On stabilization of parameterized macromodeling

On stabilization of parameterized macromodeling

Alessandro Zanco, Stefano Grivet-Talocia, Tommaso Bradde, Marco De Stefano
Dept. Electronics and Telecommunications, Politecnico di Torino, Torino, Italy
{alessandro.zanco,stefano.grivet,tommaso.bradde,marco.destefano}@polito.it

***Abstract*—We propose an algorithm for the identification of guaranteed stable parameterized macromodels from sampled frequency responses. The proposed scheme is based on the standard Sanathanan-Koerner iteration in its parameterized form, which is regularized by adding a set of inequality constraints for enforcing the positiveness of the model denominator at suitable discrete points. We show that an ad hoc aggregation of such constraints is able to stabilize the iterative scheme by significantly improving its convergence properties, while guaranteeing uniformly stable model poles as the parameter(s) change within their design range.**

I. Introduction

Signal and power integrity assessment through numerical simulation heavily relies on efficient and accurate models of all system parts that have an influence on the quality of signals. In particular, electrical interconnects and their parasitics pose significant challenges due to their complexity, both in terms of geometry, material characteristics, as well as resulting frequency responses. For this reason, macromodeling algorithms [1] have gained tremendous popularity due to their ability to produce accurate and reliable simulation models, which run very efficiently on legacy circuit solvers.

This paper focuses on a particular algorithm [2], [3] for producing linear macromodels from sampled frequency responses, while also embedding in a closed form the dependence on one or more additional parameters related to geometry or material properties. The resulting macromodels can be used to perform sensitivity, what-if, statistical, and optimization studies in a pre-layout design phase. With respect to traditional (univariate) macromodeling, the multivariate (parameterized) macromodeling schemes pose additional challenges for what concerns stability and passivity, which must be guaranteed uniformly throughout the space where the external parameters are defined.

Several approaches that are able to enforce stability and passivity by construction exist [4], [5]. Almost invariably, these methods achieve parameterization by post-processing through dedicated interpolation schemes a possibly large set of non-parameterized macromodels computed for fixed parameter configurations. Both the interpolation schemes and the coordinate system in which such interpolation is performed can indeed be optimized and targeted to preserve stability and passivity [6].

We address a different class of algorithms, which do not require such two-step procedure. A parameterized model is constructed in a single step by processing an entire set of frequency responses avaiable, e.g., from a field solver parametric sweep [2]. However, if no special countermeasures are taken, uniform stability (and passivity) are not enforced and the resulting model might be unusable. In this work, we propose a regularized iterative scheme based on the Parameterized Sanathanan-Koerner (PSK) iteration [3], [7]. In addition to provably guaranteed uniform stability, this algorithm demonstrates significantly improved convergence properties, thanks to a set of accumulated inequality constraits that are adaptively formulated and aggregated at each iteration. The performance of proposed approach is illustrated on one simple antenna structure.

II. Background and motivation

We consider a P-port electrical/electromagnetic structure parameterized by ρ independent parameters $\vartheta^1, \ldots, \vartheta^\rho$ collected in vector $\boldsymbol{\vartheta} \in \Theta \subset \mathbb{R}^\rho$. All parameters are normalized so that Θ is a ρ-dimensional hypercube. We aim at the construction of a reduced-order model whose $P \times P$ scattering responses $\mathbf{H}(s;\boldsymbol{\vartheta})$ approximate the true system response $\breve{\mathbf{H}}_{k,m} = \breve{\mathbf{H}}(s_k;\boldsymbol{\vartheta}_m)$ assumed to be available from some measurement or field solver at $\bar{k}$ discrete frequencies $s_k = \mathrm{j}\omega_k$ and $\bar{m}$ parameter samples $\boldsymbol{\vartheta}_m$.

The model is constructed by enforcing

$$\mathbf{H}(\mathrm{j}\omega_k;\boldsymbol{\vartheta}_m) \approx \breve{\mathbf{H}}_{k,m}, \quad k = 1,\ldots,\bar{k},\ m = 1,\ldots,\bar{m}, \quad (1)$$

based on the standard rational model structure

$$\mathbf{H}(s;\boldsymbol{\vartheta}) = \frac{\mathbf{N}(s,\boldsymbol{\vartheta})}{\mathsf{D}(s,\boldsymbol{\vartheta})} = \frac{\sum_{n=0}^{\bar{n}} \mathbf{R}_n(\boldsymbol{\vartheta})\,\varphi_n(s)}{\sum_{n=0}^{\bar{n}} r_n(\boldsymbol{\vartheta})\,\varphi_n(s)}, \quad (2)$$

where $\varphi_n(s) = (s-q_n)^{-1}$ is the partial fraction basis related to the n-th "basis" pole q_n. Numerator and denominator coefficients $\mathbf{R}_n(\boldsymbol{\vartheta})$ and $r_n(\boldsymbol{\vartheta})$ depend on the parameter $\boldsymbol{\vartheta}$ through suitable multivariate basis function expansions (see [2] for details), e.g. through orthogonal or trigonometric polynomials. Model identification is carried out through the so-called *Parameterized Sanathanan-Koerner* (PSK) iteration [8]

$$\min \left\| \frac{\mathbf{N}^\mu(\mathrm{j}\omega_k,\boldsymbol{\vartheta}_m) - \mathsf{D}^\mu(\mathrm{j}\omega_k,\boldsymbol{\vartheta}_m)\,\breve{\mathbf{H}}_{k,m}}{\mathsf{D}^{\mu-1}(\mathrm{j}\omega_k,\boldsymbol{\vartheta}_m)} \right\|_F^2 \quad (3)$$

where the denominator D^0 is initialized at 1 and $\mu = 1, 2, \ldots$ is the iteration index. This scheme is a simple iteratively reweighted least squares fit, which stops when the estimates of the denominator coefficients stabilize.

In its simplest form, the PSK iteration is not able to enforce model stability. In fact, even though the initial basis poles q_n are stable, the parameterization of denominator coefficients $r_n(\boldsymbol{\vartheta})$ may lead the poles of $\mathbf{H}(s;\boldsymbol{\vartheta})$ (the zeros of $\mathsf{D}(s,\boldsymbol{\vartheta})$) to have positive real part. In [3], [7] it has been demonstrated that model stability for all $\boldsymbol{\vartheta} \in \Theta$ can be guaranteed by ensuring the denominator $\mathsf{D}(s,\boldsymbol{\vartheta})$ to be a passive immittance function satisfying the following Positive-Realness (PR) condition

$$\Re\{\mathsf{D}(\mathrm{j}\omega,\boldsymbol{\vartheta})\} > 0, \quad \forall\omega, \ \forall\boldsymbol{\vartheta} \in \Theta \tag{4}$$

where operator $\Re\{\cdot\}$ extracts the real part. This property motivated the introduction of a stability-preserving PSK algorithm presented in [3], [7], which enforces (4) at discrete frequency/parameter points by embedding a set of linear inequality constraints

$$\Re\{\mathsf{D}^{\mu}(\mathrm{j}\omega,\boldsymbol{\vartheta})\} > 0, \quad \forall(\mathrm{j}\omega,\boldsymbol{\vartheta}) \in \mathcal{V}^{\mu-1} \tag{5}$$

when solving (3) at each iteration μ, where the set

$$\mathcal{V}^{\mu-1} = \{(\mathrm{j}\omega_r^{\mu-1},\boldsymbol{\vartheta}_r^{\mu-1}), \ r = 1,\ldots,\bar{r}_{\mu-1}\} \tag{6}$$

collects the location of all negative local minima of $\mathsf{D}^{\mu-1}(\mathrm{j}\omega,\boldsymbol{\vartheta})$ in $(\omega,\boldsymbol{\vartheta}) \in [0,+\infty) \times \Theta$. These are extracted in a pre-processing phase by applying a parameterized passivity check [9] to the denominator function $\mathsf{D}^{\mu-1}(s,\boldsymbol{\vartheta})$ available from previous iteration $\mu - 1$. The constraints (5) facilitate the enforcement of uniform PR conditions through the PSK iterations, by constraining the new denominator estimate being computed to be PR at these points. The effectiveness of this approach was illustrated by the several test cases documented in [3], [7].

The main problem of the above stabilized PSK iteration is the possible lack of convergence. The constraints (5) are effective only at their precise location $(\mathrm{j}\omega_r^{\mu-1},\boldsymbol{\vartheta}_r^{\mu-1})$, but nothing prevents the denominator to become negative at completely different locations at the next iteration. Moreover, it may be the case that the denominator coefficients do not stabilize, with undesired erratic behaviors through iterations. We would like each coefficient update to vanish as iterations progress,

$$d_n^{\mu}(\boldsymbol{\vartheta}) = \left|r_n^{\mu}(\boldsymbol{\vartheta}) - r_n^{\mu-1}(\boldsymbol{\vartheta})\right| \xrightarrow{\mu\to\infty} 0 \quad \forall n, \forall\boldsymbol{\vartheta}. \tag{7}$$

However, as pointed out in [10], there is no guarantee that such convergence condition occurs. The left panels of Fig. 1 illustrate this problem on an H-shaped antenna test case (see below for details on this structure). The top-left panel confirms that the denominator coefficients never stabilize through iterations. Although the model responses are very accurate (middle-left panel), the lack of uniform PR-ness of the model denominator is not able to constrain the poles to have a negative real part (bottom-left panel).

We remark that this problem is well-known even in non-parameterized passive macromodeling, for which application of non-convex formulations of the passivity constraints may lead to non-converging passivity enforcement schemes [1].

III. Robust iterations

A solution to the above-described convergence issues is available through the so-called "robust iterations", which were introduced in [11] for non-parameterized passivity enforcement (see [1] for a complete treatment). In this work, we apply a similar but more efficient procedure to the parameterized setting, with the objective of enforcing uniform model stability. The same strategy can of course be applied to the enforcement of uniform passivity [9], which is not of interest in this work.

The main idea of [11] is as follows. Let us consider again the PR denominator violation points $\mathcal{V}^{\mu-1}$ used to define the PR constraints for iteration μ, as defined in (5) and (6). Let us assume that the μ-th iteration is performed, that the PR-ness of the denominator is checked again, and that the corresponding new PR violations are collected in set $\mathcal{V}^{\mu}$. If we aggregate all constraints in a cumulative set $\mathcal{V}^{\mu-1} \cup \mathcal{V}^{\mu}$ and we use the latter instead of $\mathcal{V}^{\mu-1}$ to repeat (5), we see that this iteration μ will include constraints both where the denominator is detected to be real negative, and also where the denominator *will become* real negative. This strategy has thus a *predictive* or *look-ahead* property. The main disatvantage is however an increased (approximately threefold) computational cost, since each iteration is repeated three times.

In this work, we propose a similar strategy that aggregates constraints using a *look-behind* strategy. In essence, each iteration μ enforces PR constraints through

$$\Re\{\mathsf{D}^{\mu}(\mathrm{j}\omega,\boldsymbol{\vartheta})\} > 0, \quad \forall(\mathrm{j}\omega,\boldsymbol{\vartheta}) \in \cup_{i=0}^{\mu-1}\mathcal{V}^{i}, \tag{8}$$

implying that constraints are placed at the PR violations of the denominator at current and all previous iterations. This strategy avoids by construction oscillating behaviors, where a violation that is eliminated at some iteration reappears every two or more iterations.

The left column of Fig. 2 shows stability violation locations on the parameter space for three successive fitting iterations for the same antenna example of Fig. 1, where a red to black relative color scale is used to represent the extent of each local violation (in terms of minimum real part of the model denominator). It can be noted that, despite the constraints (5) enforced at each of the highlighted points, new violations reappear at the same locations at subsequent iterations, thus impairing convergence. Conversely, the right column of Fig. 2 depicts the accumulated constraints in case of proposed robust implementation, where each panel represents constraints from previous iterations using transparent dots. The resulting set can be interpreted as an iterative refinement of the parameter space, which zooms in the regions characterized by the stability violations that are not eliminated in one step. This provides a clear illustration that the aggregation of all constraints is equivalent to narrowing the feasible set of the optimization problem, which is obtained as the intersection of all feasible sets at all previous iterations.

IV. Results

We demonstrate the performance of the *look-behind* robust iterations on an H-shaped antenna (number of ports $P = 1$), already used above to illustrate convergence issues of the standard PSK method. The structure, whose geometry is described in full details in [12], is parameterized by its length L, which varies in the range $L = [7, \ 10]$ mm, and the aperture width $W \in [1.5, \ 1.8]$ mm. Through repeated EM simulations,

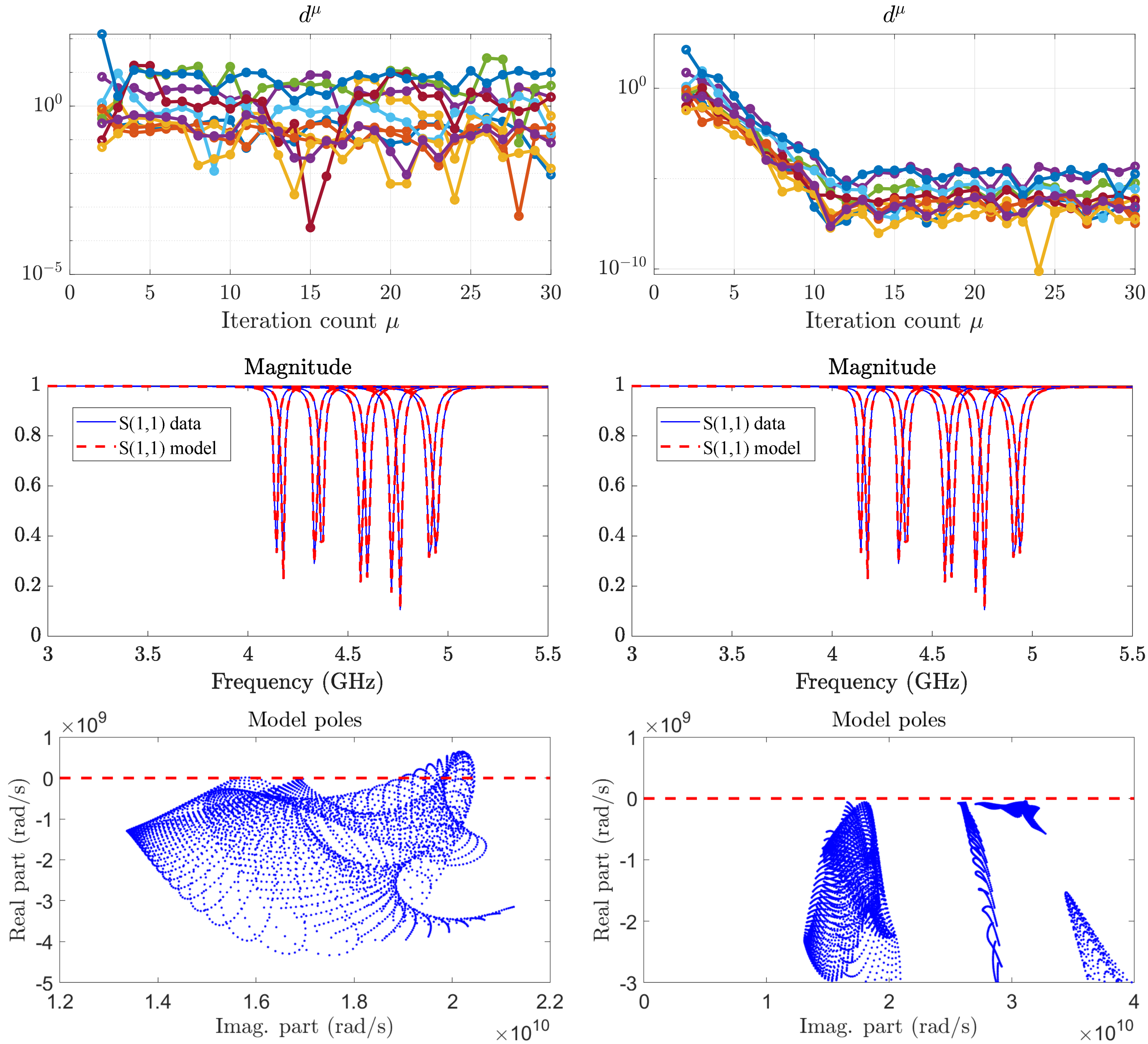

Fig. 1: Macromodeling results using standard PSK iteration (left panels) and proposed look-behind robust iterations (right panels) applied to an H-shaped antenna. Top panels: evolution of coefficient updates through iterations. Middle panels: selected model responses for few parameter configurations, compared with raw data. Bottom panels: parameterized model poles.

we gather a set of $\bar{m} = 16$ scattering responses, within the frequency band $[0,\ 5.5]$ GHz, for different parameter values uniformly distributed on a structured bi-dimensional grid. The model is synthesized with $\bar{n} = 10$ poles and Chebychev polynomials as parameter basis functions with orders $\{3,\ 3\}$ and $\{2,\ 2\}$ for numerator and denominator expansions, respectively.

The results of proposed robust implementation are depicted in the right panels of Fig. 1. The top-right panel shows that the coefficient updates at each subsequent iteration become smaller and smaller, as a clear evidence of convergence. In contrast with the results from the standard implementation (top-left panel), where denominator coefficients change significantly as μ increases, the new robust algorithm successfully converges after 11 iterations. Thanks to the aggregate PR constraints (8) the model results uniformly stable, as the bottom-right panel confirms by showing a sweep of the model poles in the parameter space. Model accuracy is not compromised, as depicted in the middle-right panel. The worst-case relative error of the stable model obtained by proposed approach is $3.6 \cdot 10^{-4}$, whereas the corresponding error for the unconstrained identification is $1.6 \cdot 10^{-4}$.

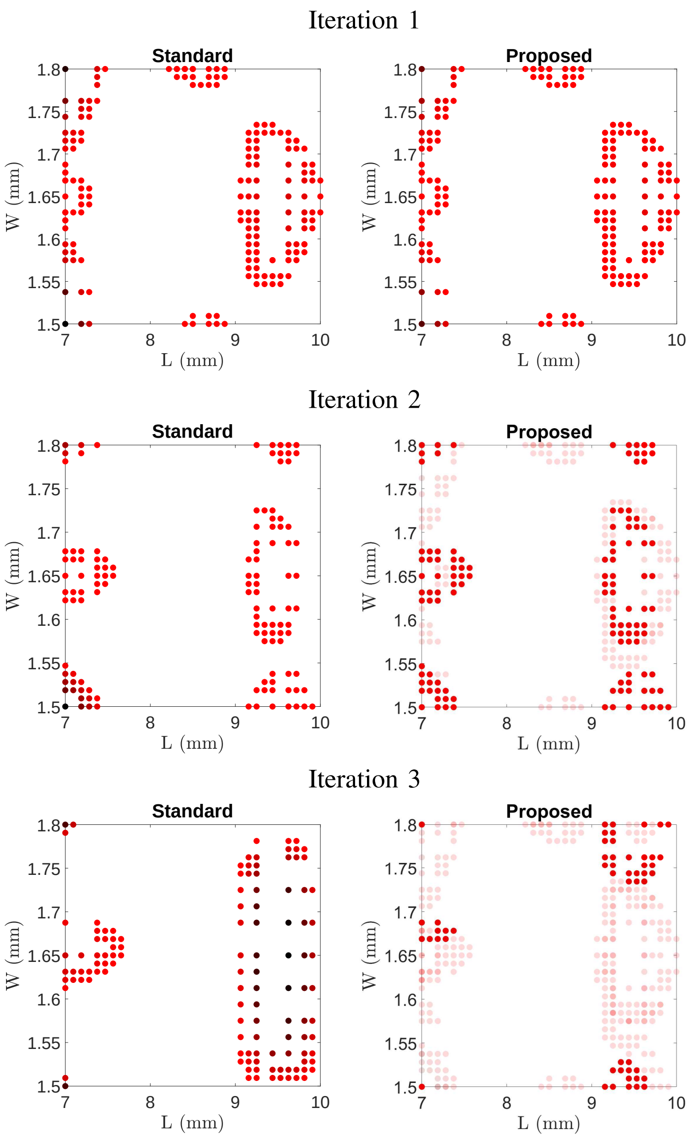

Fig. 2: Location in the parameter space Θ of stability constraints for three successive fitting iterations. Left column: standard approach of [7] with independent stability constraints at each iteration. Right column: proposed robust approach, with constraints accumulation through iterations (violation extent is represented with a red-to-black color scale, with transparent dots representing the constraints at previous iterations).

V. Conclusion

In this paper, we presented a robust, stability preserving, implementation of the well known *Parameterized Sanathanan-Koerner* iteration, which brings major improvements in terms of convergence capabilities with respect to previous works [3], [7]. The presented strategy aims at avoiding that the model coefficients undergo oscillatory behaviors as the fitting iterations proceed, undermining the overall convergence. This is achieved by stacking stability constraints as the number of iterations grows through a *look-behind* strategy. The propsed approach is illustrated on a relevant antenna test-case.

References

[1] S. Grivet-Talocia and B. Gustavsen, *Passive macromodeling: Theory and applications*. John Wiley & Sons, 2015, vol. 239.

[2] P. Triverio, S. Grivet-Talocia, and M. S. Nakhla, "A parameterized macromodeling strategy with uniform stability test," *IEEE Transactions on Advanced Packaging*, vol. 32, no. 1, pp. 205–215, 2009.

[3] S. Grivet-Talocia and R. Trinchero, "Behavioral, parameterized, and broadband modeling of wired interconnects with internal discontinuities," *IEEE Transactions on Electromagnetic Compatibility*, vol. 60, no. 1, pp. 77–85, 2018.

[4] F. Ferranti, L. Knockaert, and T. Dhaene, "Parameterized s-parameter based macromodeling with guaranteed passivity," *IEEE Microwave and Wireless Components Letters*, vol. 19, no. 10, pp. 608–610, 2009.

[5] ——, "Passivity-preserving parametric macromodeling by means of scaled and shifted state-space systems," *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, no. 10, pp. 2394–2403, 2011.

[6] E. R. Samuel, L. Knockaert, F. Ferranti, and T. Dhaene, "Guaranteed passive parameterized macromodeling by using sylvester state-space realizations," *IEEE Transactions on Microwave Theory and Techniques*, vol. 61, no. 4, pp. 1444–1454, 2013.

[7] M. De Stefano, S. Grivet-Talocia, T. Bradde, and A. Zanco, "A framework for the generation of guaranteed stable small-signal bias-dependent behavioral models," in *2018 13th European Microwave Integrated Circuits Conference (EuMIC)*. IEEE, 2018, pp. 142–145.

[8] C. Sanathanan and J. Koerner, "Transfer function synthesis as a ratio of two complex polynomials," *IEEE transactions on automatic control*, vol. 8, no. 1, pp. 56–58, 1963.

[9] A. Zanco, S. Grivet-Talocia, T. Bradde, and M. De Stefano, "Enforcing passivity of parameterized lti macromodels via hamiltonian-driven multivariate adaptive sampling," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, 2018.

[10] S. Lefteriu and A. C. Antoulas, "On the convergence of the vector-fitting algorithm," *IEEE Transactions on Microwave Theory and Techniques*, vol. 61, no. 4, pp. 1435–1443, 2013.

[11] B. Gustavsen, "Fast passivity enforcement for pole-residue models by perturbation of residue matrix eigenvalues," *IEEE Transactions on Power Delivery*, vol. 23, no. 4, pp. 2278–2285, 2008.

[12] D. Deschrijver, K. Crombecq, H. M. Nguyen, and T. Dhaene, "Adaptive sampling algorithm for macromodeling of parameterized s-parameter responses," *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, no. 1, pp. 39–45, 2011.