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Low-Order Models for Low-Frequency Combustion Instability in Hybrid Rocket Engines

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Abstract. A low-order model for a hybrid rocket engine is proposed to investigate low-frequency combustion instabilities. The present work is based on a one-way coupling between a one-dimensional thermal model to evaluate the unsteady regression rate and a zero-dimensional model which imposes mass conservation inside the rocket chamber. In this way, it is possible to estimate the unsteady pressure level inside the rocket chamber and predict the amplitude of the oscillations. Two different approaches are compared. In the first approach, the Oxidizer-Fuel (OF) ratio is directly computed from the ratio between the fuel and oxidizer mass flow rates. In this way the OF ratio strongly depends on the regression rate oscillations and, as a consequence, the pressure shows large fluctuations. An alternative approach has been investigated by computing the OF ratio from the fuel and oxidizer istantaneous densities. The two approaches are compared on a representative test case.

INTRODUCTION

The Hybrid Rocket Engine (HRE) is a promising propulsion system that combines many positive features of both Liquid Rocket Engines (LREs) and Solid Rocket Motors (SRMs): high specific and density-specific impulse, throttling capabilities, safety, reliability and low cost. However, HRE shows low frequency instabilities in the combustion process and non-linear transient response [1] that are critical issues to the actual use of HREs in real-world applications. The exact genesis of these phenomena is still unclear. However, the work of Karabeyoglu and Altman [2] shows that the low frequency instabilities in HREs are directly related to the existence of a delay between the heat flux and the regression rate, due to the inertia of the boundary layer. Analytical and numerical results available in the literature [1, 3, 4] suggest that acoustic modes could induce regression rate fluctuation at low frequency and a significant positive shift ("DC shift", [5]) in the average pressure and regression rate.
Several low order models for heat transfer and chamber dynamic have been proposed in the literature [1, 6, 7]. In a previous work [4], the effect of the boundary layer delay was investigated by a 1D thermal model which solves the heat equation in the solid grain. The output of this model is represented by the time history of the regression rate.
In the present work we propose to couple the thermal model proposed in [4] with a zero-dimensional model of the combustion chamber in order to predict chamber pressure fluctuations and evaluate the phenomena related to combustion instability. In particular, a one-way coupling approach is adopted in which the regression rate history obtained by the thermal code is given as input in the combustion chamber model.

PHYSICAL MODEL

The simulations carried out in this work are based on the interaction between two different models, a 1D thermal model for the grain and a zero-dimensional fluid model for the gases in the combustion chamber. The thermal model is based on the approach described in [4] where the unsteady heat equation is discretised in the radial direction. Particular attention is devoted to the definition of boundary conditions at the solid-fluid interface where a time delay

is introduced between the heat flux and the regression rate evolution. This delay seems to be the main responsible for the development of instabilities in HREs and influences significantly the magnitude of the regression rate fluctuations. More details on the thermal model are available in [4].

The thermal model provides the time history of regression rate and grain surface temperature. However, it does not allow to predict the amplitude of the chamber pressure fluctuations which correpond to the computed regression rate oscillations. For this reason, the following simple zero-dimensional model of the combustion chamber is introduced.
The model is based on mass conservation and the fluid is assumed to be described by the ideal gas law $p = \rho RT$ where p, ρ, R and T represent pressure, density, gas constant and temperature, respectively. The quantity RT is here evaluated by a chemical equilibrium computation based on the CEA approach [8]. In particular, the value of RT is expressed as a function of the Oxidizer-Fuel (OF) ratio. Two different versions of the model are proposed, depending on the approximation used to estimate the OF ratio.

The global mass conservation applied to the gases in the combustion chamber gives the followin equation:

$$V\frac{d\rho}{dt} = \dot{m}_{Ox} + \dot{m}_F - \frac{C_{dn}A_t p_c}{\eta c^*} \tag{1}$$

where V, t, C_{dn}, A_t, p_c, η and c^* represent chamber volume, time, nozzle discharge coefficient, throat area, chamber pressure, combustion efficiency and characteristic velocity (which is a function of the OF ratio) respectively. The volume V is here assumed constant in time: this is in line with the classical approach used for the study of instability in HREs where the geometry of the grain is assumed frozen with respect to the acoustic fluctuations.
Equation 1 can be rewritten by introducing the equation of state in order to put in evidence the chamber pressure evolution:

$$\frac{dP_c}{dt} = \frac{\dot{m}_{Ox} + \dot{m}_F - \frac{C_{dn}A_t p_c}{\eta c_*} - p_c V \frac{d1/RT}{dOF}\frac{dOF}{dt}}{\frac{V}{RT} + p_c V \frac{d1/RT}{dp_c}} \tag{2}$$

This equation represents an alternative to the approach proposed by Karabeyoglu and Dyer in [1]: in that work, a similar equation is obtained but a space time averaged RT value is introduced. In contrast, the RT fluctuations in time are taken into account in the present work since the results obtained by the thermal code showed large fluctuations of the regression rate.
Equation 2 is integrated in time by means of an explicit Euler scheme. More efficient approaches could be easily implemented but the low computational cost of the present model allows to choose a sufficiently small time step size to get the required accuracy even with the chosen first order time integration scheme. The results reported in the following are chosen after a convergence study on the time step size.
The derivatives of $1/RT$ with respect to OF ratio and chamber pressure p_c are computed numerically by means of a perturbative approach.

Mass flow based OF ratio

In the first proposed approach the OF ratio required by Equation 2 is evaluated as the ratio between the oxidizer ($\dot{m}_{Ox}$) and fuel ($\dot{m}_F$) mass flows:

$$OF = \frac{\dot{m}_{Ox}}{\dot{m}_F} \tag{3}$$

In the present work the oxidizer mass flow is assumed to be constant: this assumption is reasonable in the case of choked cavitating venturi injectors. As far as the fuel mass flow is concerned, it is directly evaluated from the regression rate:

$$\dot{m}_F = \rho_{SF} A_b \dot{r} \tag{4}$$

where ρ_{SF}, A_b and $\dot{r}$ represent solid fuel density, burning area and regression rate.

Density based OF ratio

The evaluation of the OF ratio based on the mass flow rates ratio does not take into account the inertial effects related to the capacity of the combustion chamber. In order to investigate these effects, an alternative approach is proposed. In particular, the OF ratio is here computed as

$$OF = \frac{\rho_{Ox}}{\rho_F} \tag{5}$$

where ρ_{Ox} and ρ_F represent the oxidizer and fuel density in the chamber. In order to get the time evolution of ρ_{Ox} and ρ_F it is possible to write two mass conservation equations for both fuel and oxidizer:

$$V\frac{d\rho_F}{dt} = \dot{m}_F - k_F\frac{C_{dn}A_t p_c}{\eta c_*} \tag{6}$$

$$V\frac{d\rho_{Ox}}{dt} = \dot{m}_{Ox} - k_{Ox}\frac{C_{dn}A_t p_c}{\eta c_*} \tag{7}$$

where k_F and k_{Ox} represent the mass fraction contributions in the exaust gases which are related to fuel and oxidizer, respectively. Since the sum of Equation 7 and 6 must give Equation 1 then $k_{Ox} + k_F = 1$. Here, the following assumption is chosen: $k_F = \rho_F/\rho$ and $k_{Ox} = \rho_{Ox}/\rho$. Furthermore, the following relation holds for the densities: $\rho = \rho_F + \rho_{Ox}$. For this reason, it is sufficient to integrate Equation 1 and 6 to get ρ and ρ_F. The value of ρ_{Ox} is computed as difference between these quantities.

RESULTS AND CONCLUSIONS

The proposed approach is applied to the simulation of a laboratory-scale engine which was experimentally investigated by Carmicino [9]. The engine burns hydroxil-terminated polybutadiene fuel with gaseous oxygen. The experimental measurements showed that part of the test is characterised by a significant "DC shift" phenomenon. This regime was numerically studied in [4] by using a thermal lag model with a boundary layer delay. The model described in [4] is used in the present work to predict the unsteady regression rate reported in Figure 1. In particular, the boundary layer delay which determines the instability is chosen as $\tau_{BL} = 1, 10, 25ms$ in order to investigate the influence of this parameter.

The regression rate obtained by the thermal lag model is used as input for the 0-dimensional approach described in this work in order to estimate the unsteady chamber pressure. The results are reported in Figure 2 in which the numerical prediction is compared with the experimental range observed by [9]. Both the mass flow based and density based OF ratio formulation are tested: the two approaches give very similar results.

The results put in evidence the strong influence of the boundary layer delay on the pressure fluctuations. In particular, the best agreement with the experimental results is observed in Figure 2b which corresponds to $\tau_{BL} = 10ms$.

The approach adopted in this work is based on a one-way coupling in which the regression rate history obtained by the grain thermal model [4] is imposed as a forcing term in the zero-dimensional model which describes the pressure in the combustion chamber. Recently, a parametric investigation of combustion instability with a multy-phisics model was performed by [10]. Future work will be devoted to the development of more complex approaches which include a two-way coupling in which the thermal model of the grain and the fluid dynamics model of the chamber are strongly coupled: this could give better physical insight on the phenomena which control "DC shift" in hybrid rocket engines.

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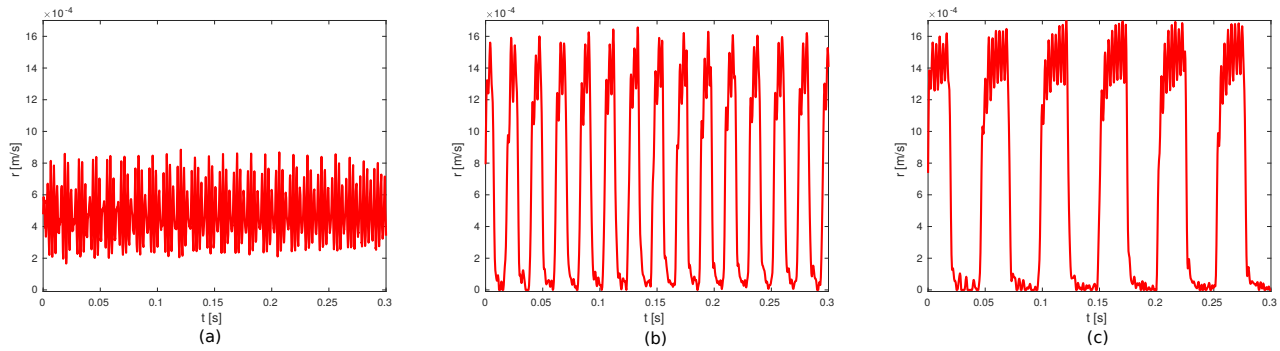

FIGURE 1. Regression rate predicted by 1D thermal code for different values of τ_{BL}: a)τ_{BL}=1 ms, b)τ_{BL}=10 ms, c)τ_{BL}=25 ms

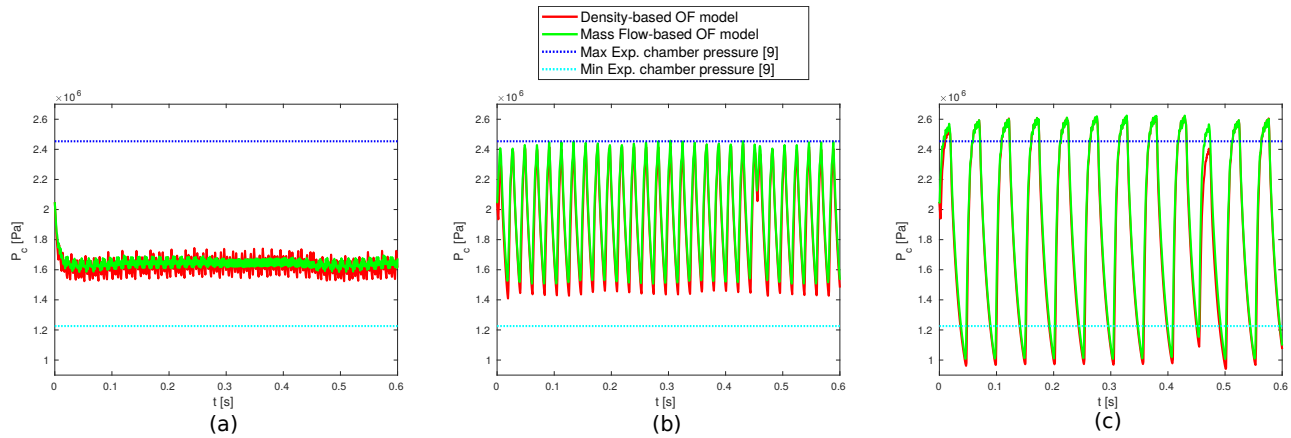

FIGURE 2. Oscillating chamber pressure during DC shift mode for different values of τ_{BL}: a)τ_{BL}=1 ms, b)τ_{BL}=10 ms, c)τ_{BL}=25 ms

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