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A 3D Passivity-Based Adaptive Algorithm for Automated Parameterized Macromodeling of Electromagnetic Structures

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***Abstract*—We present an adaptive algorithm for the automated generation of parameterized macromodels of electromagnetic structures. The proposed framework is able to select a quasi minimal distribution of simulation points over the parameter space, which can be up to three dimensions. The effectiveness of the proposed approach is demonstrated on a patch antenna.**

I. Introduction

The use of parameterized macromodels has become more and more popular in a wide variety of applications, including antenna analysis and design [1]. The optimization of electromagnetic structures, in fact, requires the evaluation of the system response for different values of its constitutive parameters, which can be related to geometry or material properties. A useful tool for this type of process is represented by parameterized macromodels, which provide an approximation of the frequency response of the structure, while reproducing at the same time its dependence on a number of parameters.

Parameterized macromodels can be obtained by applying a polynomial/rational fitting process to a number of frequency responses sampled in the parameter space. After an identification procedure, such models become a valid surrogate of the original structure for system-level analysis, allowing for a fast evaluation of the system response compared to a full-wave simulation.

An accurate approximation of a parameterized system requires a large number of full-wave reference simulations in the parameter space, which generally come from first-principle solvers. Since the computational cost and runtime of such simulations may be considerable, an important objective is to limit their number, so that only the strictly necessary data required to generate an accurate behavioral model are actually computed. Once the model has been generated, moreover, a common requirement for it is to be both stable and passive, so that it can be safely used in system-level simulations.

In this paper, we present an adaptive algorithm for the identification of a minimal set of simulation points in the parameter space. This set is used to generate a uniformly-accurate multivariate macromodel, whose stability and passivity are checked and, if necessary, enforced. The choice of simulation points is based on the evaluation of three different metrics, which take into consideration the distribution of points in space, the accuracy of the macromodel built during each iteration and the in-band passivity violations of such model. The main advancement with respect to the work presented in [6] is the extension to the three-parameter case.

II. An Adaptive Algorithm for Point Selection

The starting point for model construction is a set of frequency responses of the structure provided by a field solver, which we denote as

$$\breve{\mathbf{H}}_{k,m} = \breve{\mathbf{H}}(\mathrm{j}\omega_k; \boldsymbol{\vartheta}_q), \quad k = 1, \dots, \bar{k},\ q = 1, \dots, \bar{q}, \tag{1}$$

where $\boldsymbol{\vartheta} = [\vartheta^1, ..., \vartheta^\rho]^\mathsf{T} \in \Theta \subset \mathbb{R}^\rho$ represents a vector of ρ external parameters. In this paper we consider the case where the number of free parameters is set to $\rho = 3$. The model built from (1) is obtained by means of the Parameterized Sanathanan-Koerner (PSK) iteration [2], [3], [4], which seeks for a model in the form

$$\mathbf{H}(s;\boldsymbol{\vartheta}) = \frac{\mathbf{N}(s,\boldsymbol{\vartheta})}{\mathsf{D}(s,\boldsymbol{\vartheta})} = \frac{\sum_{n=0}^{\bar{n}} \sum_{\ell=1}^{\bar{\ell}} \mathbf{R}_{n,\ell}\, \xi_\ell(\boldsymbol{\vartheta})\, \varphi_n(s)}{\sum_{n=0}^{\bar{n}} \sum_{\ell=1}^{\bar{\ell}} r_{n,\ell}\, \xi_\ell(\boldsymbol{\vartheta})\, \varphi_n(s)} \tag{2}$$

where $\xi_\ell(\boldsymbol{\vartheta})$ denotes some parameter-space basis functions and $\varphi_n(s)$ are the frequency-domain partial fraction basis functions. Through an iterative process the numerator and denominator coefficients, $\mathbf{R}_{n,\ell}$ and $r_{n,\ell}$, are numerically determined. The iterative process stops when the model is sufficiently accurate, hence when the model-data error is below a given threshold ε

$$\mathcal{E}(\boldsymbol{\vartheta}_q) = \max_{i,j} \sqrt{\frac{1}{\bar{k}} \sum_{k=1}^{\bar{k}} \left| H_{ij}(\mathrm{j}\omega_k; \boldsymbol{\vartheta}_q) - (\breve{\mathbf{H}}_{k;q})_{ij} \right|^2} < \varepsilon \quad \forall q \tag{3}$$

In addition, the final model is required to be uniformly stable and passive, thus it must satisfy the *Uniform Bounded Realness* conditions:

a) $\mathbf{H}(s,\boldsymbol{\vartheta})$ regular for $\Re\{s\} > 0$ and $\forall \boldsymbol{\vartheta} \in \Theta$,
b) $\mathbf{H}^*(s,\boldsymbol{\vartheta}) = \mathbf{H}(s^*,\boldsymbol{\vartheta})\ \forall s \in \mathbb{C}$ and $\forall \boldsymbol{\vartheta} \in \Theta$,
c) $\mathbb{I} - \mathbf{H}^\mathsf{H}(s,\boldsymbol{\vartheta})\mathbf{H}(s,\boldsymbol{\vartheta}) \geq 0$ for $\Re\{s\} > 0$ and $\forall \boldsymbol{\vartheta} \in \Theta$.

The aim is to construct a parameterized model $\mathbf{H}(s,\boldsymbol{\vartheta})$ using a minimal number of points $\bar{q}$ in the parameter space, which translates into finding the points $\boldsymbol{\vartheta}_q$ associated to best parameter combinations allowing to fully characterize the system response. Such points $\boldsymbol{\vartheta}_q$ are obtained through an adaptive sampling algorithm, which iteratively selects the best positions in the parameter space where new points should be

added to obtain a uniformly accurate model. This approach is not new, as it was already presented in [6], [7]. The main novelty in this paper is the extension to the tri-variate case.

The algorithm starts by selecting a small set of q_0 points $\mathcal{Q}_0 = \{\boldsymbol{\vartheta}_1, \ldots, \boldsymbol{\vartheta}_{\bar{q}_0}\}$ randomly distributed on the parameter space Θ, where each point $\boldsymbol{\vartheta}_q = [\vartheta_q^1, \vartheta_q^2, \vartheta_q^3]$ corresponds to a specific combination of parameter values, in turn associated to a specific configuration of the structure under analysis. The aim of the algorithm is to identify at each *i*-th iteration a set of new simulation points $\mathcal{P}_i$, so that the total set becomes $\mathcal{Q}_i = \mathcal{Q}_{i-1} \cup \mathcal{P}_i$. In order to do so, at each *i*-th iteration of the algorithm the space is subdivided into its Voronoi tessellation [5] using the available simulation points $\boldsymbol{\vartheta}_q$ as seed points, giving rise to a set of disjoint cells $C_{i;q}$. The cells that will host the new points $\boldsymbol{\vartheta}_q \in \mathcal{P}_i$ are identified by means of three metrics, which at every iteration are evaluated and properly combined for an adaptive choice of new points. The selected cells, in fact, are those where the weighted sum of the three metrics is higher. A detailed description of such metrics is given below.

A. Exploration

The *exploration* metric aims at positioning new points in the undersampled areas, which have not been explored yet and might need further simulation points [5]. This is measured in practice by considering the normalized volume of each cell $C_{i;q}$, meaning the relative portion of space occupied by each cell with respect to the parameter space Θ. The exploration metric can then be defined as

$$\Lambda'_{i;q} = \frac{|C_{i;q}|}{|\Theta|}\,. \tag{4}$$

B. Model-data Error

At each *i*-th iteration, once the set of new points $\mathcal{P}_i$ has been identified, an intermediate macromodel is constructed exploiting the modeling procedure presented in (2). This model is then used to calculate the second metric, which consists of the error between the intermediate model and the simulation data at each point $\boldsymbol{\vartheta}_q$. The *model-data error* metric can be formulated as

$$\Lambda''_{i;q} = \frac{\mathcal{E}_i(\boldsymbol{\vartheta}_q)}{\sum_{q=1}^{\bar{q}_i} \mathcal{E}_i(\boldsymbol{\vartheta}_q)}. \tag{5}$$

where the error $\mathcal{E}(\boldsymbol{\vartheta}_q)$ is the one reported in (3). The use of such metric is based on the assumption that new points must be added in the regions where the intermediate model is less accurate [6], [7]. Since the algorithm takes care that proper orders are selected both for the frequency-domain and the parameter-space bases, an inaccurate model is most probably the result of an insufficient amount of data, leading to poor fitting.

C. Passivity violations

The two metrics presented above ensure that the new simulation points are selected targeting a final model which is uniformly accurate over the parameter space. However, this is not sufficient to ensure also the stability and passivity of the model, which is the final goal in this paper. Assuming that simulation data coming from a field solver are passive, the intrinsic nature of the fitting process as an approximation procedure can lead to a non-passive model. Moreover, the model error metric presented in II-B has a strong limitation, due to the fact that it can be evaluated only in those points where simulation data are present, while no value can be inferred on the remaining parameter space. These are the main motivations behind the use of a third metric, which is based on the evaluation of passivity violations of the intermediate macromodels [6], [7]. A model built starting from passive data, in fact, turns out to be non-passive in those regions where the model is physically inconsistent, thus highly inaccurate: passivity violations can then be used to detect such regions, which consequently need further sampling. The third metric, thus, consists in evaluating the extent of passivity violations, directly linked to the extent of model inaccuracy.

Recalling the *Uniform Bounded Realness* conditions presented in Sec. II, it can be inferred that a model is passive at $(\mathrm{j}\omega, \boldsymbol{\vartheta})$ if $\sigma_{\max}(\mathbf{H}(\mathrm{j}\omega, \boldsymbol{\vartheta})) \leq 1$, where $\sigma_{\max}$ indicates the maximum singular value of model response. If a passivity violation occurs, it can be located at a frequency ω inside the modeling band Ω, or it can be out-of-band. For our purpose, only in-band violations are relevant, as they directly come from poor model accuracy. Thus, the extent of a passivity violation can be expressed as

$$S(\boldsymbol{\vartheta}_p) = \max_{\omega \in \Omega} \sigma_{\max}(\mathbf{H}_i(\mathrm{j}\omega, \boldsymbol{\vartheta}_p)). \tag{6}$$

The engine that we use to find the worst-case passivity violations is the Skew-Hamiltonian/Hamiltonian (SHH) adaptive sampling scheme presented in [8], [9]. This is based on the construction of the SHH matrix pencil $(\mathbf{M}_S(\boldsymbol{\vartheta}), \mathbf{K}_S)$, defined as

$$\begin{aligned} \mathbf{M}_S(\boldsymbol{\vartheta}) &= \begin{bmatrix} \mathbf{A}(\boldsymbol{\vartheta}) & \mathbf{B}\mathbf{B}^{\mathsf{T}} \\ -\mathbf{C}^{\mathsf{T}}(\boldsymbol{\vartheta})\mathbf{C}(\boldsymbol{\vartheta}) & -\mathbf{A}^{\mathsf{T}}(\boldsymbol{\vartheta}) \end{bmatrix} \\ \mathbf{K}_S &= \begin{bmatrix} \mathbf{E}^{\mathsf{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} \end{bmatrix} \end{aligned} \tag{7}$$

and on the extraction of its purely imaginary eigenvalues. If any such eigenvalues are found, their imaginary part corresponds to the frequencies at the onset of passivity violations. The algorithm in [9] is here used to detect the location of the non-passive points $\hat{\boldsymbol{\vartheta}}_p$ across the entire parameter space, corresponding to local passivity violation maxima. Local in-band passivity violations are then defined as

$$\Delta_i(\hat{\boldsymbol{\vartheta}}_p) = \max\{S(\hat{\boldsymbol{\vartheta}}_p) - 1, 0\} \tag{8}$$

Each such point $\hat{\boldsymbol{\vartheta}}_p$ such that $\Delta_i(\hat{\boldsymbol{\vartheta}}_p) > 0$ is associated to the Voronoi cell $C_{i;q}$ to which it belongs. Finally, the passivity violation metric for each cell $C_{i;q}$ is defined as the largest passivity violation extent among all its internal points

$$\Lambda'''_{i;q} = \max_{\hat{\boldsymbol{\vartheta}}_p \in C_{i;q}} \Delta_i(\hat{\boldsymbol{\vartheta}}_p). \tag{9}$$

D. Global Metric and Grid Refinement

The three above metrics are combined in a global metric

$$\Lambda_{i;q} = w'\Lambda'_{i;q} + w''\Lambda''_{i;q} + w'''\Lambda'''_{i;q} \tag{10}$$

where the weights w', w'', w''' are used to adjust the contribution of the single metrics to the general criterion for point selection. Cells $C_{i;q}$ are ranked based on the value of metric $\Lambda_{i;q}$, and a fraction β of the cells with high ranking will be chosen to host the new points at the next iteration. In the examples provided in this paper, β has been set equal to $1/3$, in order to provide a good trade-off between algorithm adaptivity and number of iterations. Once a cell $C_{i;q}$ has been selected, the new point will be placed in the centroid of the largest polyhedron obtained by joining the simulation point $\boldsymbol{\vartheta}_q$ to all cell vertices.

III. Enforcing Uniform Passivity

The iterative procedure for point selection stops when the number of simulations is sufficient to obtain a uniformly accurate model. There is no guarantee, however, that such model is stable an passive. For this reason both model stability and passivity are first checked and, if necessary, enforced. Further details on the perturbation-based procedures used for these operations can be found on [8], [9] and [10].

IV. Examples

A. Patch antenna

We validate the proposed method on a patch antenna, whose representation is reported in Fig. 1. The patch has a width of 16 mm, while the height is 12.5 mm. The dielectric layer is 0.794 mm thick ($\varepsilon = 2.213$). The structure has been parameterized, in particular ϑ^1 corresponds to the stub length $L \in [4 - 10]$ mm, ϑ^2 to the stub width $W \in [2.5 - 4]$ mm and ϑ^3 to the stub position $S \in [7 - 9]$ mm. The system responses are retrieved from the EM simulator ADS-Momentum, with $\bar{k} = 250$ frequency samples for each configuration of the structure.

The algorithm starts with an initial set of $q_0 = 23$ points, 15 of which are randomly scattered in space, while the remaining 8 are placed on the vertices of the space. The framework requires 8 iterations of the algorithm, for a total of 220 points, to obtain a uniformly accurate macromodel. The intermediate results of the selection process can be seen in Fig. 2, where the Voronoi tessellation of the parameter space is represented at different algorithm iterations, on a color scale representing the logarithm of the model error in each simulation point: it can be noticed how the overall model accuracy progressively increases, tending toward a uniform accuracy across the space. The final model has been validated on a set of 10 model responses (not used for model construction) swept along the diagonal of the space. The results are reported in Fig. 3.

The final model has $\bar{n} = 20$ poles and Chebychev polynomial bases of order $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}_3 = 2$ for both numerator and denominator.

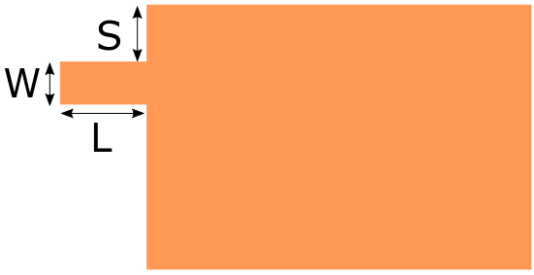

Fig. 1: A top-view of the patch antenna.

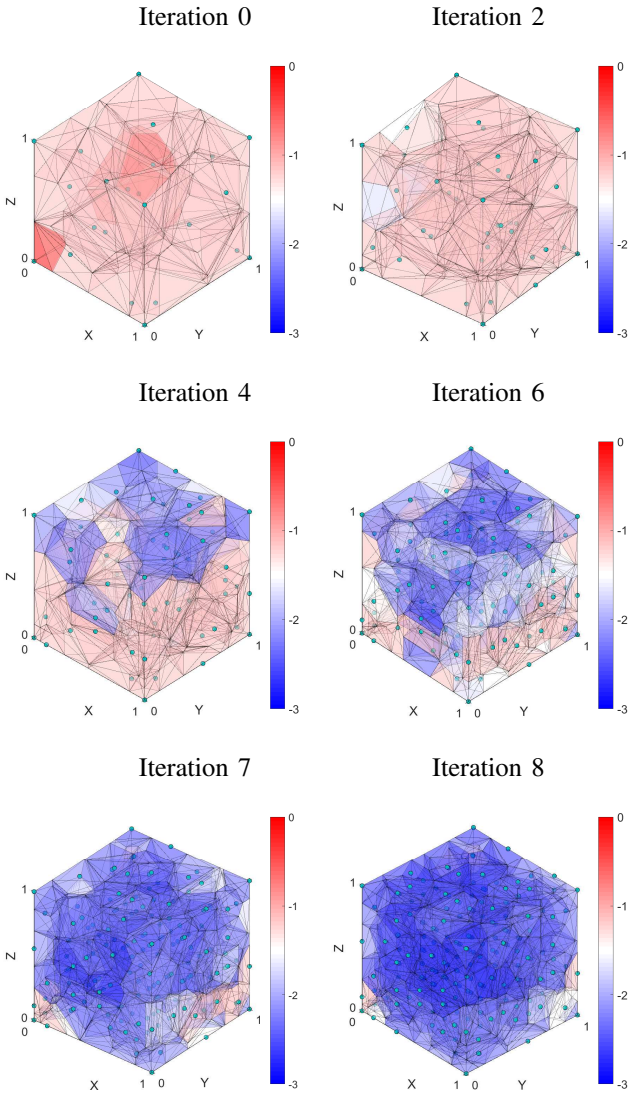

Fig. 2: Voronoi diagram of the parameter space at different iterations for the single patch antenna example.

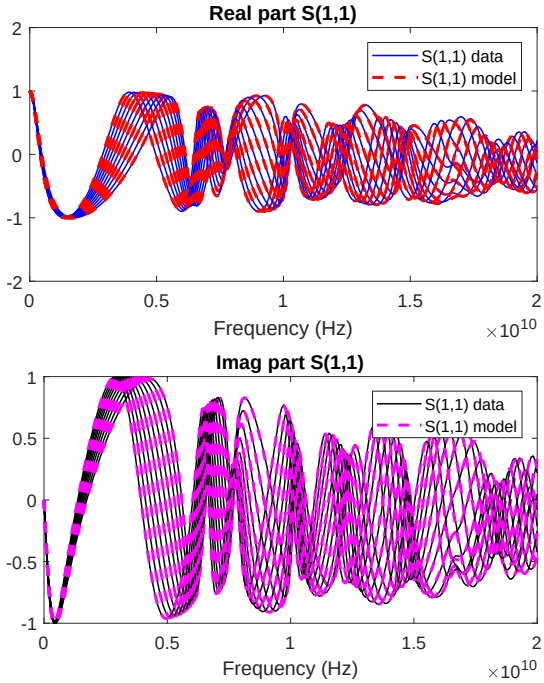

Fig. 3: Comparison between passive model and experimental data of the single patch antenna for a selected set of responses.

V. Conclusions

We presented a fully automated adaptive algorithm for the generation of passive tri-variate macromodels. The algorithm starts from a small subset of simulation points in the parameter space and, through the evaluation of three different metrics, is able to iteratively select the best position for new points in the parameter space. The selected simulation points are used for the generation of a uniformly accurate tri-variate macromodel, whose stability and passivity are checked and eventually enforced through a perturbation process. The effectiveness of the proposed method is illustrated on a single antenna design.

References

[1] S. Grivet-Talocia and B. Gustavsen, *Passive macromodeling: Theory and applications*. John Wiley & Sons, 2015, vol. 239.

[2] P. Triverio, S. Grivet-Talocia, and M. S. Nakhla. A parameterized macromodeling strategy with uniform stability test. *IEEE Trans. Advanced Packaging*, 32(1):205–215, Feb 2009.

[3] C. Sanathanan and J. Koerner. Transfer function synthesis as a ratio of two complex polynomials. *Automatic Control, IEEE Transactions on*, 8(1):56–58, jan 1963.

[4] D. Deschrijver, T. Dhaene, D. De Zutter, "Robust parametric macromodeling using multivariate orthonormal vector fitting", IEEE Trans. Microw. Theory Tech., vol. 56, no. 7, pp. 1661-1667, Jul. 2008

[5] D. Deschrijver, K. Crombecq, H. M. Nguyen, and T. Dhaene, "Adaptive algorithm for macromodeling of parameterized S-Parameter Responses," IEEE Transactions on Microwave Theory and Techniques, vol. 59, no.1, pp. 39–45, 2011.

[6] E. Fevola, A. Zanco, S. Grivet-Talocia, T. Bradde, M. De Stefano, "An Adaptive Sampling Process for Automated Multivariate Macromodeling Based on Hamiltonian-based Passivity Metrics, " *to appear in the IEEE Transactions on Components, Packaging and Manufacturing Technology*,

[7] E. Fevola, A. Zanco, S. Grivet-Talocia, T. Bradde, M. De Stefano, "An Adaptive Algorithm for Fully Automated Extraction of Passive Parameterized Macromodeling, " *2019 IEEE MTT-S International Conference on Numerical Electromagnetic and Multiphysics Modeling and Optimization, May 29-31 2019, Cambridge, MA, USA*,

[8] S. Grivet-Talocia, "Passivity enforcement via perturbation of hamiltonian matrices," IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 51, no. 9, pp. 1755–1769, 2004.

[9] A. Zanco, S. Grivet-Talocia, T. Bradde, and M. De Stefano, "Enforcing passivity of parameterized LTI macromodels via Hamiltonian-driven multivariate adaptive sampling," IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 2018.

[10] Grivet-Talocia, Stefano, "A perturbation scheme for passivity verification and enforcement of parameterized macromodels," *IEEE Transactions on Components, Packaging and Manufacturing Technology*, vol. 7, no. 11, pp. 1869–1881, 2017.