Simplified cracking analysis of RC ties taking account of the effect of secondary cracks

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ABSTRACT
A simplified model based on the assumption that bond stresses are linearly distributed along the transmission length is here set up to study the cracking behaviour of an RC tie subjected to monotonic loading, taking into account the effects of the so-called Goto cracks, or internal secondary cracks. At the stabilized cracking stage, under the assumption that the crack spacing is maximum, the relative mean strain formula proposed in *fib* Model Code 2010 and Eurocode 2 is modified by introducing a coefficient η, depending only on the acting axial force-to-cracking force ratio, which allows the effect of internal secondary cracks to be considered.

Key words: average bond stress, cracking, maximum and minimum crack width, crack spacing.

1. INTRODUCTION

A simplified approach, named linear method, is here formulated to study the stabilized cracking stage of an RC tie. The method avoids the need of iterative procedures for the stabilized cracking stage. It is based on the assumption that bond stresses are linearly distributed along the transmission length, L_s. Here, only the situation of maximum crack spacing is considered, equal to twice the transmission length, but the study can be performed also in case of minimum crack spacing [1]. The formula for the calculation of the maximum crack width in the stabilized cracking stage is, then, obtained. Finally, the theoretical results are compared with the experimental data concerning tests on RC ties.

2 SIMPLIFIED ANALYSIS OF CRACKING BEHAVIOUR

Assuming a linear elastic behaviour of the materials and on the basis of the equilibrium conditions of forces acting on the concrete and steel, the following second-order differential equation of the slipping contact between steel and concrete can be obtained:

$$\ddot{s}_s(x) = \frac{4 \cdot \tau_{bs}}{E_s \cdot \phi_s} \cdot \left(1 + \alpha_e \cdot \rho_{s,ef}\right) \qquad (1)$$

Two stages can be distinguished. First, a crack formation stage takes place. When the axial force increases, other cracks occur randomly along the axis of the member till a certain situation, named stabilized cracking stage, in which the crack pattern stabilizes.
With the general method [2], the solution of the second-order differential equation (1) can only be expressed in closed form for the crack formation stage, while an iterative procedure is needed to solve the problem when the crack pattern stabilises.
In order to avoid iterative calculations, a simple as possible approach for the direct calculation of the crack width at the stabilized cracking stage, based on the assumption that the bond

stresses are linearly distributed along the transmission length, taking account of the effect of internal secondary cracks, is here proposed. Bond stresses and slips are correlated through the *fib* Model Code 2010 bond law only at the section where bond stresses reach the maximum value. In the other sections, bond stresses and slips are in compliance with equation (1).
When the crack distance from the adjacent cracks on each side of the crack is equal to twice the transmission length, the crack presents an upper bound width, w_{max}:

$$w_{\max}^{(\,stab.cracking\,)} = s_{r\,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) = \frac{2 \cdot L_s}{E_s} \cdot \left[\sigma_{s2} - \frac{2}{3} \cdot \frac{f_{ct}}{\rho_s} \cdot (1 + \alpha_e \cdot \rho_s) \cdot \left(1 - \frac{\ell_{sc}}{2 \cdot L_s} \right) \right] \qquad (2)$$

In equation (2) the term in square brackets assumes a form that is similar to the formula of the relative mean strain proposed in *fib* Model Code 2010 or in Eurocode 2 for the calculation of the crack width. This similarity occurs unless for the term (1 – η / 2), in which $\eta = \ell_{sc} / L_s$ and ℓ_{sc} is the length of secondary cracks. It results that, when the axial force increases, the secondary cracks reduce the effect of tension stiffening on the relative mean strain, ε_{sm}-ε_{cm}, through a factor (1 – η / 2) which depends only on the axial force, diminishes from 1 and 0.5 as the axial force increases, and is independent on bar diameter, reinforcement ratio and concrete strength.

3 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL DATA

The linear method can be applied for a comparison with the experimental data, concerning short-term tests on RC ties, available in the literature. It results that the experimental data are mostly smaller than the theoretical upper bounds of crack width.

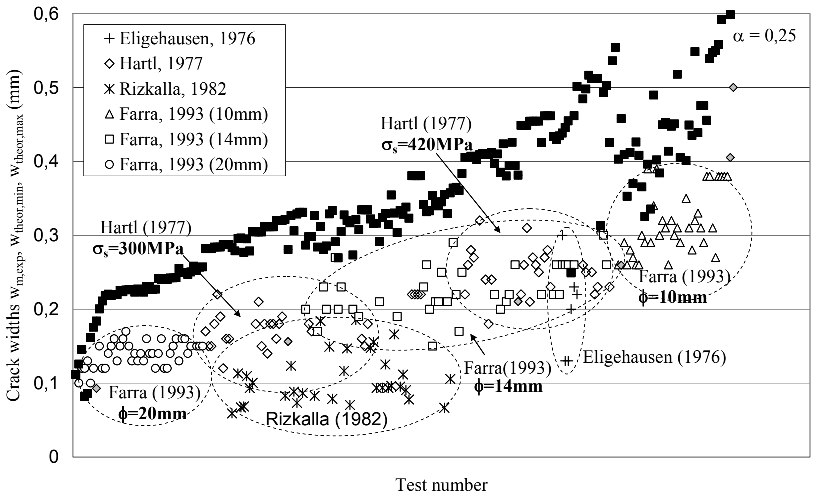

Figure 1 – Comparison between experimental results and theoretical values of maximum crack width (square black boxes).

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