessness of the rock in the plastic area) and case 2 (proposed solution).

Fig. 6. Trend of the bending moments along half of the support obtained with the hyperstatic reaction method, applying the vertical and horizontal loads derived from the analysis with the method of the characteristic curves and the solution proposed in this work.

Fig. 7. Trend of normal forces along half the support obtained by the hyperstatic reaction method, applying the vertical and horizontal loads derived from the analysis with the method of the characteristic curves and the solution proposed in this work.

Fig. 8. A). Procedure for the determination of the load values on the support ($\sigma_{R,crown}$ and u_{crown}) from the convergence-confinement curve referred at the condition at crown (CCC crown) and from that referred at the condition at sides (CCC sides). Key: p is the internal

pressure in the tunnel; u is the radial displacement of the tunnel wall; p_0 is the natural lithostatic load at the tunnel depth; p_{cr} is the internal pressure below which a plastic region at the tunnel contour is formed; $\sigma_{R,crown}$ and $\sigma_{R,sides}$ vertical load acting on the support respectively at the crown and at sides; u_{crown} and u_{sides} are the displacement support respectively at the zone of crown and sides; B) Contact stresses at lining-rocks interface.