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Original
A Hierarchical Approach to the Stochastic Analysis of Transmission Lines via Polynomial Chaos / Manfredi, P.; Trinchero, R.. - ELETTRONICO. - (2019), pp. 1-4. (Intervento presentato al convegno 23rd IEEE Workshop on Signal and Power Integrity (SPI 2019) tenutosi a Chambery (Francia) nel 2019) [10.1109/SaPIW.2019.8781644].

Availability:
This version is available at: 11583/2759718 since: 2019-10-09T15:17:36Z

Publisher:
Institute of Electrical and Electronics Engineers Inc.

Published
DOI:10.1109/SaPIW.2019.8781644

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24 September 2024

A Hierarchical Approach to the Stochastic Analysis of Transmission Lines via Polynomial Chaos

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***Abstract*—Polynomial chaos-based techniques recently became popular tools for signal integrity investigations that include the effects of parameter variability. Most of the available approaches are limited by the "curse of dimensionality", apply only to Gaussian correlations, or they are hindered by the lack of explicit parametrization or knowledge of the input random parameters. This paper presents a hierarchical approach for transmission line analysis, according to which line voltages and currents are modeled as polynomial chaos expansions that are function of the per-unit-length parameters, rather than of the underlying geometrical and material parameters. This new approach exhibits some useful advantages such as non-parametricity (with respect to physical parameters), higher accuracy for low expansion orders, and a potential for dimensionality reduction. An application example involving the transient analysis of a stripline interconnect is used to illustrate the feasibility of the advocated approach and discuss its performance.**

***Index Terms*—Non-Gaussian correlation, polynomial chaos, stochastic Galerkin method, transmission lines, variability analysis, uncertainty quantification.**

I. Introduction

Polynomial chaos (PC) became widely popular for circuit simulations that include the effects of parameter variability [1]. PC expands stochastic quantities in terms of orthogonal polynomials and exhibits superior accuracy and efficiency compared to the blind Monte Carlo (MC) method. In particular, PC was extensively applied to transmission line analysis and signal integrity investigations [2]–[6].

Generally speaking, there are several longstanding limitations to the effective application of PC to real-life problems. The so-called "curse of dimensionality", i.e., the efficiency decrease with increasing number of uncertain parameters, is probably the best known. Moreover, independency of parameters is a fundamental requirement. Straightforward extensions to correlated parameters are available, but only for when they are Gaussian distributed. Another fundamental requirement is the preliminary *parametrization* of the uncertainty: an explicit, one-to-one correspondence is needed between input uncertain parameters and stochastic outputs of interest, and it must be possible to compute the latter for a given realization of the former. This is not always the case, for example when the circuit configurations are inherently non-parametrizable or come, e.g., from measured data.

Tensor-based [7] and hierarchical [8] approaches were proposed to mitigate the dimensionality issue. In [9], suitable multivariate orthogonal polynomials for arbitrary distributions were numerically computed by means of a Gram-Schmidt orthogonalization. The parametrization issue was instead never addressed within the framework of PC. A non-parametric approach to generate interconnect responses in a MC-like fashion, starting from a limited set of input data, was presented in [10] and [11].

This paper presents a hierarchical PC (HPC) approach for the stochastic analysis of transmission lines that helps cope with the aforementioned limitations. The main novelty lies in the fact that voltages and currents are expanded w.r.t. the line per-unit-length (p.u.l.) parameters, rather than to the underlying physical parameters. This provides manifold advantages, such as: 1) a one-to-one correspondence between p.u.l. and geometrical/material parameters is no longer required; 2) the problem dimension is reduced if the number of distinct p.u.l. parameters is smaller than the number of physical random parameters; 3) a more accurate expansion can be obtained for low orders, because the relationship between voltages and currents and p.u.l. parameters is smoother. However, since the p.u.l. parameters are not independent and turn out to have a non-standard correlated distribution, the hierarchical method in [8] cannot be used, but rather the technique in [9] is leveraged to compute suitable basis functions.

The present paper briefly outlines the proposed HPC approach. An expanded paper will provide a deeper theoretical insight and further validations.

II. Stochastic Transmission Line Analysis

For the sake of illustration, and without loss of generality, the discussion is based on a stripline interconnect, whose cross-section is depicted in the left frame of Fig. 1. The geometrical and material parameters therein indicated are all considered as uncertain parameters, leading to a total number of ten random variables, which are collected into vector $\boldsymbol{\xi}$.

Figure 1 illustrates in the top diagram the traditional simulation workflow, denoted as "classical PC" (CPC): the uncertainty of the underlying physical parameters translates into a variability of the p.u.l. parameters and of the voltages and currents. The latter are represented by PC expansions that are function of the uncertain parameters $\boldsymbol{\xi}$, i.e.,

$$\begin{cases} \boldsymbol{v}(z,t,\boldsymbol{\xi}) \approx \sum_{k=1}^{K} \boldsymbol{v}_k(z,t)\varphi_k(\boldsymbol{\xi}) \\ \boldsymbol{i}(z,t,\boldsymbol{\xi}) \approx \sum_{k=1}^{K} \boldsymbol{i}_k(z,t)\varphi_k(\boldsymbol{\xi}), \end{cases} \tag{1}$$

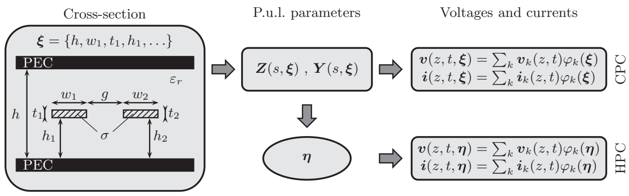

Fig. 1. Workflow of PC-based simulations. In the CPC, line voltages and currents are expanded w.r.t. the underlying physical uncertain parameters $\boldsymbol{\xi}$. In the proposed HPC, they are expanded w.r.t. the intermediate parameters $\boldsymbol{\eta}$, associated to the line p.u.l. parameters.

where the basis functions φ_k are a suitable set of polynomials orthogonal to the joint distribution of $\boldsymbol{\xi}$, whereas $\boldsymbol{v}_k$ and $\boldsymbol{i}_k$ are the corresponding voltage and current coefficients, respectively. Thanks to orthogonality, voltage average and standard deviation are readily estimated as

$$\mathrm{E}\{\boldsymbol{v}(z,t,\boldsymbol{\xi})\} \approx \boldsymbol{v}_1(z,t), \tag{2}$$

$$\mathrm{Std}\{\boldsymbol{v}(z,t,\boldsymbol{\xi})\} \approx \sqrt{\textstyle\sum_{k=2}^{K} \boldsymbol{v}_k^2(z,t)}, \tag{3}$$

and similarly for the currents.

Several approaches are available for the determination of the expansion coefficients [5]. Galerkin-based approaches construct a deterministic, yet augmented transmission line equation:

$$\begin{cases} \dfrac{d}{dz}\widetilde{\boldsymbol{V}}(z,s) = -\widetilde{\boldsymbol{Z}}(s)\widetilde{\boldsymbol{I}}(z,s) \\ \dfrac{d}{dz}\widetilde{\boldsymbol{I}}(z,s) = -\widetilde{\boldsymbol{Y}}(s)\widetilde{\boldsymbol{V}}(z,s), \end{cases} \tag{4}$$

where z denotes the longitudinal coordinate, s is the Laplace variable, whereas $\widetilde{\boldsymbol{V}} = (\boldsymbol{V}_1^\mathsf{T}, \cdots, \boldsymbol{V}_K^\mathsf{T})^\mathsf{T}$ and $\widetilde{\boldsymbol{I}} = (\boldsymbol{I}_1^\mathsf{T}, \cdots, \boldsymbol{I}_K^\mathsf{T})^\mathsf{T}$ collect the Laplace-domain counterparts of the voltage and current coefficients in (1). The pertinent augmented p.u.l. matrices $\widetilde{\boldsymbol{Z}}$ and $\widetilde{\boldsymbol{Y}}$ are constructed from the coefficients of the PC expansion of the original p.u.l. matrices $\boldsymbol{Z}$ and $\boldsymbol{Y}$. In this paper, a numerical inverse Laplace transform (NILT) [12] is used to obtain the time-domain coefficients after solving (4).

III. Proposed Hierarchical Approach

The state-of-the-art approach, in which stochastic voltages and currents are expanded in terms of the uncertain physical parameters, is sometimes inconvenient. An example is when the number of such parameters is larger than the number of distinct p.u.l. parameters. In some cases, data of p.u.l. parameters could be available without explicit knowledge of the physical parameters they were computed for.

For the abovementioned reasons, a hierarchical approach is proposed in this paper. As illustrated in the bottom diagram of Fig. 1, voltages and currents are expanded w.r.t. a new set of intermediate variables $\boldsymbol{\eta}$, associated to the p.u.l. parameters, disregarding the underlying physical parameters $\boldsymbol{\xi}$. The main difficulty that arises is that the entries of the p.u.l. matrices are related to each other, and therefore these parameters cannot be considered to be independent. This makes the standard PC, as well as other hierarchical approaches like [8], inapplicable.

To solve this issue, suitable orthogonal polynomials for the p.u.l. variables are computed based on a Gram-Schmidt orthogonalization [9], which generically applies to arbitrary correlated distributions. The starting point is a set of linearly independent multivariate *monomials*:

$$\mathcal{M} = \{\Psi_1(\boldsymbol{\eta}), \Psi_2(\boldsymbol{\eta}), \ldots\} = \{1, \eta_1, \eta_2, \ldots, \eta_1^2, \eta_1\eta_2, \ldots\}. \tag{5}$$

The orthogonal polynomials are recursively computed as

$$\begin{cases} \hat{\varphi}_k(\boldsymbol{\eta}) & = \Psi_k(\boldsymbol{\eta}) - \displaystyle\sum_{j=1}^{k-1} \mathrm{E}\{\Psi_k(\boldsymbol{\eta})\varphi_j(\boldsymbol{\eta})\}\varphi_j(\boldsymbol{\eta}) \\ \varphi_k(\boldsymbol{\eta}) & = \dfrac{\hat{\varphi}_k(\boldsymbol{\eta})}{\sqrt{\mathrm{E}\{\hat{\varphi}_k^2(\boldsymbol{\eta})\}}} \end{cases} \tag{6}$$

for $k > 1$, starting from $\varphi_1 = 1$. In (6), the expectations are computed based on the multivariate joint distribution of the new variables $\boldsymbol{\eta}$ by means of MC integration. This is computationally tractable, as only analytical functions need to be evaluated in this process. The distribution of the variables $\boldsymbol{\eta}$ is estimated from the available samples of the p.u.l. parameters. In order to have a convenient closed form, the actual distribution is fitted with a mixture of Gaussians, which is a flexible model that allows reproducing any arbitrary distribution shape. Once the basis functions are computed, they are used in (1) to expand voltages and currents in terms of the new variables $\boldsymbol{\eta}$. The rest of the PC framework is unvaried.

As a by-product of the advocated approach, the HPC expansion is expected to be more accurate for a given order, as voltages and currents are modeled directly in terms of the intermediate p.u.l. parameters and their distribution, regardless of their further relationship with the underlying physical parameters.

IV. Variable Definition

This section discusses the definition of the intermediate variables $\boldsymbol{\eta}$. For the stripline interconnect depicted in Fig. 1, the frequency-dependent p.u.l. impedance matrix is expressed as follows:

$$\boldsymbol{Z}(s,\boldsymbol{\xi}) = \boldsymbol{R}_{\mathrm{dc}}(\boldsymbol{\xi}) + \sqrt{s/\pi}\,\boldsymbol{R}_{\mathrm{hf}}(\boldsymbol{\xi}) + s\boldsymbol{L}_{\mathrm{dc}}(\boldsymbol{\xi}), \tag{7}$$

where the stochastic and frequency-independent resistance and inductance matrices $\boldsymbol{R}_{\mathrm{dc}}$, $\boldsymbol{R}_{\mathrm{hf}}$, and $\boldsymbol{L}_{\mathrm{dc}}$, depend on the random physical parameters $\boldsymbol{\xi}$, and they are therefore random themselves. Their entries become the new random variables $\boldsymbol{\eta}$, w.r.t. which the line voltages and currents are expanded. These new variables are defined as

$$\boldsymbol{R}_{\mathrm{dc}} = \begin{pmatrix} R_{\mathrm{dc},11} & 0 \\ 0 & R_{\mathrm{dc},22} \end{pmatrix} = \begin{pmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{pmatrix},$$

$$\boldsymbol{R}_{\mathrm{hf}} = \begin{pmatrix} R_{\mathrm{hf},11} & R_{\mathrm{hf},12} \\ R_{\mathrm{hf},21} & R_{\mathrm{hf},22} \end{pmatrix} = \begin{pmatrix} \eta_3 & \eta_4 \\ \eta_4 & \eta_5 \end{pmatrix},$$

$$\boldsymbol{L}_{\mathrm{dc}} = \begin{pmatrix} L_{\mathrm{dc},11} & L_{\mathrm{dc},12} \\ L_{\mathrm{dc},21} & L_{\mathrm{dc},22} \end{pmatrix} = \begin{pmatrix} \eta_6 & \eta_7 \\ \eta_7 & \eta_8 \end{pmatrix},$$

where it has been considered that $R_{\mathrm{dc},12} = R_{\mathrm{dc},21} = 0$ under the assumption of PEC ground planes, and that $R_{\mathrm{hf},12} = R_{\mathrm{hf},21}$ and $L_{\mathrm{dc},12} = L_{\mathrm{dc},21}$ because of reciprocity.

Under the assumption of negligible dielectric losses, the p.u.l. admittance is simply given by

$$\boldsymbol{Y}(s, \boldsymbol{\xi}) = s\boldsymbol{C}_{\mathrm{dc}} \tag{8}$$

where, owing to the line homogeneity,

$$\begin{aligned}\boldsymbol{C}_{\mathrm{dc}} = \begin{pmatrix} C_{\mathrm{dc},11} & C_{\mathrm{dc},12} \\ C_{\mathrm{dc},21} & C_{\mathrm{dc},22} \end{pmatrix} &= \mu_0 \varepsilon_0 \varepsilon_r \boldsymbol{L}_{\mathrm{dc}}^{-1} \\ &= \frac{\mu_0 \varepsilon_0 \cdot \eta_9}{\eta_6 \eta_8 - \eta_7^2} \begin{pmatrix} \eta_8 & -\eta_7 \\ -\eta_7 & \eta_6 \end{pmatrix}.\end{aligned} \tag{9}$$

In (9), an additional variable η_9 is associated to the random relative permittivity ε_r. The total number of random variables $\boldsymbol{\eta}$ in thus nine.

V. Numerical Results

For the validation, a stripline with the following nominal parameters is considered: $h = 20$ mil, $w_1 = w_2 = 5$ mil, $t_1 = t_2 = 0.6$ mil, $h_1 = h_2 = 10$ mil, $g = 5$ mil, $\varepsilon_r = 4$, $\sigma = 58$ MS/m (copper). One of the two traces is excited at the near end with a pulse voltage source with an internal resistance of 100 Ω, an amplitude of 1 V, rise/fall times of 200 ps, and a duration of 2.8 ns. The other trace is terminated with a 100-Ω resistor. Both traces are terminated by 1-pF capacitances at the far end, and a length of 10 cm is considered. The p.u.l. parameters are computed by means of a field solver and, for the HPC simulation, a mixture of Gaussians distribution is fitted to the intermediate variables defined in Section IV. Reference MC results are obtained by means of transient SPICE simulations. For the CPC and HPC results, a NILT is applied to the Laplace-domain solution of (4).

Figure 2 illustrates the variability of the far-end transmitted voltage (left panels) and crosstalk (right panels) by considering a 5% relative standard deviation of the random physical parameters. The top panels show a subset of MC samples (gray lines) highlighting the variability of the voltages as a result of the uncertainty, as well as the average voltage computed from 5000 MC samples (solid blue lines), or with the CPC expansion (dashed red line) and proposed HPC expansion (dotted green line). The excellent agreement is further confirmed by the central and bottom panels comparing the standard deviations. In particular, the bottom panel shows that a first-order HPC expansion is already very accurate, as opposed to a CPC expansion of the same order. This can be understood by the fact that line voltages are smoother functions of the p.u.l. parameters rather than of the geometrical and material parameters, as already noted. A higher accuracy of the HPC is also observed for a second-order expansion (central panels).

For a more quantitative assessment of the modeling accuracy, the following error measure on the standard deviation is introduced:

$$\mathcal{E} = \frac{1}{T} \int_0^T |\mathrm{Std}_{\mathrm{PC}}(t) - \mathrm{Std}_{\mathrm{MC}}(t)|\, dt, \tag{10}$$

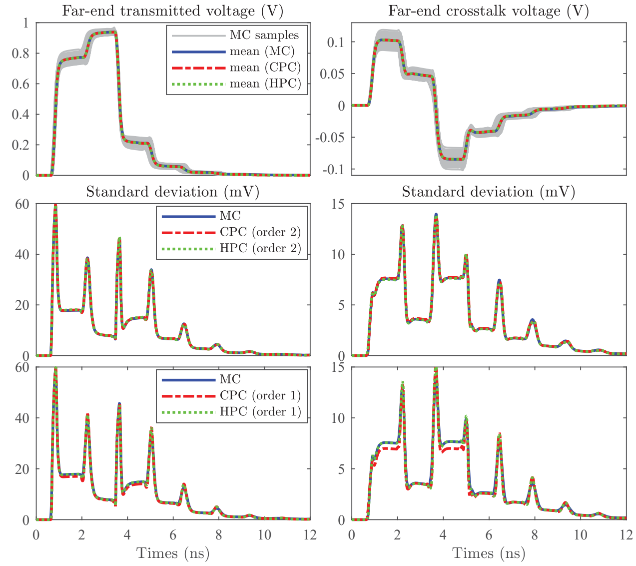

Fig. 2. Variability of the far-end transmitted (left panels) and crosstalk (right panels) voltages for a 5% parameter variation. Top panels: subset of MC samples (gray lines) and average computed from the MC samples (solid blue lines), CPC expansion (dashed red lines) and HPC expansion (dotted green lines). Central and bottom panels: standard deviation for different PC orders (the same color code is used).

where T is the maximum observation time (here set to 12 ns). Briefly speaking, the error is defined as the integral of the area between the PC curve and reference MC result. A Simpson's quadrature rule is used to evaluate the integral. Table I collects the errors for the terminal voltages, including the near-end crosstalk (not shown in Fig. 2 due to the lack of space). The figures confirm the superior accuracy of the proposed HPC.

Next, the relative standard deviation of the random parameters is increased to 10%. Figure 3 shows the updated plots for the far-end terminal voltages. The larger uncertainty causes a much broader variability of the voltage levels. Nevertheless, the HPC expansion is still remarkably accurate, even for order one. In this case, 15000 MC samples are used to calculate the reference results. Table I collects the errors of the CPC and HPC approaches also for this second scenario. The figures corroborate the previous conclusions, although in this case the HPC is not always more accurate than the CPC for the second-order expansion. The reason is probably a numerical inaccuracy in the evaluation of the basis functions and expansion coefficients.

The computational times are similar for the two scenarios considered and they are collected in Table II. The HPC has the additional overhead of numerically computing the basis functions, which are instead standard and available for the CPC. The calculation of the augmented p.u.l. matrices in (4) takes similar time for the two methods. Instead, the solution of (4) is slightly faster for the second-order HPC simulation, because the number of uncertain parameters is reduced from ten to nine, leading to a smaller dimension of the Galerkin problem. The achievement is however significant considering that a first-order HPC expansion has comparable accuracy as

TABLE I
ERROR BETWEEN CPC/HPC EXPANSIONS AND MC RESULTS.

variation	quantity	order 1		order 2	
		CPC	HPC	CPC	HPC
5%	far-end transmission	2.9915e-04	1.9635e-04	8.6582e-05	8.1490e-05
	near-end crosstalk	4.1755e-04	2.4029e-04	1.2283e-04	1.1526e-04
	far-end crosstalk	2.6472e-04	1.3034e-04	6.7092e-05	5.9144e-05
10%	far-end transmission	9.3660e-04	5.8507e-04	3.5322e-04	4.4294e-04
	near-end crosstalk	2.6854e-03	1.1444e-03	9.2660e-04	6.8755e-04
	far-end crosstalk	1.0048e-03	4.6998e-04	2.8028e-04	3.7999e-04

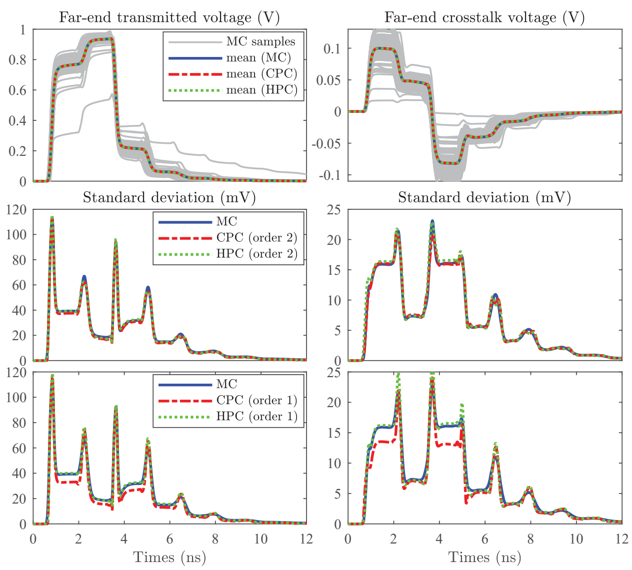

Fig. 3. Variability of the far-end transmitted and crosstalk voltages for a 10% parameter variation.

TABLE II
COMPUTATIONAL TIMES FOR THE CPC AND HPC APPROACHES.

method	basis functions	augmented matrices	line solution
CPC (order 2)	—	49.7 s	57.2 s
HPC (order 2)	75.0 s	45.5 s	44.5 s
CPC (order 1)	—	8.0 s	2.3 s
HPC (order 1)	3.2 s	0.3 s	2.4 s

a second-order CPC expansion, leading to a 94.5% reduction in the overall simulation time. The speed-up becomes more relevant when the dimensionality reduction in the hierarchy is more substantial.

VI. CONCLUSIONS

This paper discussed a HPC approach for the stochastic analysis of transmission lines. As opposed to state-of-the-art implementations, the PC expansion of the line voltages and currents is computed w.r.t. the line p.u.l. parameters, rather than of the underlying physical parameters. The technique was illustrated by means of its application to the time-domain analysis of a stripline interconnect with frequency-dependent losses. It is shown that it is a viable solution that can achieve good accuracy with a lower expansion order. The approach is beneficial whenever the number of p.u.l. parameters is smaller than the number of random physical parameters, or when the latter cannot be explicitly defined. Further examples and validations will be provided in an expanded paper.

REFERENCES

[1] A. Kaintura, T. Dhaene, and D. Spina, "Review of polynomial chaos-based methods for uncertainty quantification in modern integrated circuits," Electronics, vol. 7, no. 3, p. 30:1–21, Feb. 2018.

[2] M. R. Rufuie, E. Gad, M. Nakhla, and R. Achar, "Generalized Hermite polynomial chaos for variability analysis of macromodels embedded in nonlinear circuits," *IEEE Trans. Compon. Packag. Manuf. Technol.*, vol. 4, no. 4, pp. 673–684, Apr. 2014.

[3] Y. Ye, D. Spina, P. Manfredi, D. Vande Ginste, and T. Dhaene, "A comprehensive and modular stochastic modeling framework for the variability-aware assessment of signal integrity in high-speed links," *IEEE Trans. Electromagn. Compat.*, vol. 60, no. 2, pp. 459–467, Apr. 2018.

[4] J. B. Preibisch, T. Reuschel, K. Scharff, and C. Schuster, "Impact of continuous time linear equalizer variability on eye opening of high-speed links," in *Proc. IEEE 20th Workshop Signal Power Integr. (SPI)*, Turin, Italy, May 2016, pp. 1–4.

[5] P. Manfredi, D. Vande Ginste, I. S. Stievano, D. De Zutter, and F. G. Canavero, "Stochastic transmission line analysis via polynomial chaos methods: an overview," *IEEE Electromagn. Compat. Mag.*, vol. 6, no. 3, pp. 77–84, 2017.

[6] A. K. Prasad and S. Roy, "Reduced dimensional Chebyshev-polynomial chaos approach for fast mixed epistemic-aleatory uncertainty quantification of transmission line networks," *IEEE Trans. Compon., Packag., Manuf. Technol.* (early access).

[7] Z. Zhang, T. Weng, and L. Daniel, "Big-data tensor recovery for high-dimensional uncertainty quantification of process variations," *IEEE Trans. Compon. Packag. Manuf. Technol.*, vol. 7, no. 5, pp. 687–697, May 2017.

[8] Z. Zhang, T. A. El-Moselhy, I. M. Elfadel, and L. Daniel, "Calculation of generalized polynomial-chaos basis functions and Gauss quadrature rules in hierarchical uncertainty quantification," *IEEE Trans. Comput.-Aided Des. Integr. Circuits Syst.*, vol. 33, no. 5, pp. 728–740, May 2014.

[9] C. Cui and Z. Zhang, "Stochastic collocation with non-Gaussian correlated process variations: theory, algorithms and applications," *IEEE Trans. Compon. Packag. Manuf. Technol.* (early access).

[10] S. De Ridder, P. Manfredi, J. De Geest, D. Deschrijver, D. De Zutter, T. Dhaene and D. Vande Ginste, "A generative modeling framework for statistical link analysis based on sparse data," *IEEE Trans. Compon. Packag. Manuf. Technol.*, vol. 8, no. 1, pp. 21–31, Jan. 2018.

[11] X. Ma, M. Raginsky, and A. C. Cangellaris, "A machine learning methodology for inferring network S-parameters in the presence of variability," in *Proc. IEEE 22nd Workshop Signal Power Integr. (SPI)*, Brest, France, May 2018, pp. 1–4.

[12] R. Trinchero, P. Manfredi, and I. S. Stievano, "Self-validated time-domain analysis of linear systems with bounded uncertain parameters," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 65, no. 11, pp. 1499–1503, Nov. 2018.