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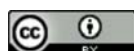
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GEOMETRY OF VAULTED SYSTEMS IN THE TREATISES BY GUARINO GUARINI

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Abstract

Guarini first develops a rigorous and systematic discourse on vaulted systems. In three treatises: Architettura Civile, Euclides adauctus, and Modo di misurare le fabriche, he described the geometric nature of the vaults, the principles of geometry and their practices in the stereotomy, and the measurement of the vaults' surfaces and volumes, respectively. In this paper, moving from previous studies, the significant relationships between the Architettura Civile and Modo di misurare le fabriche are deepened, also in light to the theoretical bases established in the Euclides adauctus. Graphical analyses and reconstructive digital models, linking the texts to the original diagrams and drawings of the treatises, allow to highlight the role of geometry in Guarini's theorization and the logic of shapes' composition at the basis of his inventions, and constitute a knowledge base to recognize and interpret the geometric structure of the vaults in Baroque built heritage.

Keywords: Vaults, Baroque architecture, Graphical analysis, Reconstructive digital modelling.

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1. INTRODUCTION

"The vaults are the main part of the buildings, and the authors who wrote about architecture debate them so briefly, that someone does not even mention them, because they are the hardest to ideate, draw, and build" (Guarini 1737: 183).

Guarino Guarini (1624-1683), born in Modena (Italy), was an abbot of the Theatines order and an eminent scholar with wide-ranging interests, mainly in architecture, mathematics and philosophy. He lived and worked in the 17th century, a period imbued by the *esprit de géométrie*, which was fueled by the discoveries and studies in the mathematical field, in particular of pure geometry. Indeed, at that time a leading position of geometry within the philosophical method established, through the theories of Galileo, Bacon, Descartes, Malebranche, Desargues.

Guarini, after a series of journeys in Italy and France, during which he focused on theoretical studies and architectural design, in 1666 settled in Turin. Here, he published some of his most important treatises and created several religious and civil Baroque architectures.

Guarini first develops a rigorous and systematic discourse on vaulted systems.

In three treatises: *Architettura Civile* (posthumous 1737), *Euclides adauctus* (1671), and *Modo di misurare le fabriche* (1674), he described the geometric nature of the vaults, the principles of geometry and their practices in the stereotomy, and the measurement of the vaults' surfaces and volumes, respectively.

Among these books, the *Euclides adauctus* takes the role of theoretical reference based on geometry, with respect to the approach of the other two treatises, constantly referred to the architectural works.

In this paper the significant relationships between the *Architettura Civile* and *Modo di misurare le fabriche*, whose complementarity also emerges in the continuous reference to the binomial Geometry-Architecture, are deepened, also in light to the theoretical bases established in the *Euclides adauctus*.

Moving from previous studies (Spallone 2016, Spallone 2017, Spallone 2019a, Spallone 2019b), a logical sequence of interpretation which combines Guarini's contribution to the geometric definition of vaulted systems in an organic discourse, is proposed.

Moreover, the texts were linked to the original diagrams and drawings of the treatises by means of graphical analysis and reconstructive digital modeling, personally carried out.

The graphical analysis and digital modeling of surfaces allowed: the recognition of the geometric primitives underlying simple and complex vaults, the check of consistency between the orthographic projections of the models of vaults proposed for civil architecture, the construction of a vocabulary of shapes, and the reconstruction of the compositional logics recognizable in several Baroque architectures.

2. REASONING ABOUT THE VAULTS IN GUARINI'S TREATISES

The main contributions from *Euclides adauctus* to the discourse about the vaults, developed in the *Architettura Civile* and *Modo di misurare le fabriche*, are present in the *Tractatus XXIV* focused on the conics, in the *Tractatus XXXI* on the surfaces calculations, and in the *Tractatus XXXII* linked to stereotomy through the explanation of intersections between solids, between solids and planes, and their developments on a plane.

The references to the *Euclides adauctus* are very numerous in the other two treatises, demonstrating Guarini's will to connect the theoretical foundations and their applications in the ideation, design, and construction phases.

Architettura civile, written in Italian in the last years of Guarini's life and unpublished at his death, but definitive in the drafted parts including the original plates (Kruft 1999: 125), was finally edited in 1737 by Bernardo Vittone.

The text, divided into five "treatises", counts 308 pages with, at the end, 44 plates related to the text and 34 coming from the collection of *Dissegni d'Architettura civile et ecclesiastica* (1686).

The topic of the vaults is inserted at the end of the *Trattato III* devoted to the *Ortografia elevata* (orthographic elevation) and is developed along eight pages and by two plates.

In particular, in the *Capo Vigesimosesto* (Chapter XXVI) entitled "About vaults, and various ways to make them", the author performs an unusual vaults' classification based on a geometric criterion, identifying their elements and proposing some combinations that are suitable for roofing in the field of civil architecture.

Guarini explains his will to illustrate personal inventions and overcome the classifications used in the past: "But now I will divide the species, I will propose different methods and inventions of vaults, and finally I will treat the way to build them" (Guarini 1737: 183). As Edoardo Piccoli points out considering both the drawings, the treatise, and the buildings by the Theatine, Guarini aims to review the whole process of vaults design and construction, reformulating it from the geometric principles, materials and procedures (Piccoli 2006: 43).

In the two autograph plates, the XIX and the XX, Guarini represents in parallel oblique projection and in double orthographic projection the formal genesis of the vaults, even invented by himself, which was born from the combination of "six round bodies" (Guarini 1737: 183) through cuts and intersections.

The text and the plates relate to rigorously and systematically describe a repertoire of elements that, differently composed, give rise to a variety of vaulted systems.

Modo di misurare le fabriche, written in Italian too, is a relatively small book: 208 pages in 10.5x18.5 cm format.

Referring continually to the *Euclides adauctus*, Guarini expresses the desire to make the geometric principles developed therein applicable and accessible. The purpose of *Modo di misurare le fabriche*, stated in the dedication to the Superintendent of the Royal Finances of the Savoy, is to provide methods of measurement of works in buildings that were under construction in the second expansion of the Baroque city, started in 1673 under the direction of Amedeo di Castellamonte

The themes that characterize the book are preceded by the Prelude in which the author exposes the main rudiments of Arithmetic, aimed at carrying out the calculations proposed in the text. Then, the book describes methods of survey and expresses the intention to establish as an operative complement of the demonstrations presented in the *Euclides adauctus*.

The treatise is structured in three parts, which are dedicated to the rules for the calculation, in sequence, of flat Figures, 3-dimensional surfaces, and volumes. In particular, the second part constantly refers to the intradosal surfaces of the vaults. Thirty-six propositions make up this second part; among these, 19 refer to the calculation of the intradosal surface of the vaults; further evidence of the importance that the author, first in the architectural treatises, attributes to the theme.

Along these propositions the constructions and surface measurements of more than 30 shape variations applicable to the vaults are proposed; twenty of them add new opportunities to the casuistry exposed in the *Architettura civile*.

Each proposition is illustrated by a xylography interspersed in the text that represents by a diagram in pseudo-axonometry, traced with some incertitude, the measurement problem.

3. FROM THE "SIX ROUND BODIES" TO THE VAULTS ELEMENTS

In the *Architettura civile*, Guarini focuses on the geometric genesis of the various types of vaults that all derive from six round bodies: cylinder, cone, conoid (i.e. cone that does not end in a vertex, but in a straight line), sphere, rotating ellipsoid or oval (i.e. elliptical or ovate surface that cut in the middle has a circular section) and scalene ellipsoid or oval (i.e. lenticular surface that cut in the middle has an elliptic or oval section) (Fig. 1).

He geometrically demonstrates the possible decompositions by cuts and compositions of elements. Then, he shows the variants deriving from the different elementary surfaces, and proposes different types and inventions by specifying the right ratios of the rooms to be vaulted.

Lastly, he describes the methods of graphic tracing and proportioning the vaults, and finally "to build them in bricks as well as marble" (Guarini 1737: 183).

As anticipated, cutting each one of the six primitives by vertical planes, he explains how to generate elements for the different vaults.

Summarizing:

- groins, coves, *fascie* (i.e. arches), *trombe* (i.e. portions of barrel vaults defined by planes not parallel to each other) derive from cylinders (Fig.3).

- *fascie a squarcio* (obtained through the cut of conical surfaces with planes parallel to the directrix) derive from cones;

- domes, pendentives, and sails derive from spheres, rotational ellipsoids or ovals, and scalene ellipsoids or ovals.
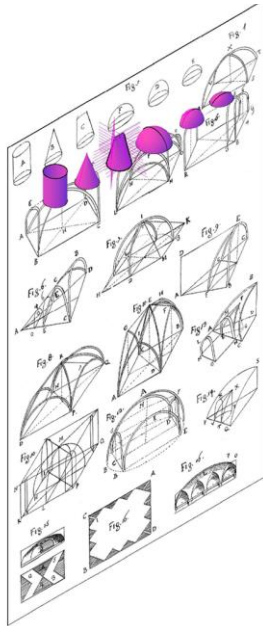
4. THE COMPOSITION OF ELEMENTS AND THE VARIANTS

The elements described in the *Architettura civile* create a real vocabulary of elementary surfaces to be used for creating a lot of simple and composed vaults that feature the covering of Baroque architecture.

Fig. 1. Guarini, the six round bodies. (Source: Guarini, 1737, plate XIX. Modelling: author).

Fig. 2. Guarini, elements and vaults arising from cylinders. (Source: Guarini, 1737, plate XIX. Modelling: author).
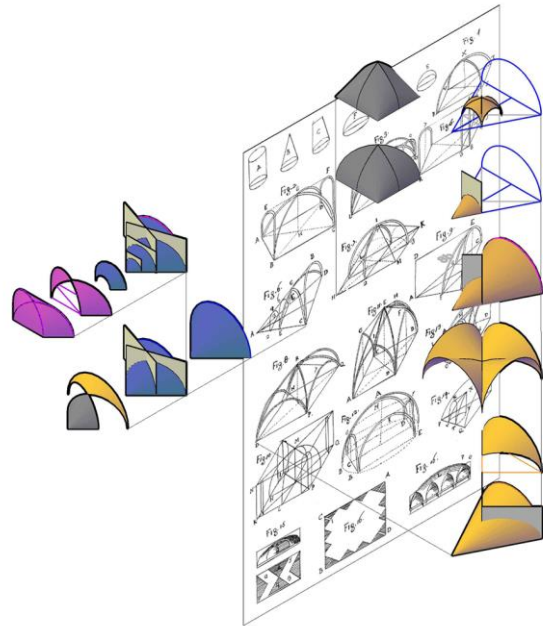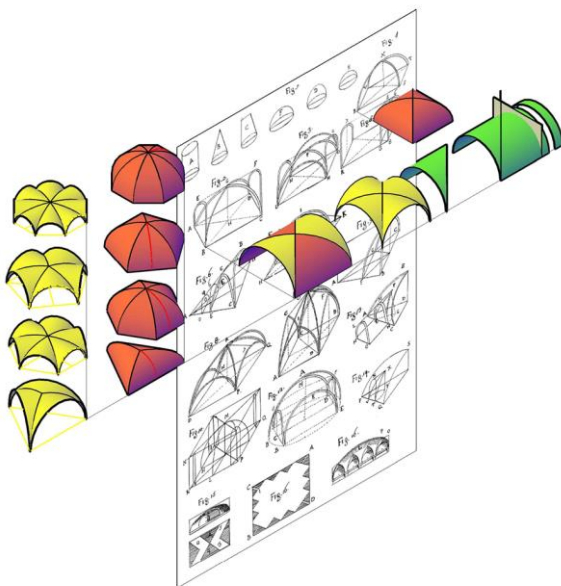

Fig. 3. Guarini, elements and vaults arising from cones. (Source: Guarini, 1737, plate XIX. Modelling: author).
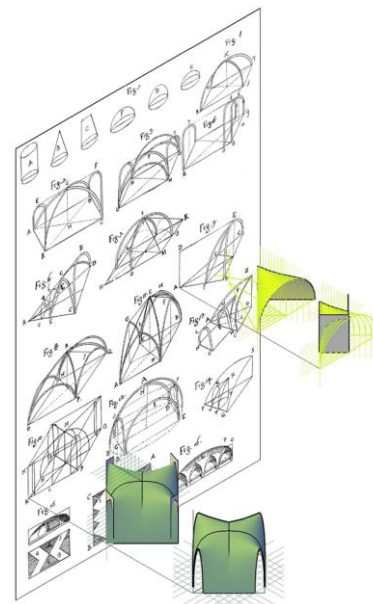

Fig. 4. Guarini, elements and vaults arising from conoids. (Source: Guarini, 1737, plate XIX. Modelling: author).

Indeed, can be obtained:

- barrel vaults, cutting cylinders and cones;

- cross and cloister vaults (alsohaving different polygonal plans), combining groins and coves derived from the cuts of cylinders, cones, and conoids (Figg. 2, 3, 4);

- domes, sail vaults, and pendentives cutting spheres, rotational ellipsoids or ovals, and scalene ellipsoids or ovals (Fig. 5).
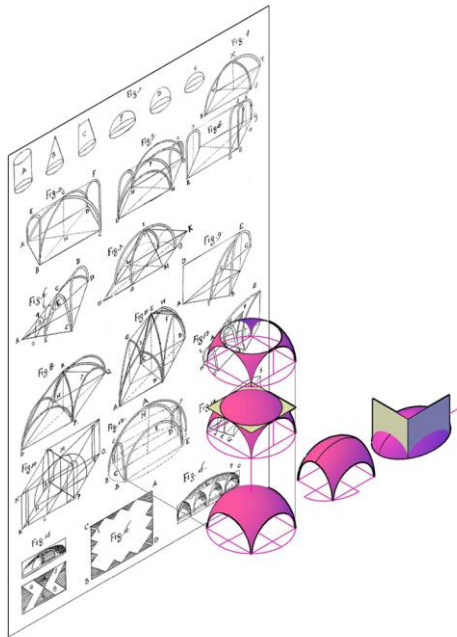
Guarini devotes special attention in comparison with the treatisers of the time to the so-called Gothic vaults, as variants of the Roman groin vault. They may come from different surfaces (cylindrical and conical with circular or elliptical directrices) and, in the words of the author, even if they are not in use, they may sometimes become useful (Guarini 1737: 186) (Fig. 6).

About the vaults originated from cones and conoids, Guarini claims the invention of, and reminds the application of one of the possible variants in the design of the Castello di Racconigi. This last, a kind of cloister that is formed by cone's portions with the vertex in the corners, is not today existent and is not identifiable within the autograph drawings classified by Augusta Lange (Lange 1970).

About the domes, Marco Boetti notes that starting from them ever-changing creative approaches achieve spherical or elliptical shell, cut until to form a series of interwoven arches. In this way, vaults like those represented in the treatise (e.g., those in the chapel of the Holy Shroud and the churches of San Lorenzo in Turin, Sainte-Anne-la-Royale in Paris, San Gaetano in Nice, the Somaschi in Messina) originate (Boetti 2006: 35).

Fig. 5. Guarini, elements and vaults arising from spheres. (Source: Guarini, 1737, plate XIX. Modelling: author).
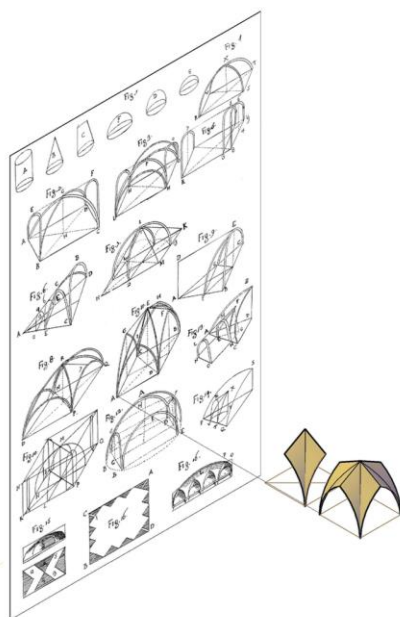

Fig. 6. Guarini, elements and vaults arising from ogives. (Source: Guarini, 1737, plate XIX. Modelling: author).
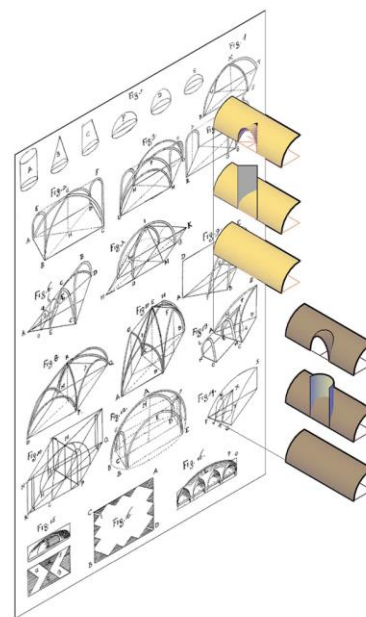

Fig. 7. Guarini, lunette vaults arising from ogives. (Source: Guarini, 1737, plate XIX. Modelling: author).

The definition of lunette vaults deriving from the cut of one of the vaults previously described (barrel, cross, cloister, dome, sail), concludes the description of the elements to shape the vaults. The cuts that do not arrive in the middle of the main vault can be practiced by vertical planes, arising from a triangular, circular or square plan. Then, as in Guarini's words "that void is filled with a piece of cylinder cut triangularly with the same angle of the cut, and as high as the same cut" (Guarini 1737: 187).

The diagrams of such vaults show two variants, i.e. groins with a triangular and circular plans inserted into a cylindrical cove.

In Guarini's explanation there is an evident geometric inconsistency, because to guarantee the joint between the main vault and the groin, this groin cannot be geometrically described as cylindrical, but it must be generated by parallel and variable sections (e.g. arcs of circumference or ellipse) (Fig. 7).

In the *Modo di misurare le fabriche* the explanatory sequence takes up the systematic organization of the *Architettura civile*, according to geometric primitives, here reduced to four (since conoid and scalene ellipsoid are absent): i.e. cylinder, cone, sphere, and rotational ellipsoid and widening to their sections. Moreover, the same typological sequence (barrel, conical, cloister, and cross vaults, domes and pendentives) is used. For this reason, they are in the following grouped with the same sequence found in the *Architettura civile*. Moreover, their shape is compared to built realities.

Throughout the text, Guarini tends to focus on conics, which had been the subject of Treatise XXIV, De Sectionibus Conicis, of the *Euclides adauctus*, and which here take on the role of directrix of domes, groin, and cloister vaults. In the 17th century, the conics, considered as a projection of the circle on a plane, were the subject of Desargues' writings, which Guarini might have known (Bianchini 2008: 15).

The calculation of the surface of a cylinder (Proposition 2) presents a Corollary referred to a barrel vault, suggesting its application to the covering of corridors. The measures given in the text (15 feet for the diameter, 50 for the length; 1 foot is about 0.29m) are effectively consistent with the development of a corridor (about 4.35 × 14.50 m; 1 span is about 0.25 m).

The surface of a truncated cone (Proposition 8), in the Corollary, gives rise to a conical vault. Its two diameters respectively measure 16 and 10 spans, the generatrix 9 span (about 4.00, 2.50, 2.25 m). The measures can refer to a connecting vault between a main central vault and the perimeter walls.

The calculation method for a semi-cove of a round arched cloister vault based on a square plan (and a portion of it, designed to constitute a lowered cloister) are developed in Proposition 10. (Fig. 8). The repertoire of the coves that generate cloister vaults ranges among the plan's various Figures and various directrices.

So, Proposition 11 is focused on the calculation of coves on five, six, and eight-sided plans, and Proposition 22 on a semi-cove with an elliptical directrix of a square base cloister vault (and a portion of it). Moreover, Proposition 12 proposes the calculation of a cove (and a portion of it), as an element of a cloister vault on a rhombus or rhomboid plan (Fig. 8). An application of this vault, in the lowered version, can be found in the corner vaults of the atrium of Palazzo Provana di Collegno, designed by Guarini himself.
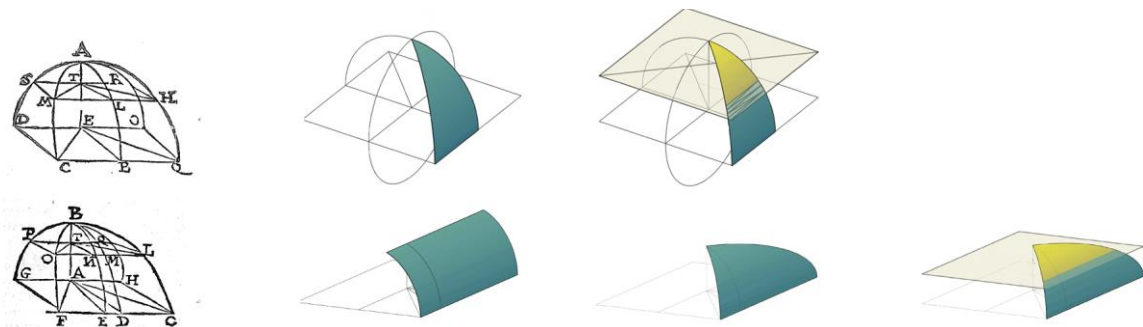

Fig. 8. Guarini, coves with circular and elliptic directices. (Source: Guarini, 1674, Propositions 10 and 12. Modelling: author).

The method for finding the surface of a circular-section groin, designed as part of a groin vault on a square plan, is described in Proposition 14. Also, it is cut by two vertical planes parallel to the keystone line and by a plane parallel to the directrix. In the Baroque age, this extremely lowered lunette, whose directrix is an arc of circumference, is very widespread to give light in low-pitched vaulted rooms, such as mezzanines and basements. (Fig. 9). Also, the groins repertoire widens to the different directrix. Indeed, Proposition 23 extends the case studies to elliptic groins and their portions. These are also considered elements of a groin vault on a square plan. In the graphics scheme, the groin's height is given by the major semi-axis, but the method is

also valid for the smaller one, which is prevalent in the built vaults; just think of the lunettes in the atria of the palaces Carignano and Provana di Collegno, designed by Guarini (Fig. 9). The discussion on the groins is concluded through an original construction (Proposition 24), generated by the lateral half-faces of a concave pyramid on a square base, or by portions of them. The semi-face, rotated by 180°, can constitute half of an ogival groin (Fig. 9).

Propositions 17 and 18 develop, on the basis of Archimedes' teachings, the calculation of the spherical surface and a part thereof, applying it to the description of a spherical dome and a calotte dome.
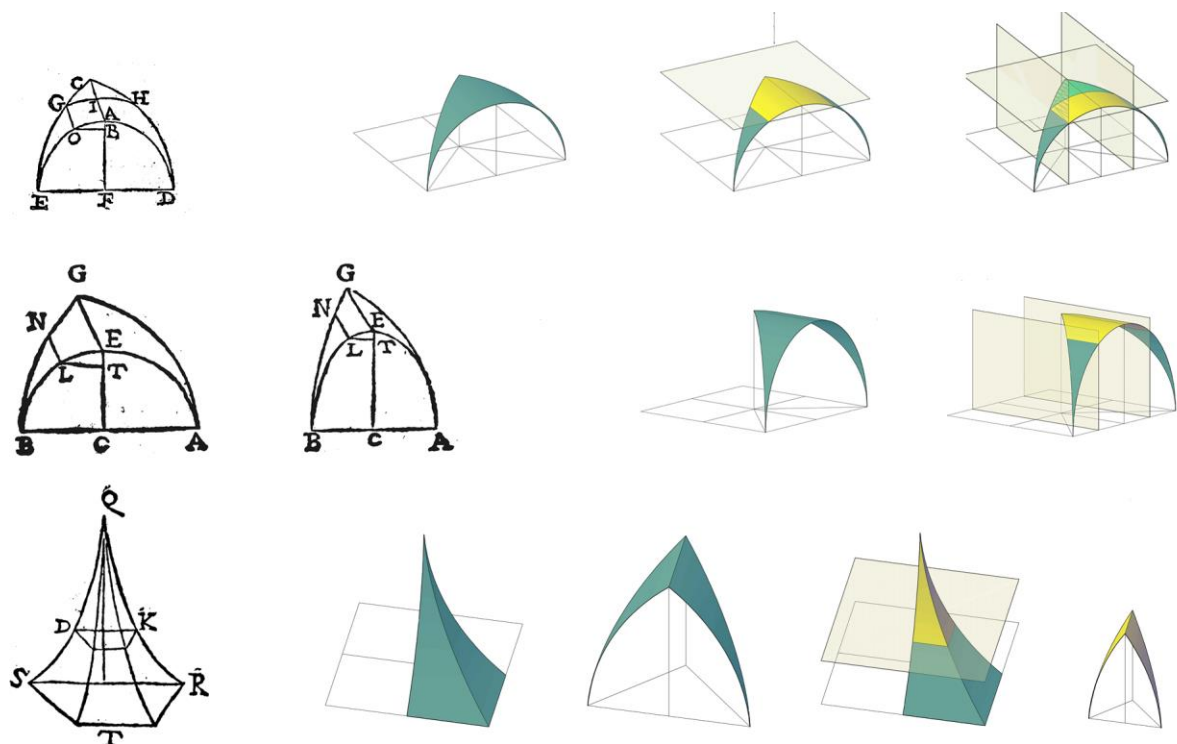

Fig. 9. Guarini, groins with circular, elliptic, and ogive directices. (Source: Guarini, 1674, Propositions 14, 23 and 24. Modelling: author).

The dimensional reference to the radius of the sphere (7 feet ≈ 2.03 m) and the generatrix bowstring of the calotte (9 spans ≈ 2.25 m) suggest that Guarini is thinking of the small central domes of the vaults, like those present in the rooms of Palazzo Carignano. (Fig. 10). Proposition 20 describes the calculation of an ellipsoid surface, applying it to the ellipsoidal dome. The measures of the axes: 50 and 82 spans, about 12.50 and 20.50 m, bring to mind a

vault of considerable size, much wider than that in the atrium of Palazzo Carignano (about 10 × 14.65 m). (Fig. 10). This case gives rise to the elliptical calotte proposed in Proposition 21.

The case of an ogival dome, similar, as Guarini affirms to the ancient pseudo-cupola is developed in Proposition 26. Therefore, its cross-section is an acute arc (Fig. 10)
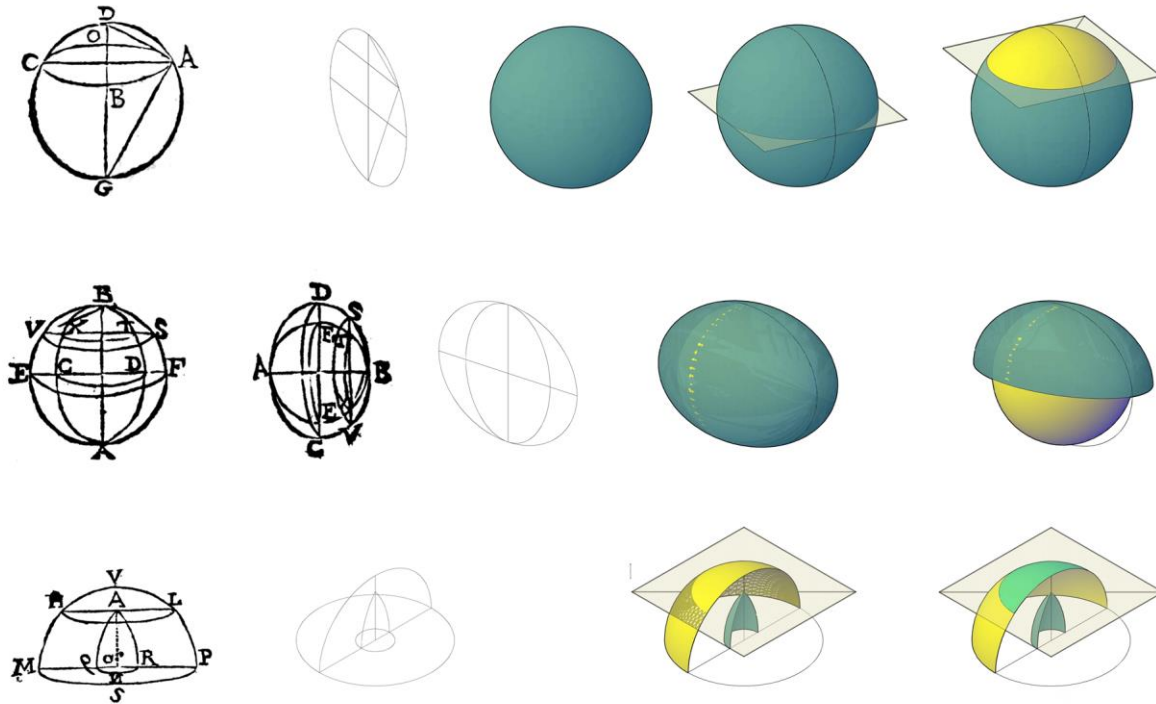
Fig. 10. Guarini, domes with circular, elliptic, and ogive directices. (Source: Guarini, 1674, Propositions 18, 20 and 26. Modelling: author).
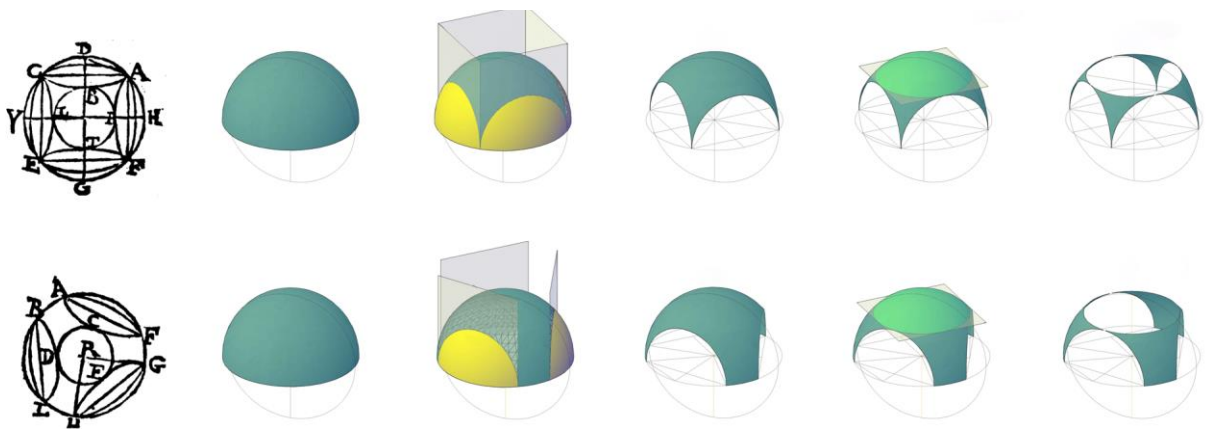
Fig. 11. Guarini, "triangular" and "quadrangular" pendentives. (Source: Guarini, 1674, Proposition 19. Modelling: author).

The calculation of the pendentives, is particularly articulated, starting from two cases (Proposition 19). In the first, the cut of the hemisphere with four two-by-two orthogonal planes generates a sail vault, which, cut with a tangent plane to the arches keystone, forms four "triangular" pendentives with impost in a point. In the second, the hemisphere is intersected by three planes that first generate a triangular base sail with wide circular imposts, then cut with a tangent plane to the arches keystone, result in three "quadrangular" pendentives. (Fig. 11).

Proposition 16 shows a particular problem related to the lunette vaults: the calculation of the surface to be subtracted to the main surface consisting of quarter of a cylinder after cutting with vertical planes. The author explains the necessity of such a calculation to achieve a correct measurement of the lunette vaults: "As it is reasonable to measure the lunettes, that

certainly make greater the surface of a vault, so it is convenient to remove that surface from the same vault, which occupies the space of the lunette" (Guarini 1674: 109). This case establishes an evident relationship with the built architecture. Indeed, Guarini explains how to survey the position of some particular points and the horizontal plane tangent to the vertex of the groin, finalizing these operations to the calculation (Fig. 12). In the empty space a lunette must be inserted. Also in this case the issue of the geometric joint between the two surfaces is not solved.

Fig. 12. Guarini, surface to be subtracted for inserting a lunette. (Source: Guarini, 1674, Proposition 16. Modelling: author).

5. FROM THE GEOMETRIC SHAPES TO THE DESIGN SHAPES

In the *Architettura Civile*, from the Observation 8, Guarini deals with the way to design the vaults (Guarini 1737: 188) passing to the architectural implementations, supported by his experience as designer.
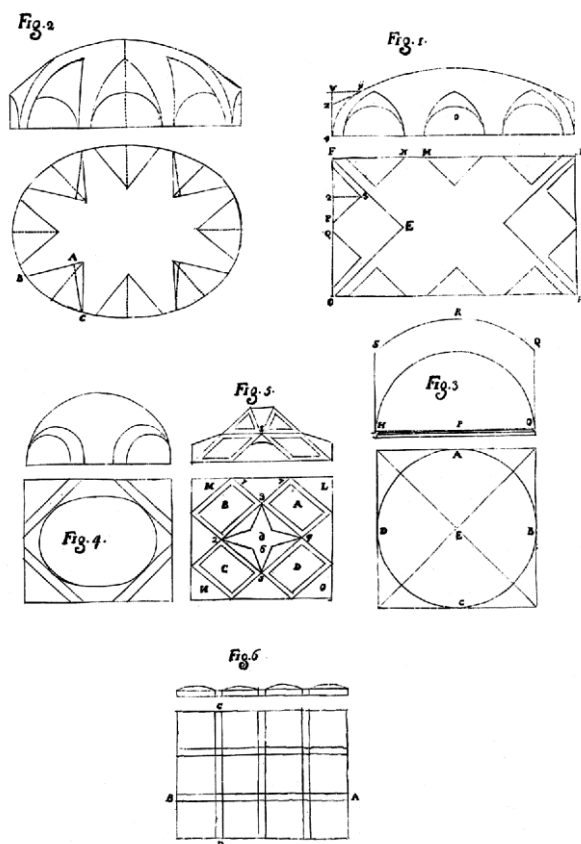

Fig. 13. Guarini, patterns of lunette vaults, sail vault, and *a fascie* vaults. (Source: Guarini, 1737, plate XX. Modelling: author).

The passage is also highlighted by the change of graphic representation method. The double orthographic projections replace the three-dimensional diagrams. This change of method seems to conceptually indicate the gap between the geometrical patterns previously described, which are differently applicable, and the vaults' models often inspired to his architectural creations (Fig. 13). Approaching the architectural design, Guarini uses the language of the technical drawing, where the elevation "implies the plan and raises from it the drawing" (Guarini 1737: 73)

In this part of the text, initially Guarini treats the drawing as a geometric tracing: "The vaults drawing does not involve much difficulty, because the circle could be traced by the compass, about the ovoidal and elliptical vault... I stated different rules for giving them the rise" (Guarini 1737: 188). Nevertheless, the term drawing assumes a more pregnant value in the following of the discourse. Here, it takes both a projectual value, identifying drawing with design, and a perceptive value, since, as in the words of the Theatine "in each type of vaults certain devices could be adopted to make them more grateful to the eye" (Guarini 1737: 188). At the end of the discussion, 'drawing' also takes a constructive value.

In the transition, from the definition of vault typologies according to the pure geometries of the intradosal surfaces, to the specifications of the constructive features to be adapted to the civil buildings, appears the gap between geometric criteria of ideation, and proportional criteria aimed to constructive dimensioning. This gap appears only in the text, not in the final eight orthographic drawings, in which the will to

regularize the schemes prevails even in the elevations (using regular geometric Figures as rotational main surfaces and round arcs).

The description about designing the vaults therefore focuses on the rise in relation to the planar ratios and, about the lunette vaults, on the number of lunettes in relation to the dimensions of the rooms. In particular, Guarini asserts that the cloister vaults (also named *a conca*) shall be more beautiful if they have a lower rise. Indeed, in the ordinary rooms that are not so much high, if they are built with a little rise, they will make the room airier. The useful ratio for the rise is a quarter or at less one fifth of the shorter side of the plan, and if the designer wants to make them look flat, he can create the cornice on which the vaults' imposts arise (Guarini 1737: 188).

The proposed typologies of vaults to be chosen, are related to the shape and the ratios in plan of the rooms they have to cover. Guarini suggests: groin vaults, for the rectangular rooms, even oblong, lunette vaults, having the groins inserted on cloisters and barrel with cloister heads vault, for the rectangular halls, and rotational domes for elliptical or oval plans.

Finally, Guarini attributes to himself the ideation of the *a fascie* (arched or architraved) vaults: "this kind of particular vaults is mine, and I created them in many variations and satisfaction of the people… They are very refined to behold, and leave great areas for painting" (Guarini 1737: 189). These are designed to cover very wide rooms with single vaults.

Several inconsistencies between the plans and the vertical sections, particularly in the plate XX, have been noted. For their interpretation, it has been followed the remind by Piccoli, who observed the guide role that the plan tends to assume in Guarini's design method (Piccoli 2006: 47).

In support of this hypothesis, it should also remember that Guarini emphasizes the relevance of the plan in several parts of the *Architettura Civile*. Indeed, he points out that the design starts and develops from it: "Knowing how to perfectly draw the plans depends on the elevations, and this last depends on the first one…. Just because you have to start from any one of them, here I have intended to give those notions aimed to draw the plans, which are more general, and more

independent from any other knowledge" (Guarini 1737: 62).

For these reasons, the methods of graphic analysis and three-dimensional modeling carried out in relation to the orthographic drawings of these last models of vaults has been differently set. Indeed, it starts from the plans' analyses and makes checks and congruent corrections of the vertical sections.

The geometric interpretation method has been suggested by the comparison between Guarini's text and the drawings, and supported by the experience about three-dimensional digital reconstruction of the multiple schemes present in the plate XIX, which, as previously said, allowed to discover the formal vocabulary he proposed using simple geometries.

This cognitive approach guided the graphic analysis operations that, starting from the plans and sections drawn by Guarini, identified their proportional relationships, symmetries, geometric constructions, primitives, and their compositional criteria.

Three-dimensional modeling was carried out according to the sequences and concatenations that are typical of Guarini's *modus operandi*. This is described by Henry Millon, who, referring to the work by Borromini and Guarini, states that the building shapes designed by them were set on some basic geometric principles. Both developed their schemes adhering to a general concept of geometric evolution, which subsequently determines the position of the main design components (Millon 1970: 39).

The analysis of the Theatine's drawings also involved the use of a particular process of work, through which the two vertical sections are automatically generated cutting the 3D models: the first one is superimposed on the original drawing, allowing the check of the projective correspondence, and the second one completes the representation with the lateral projection.

6. PRACTICAL APPLICATIONS: GROIN, LUNETTES, *"A FASCIE"* VAULTS IN BAROQUE ARCHITECTURE

model, drawn in plate XIX, consists of a cross vault for covering rectangular and oblong rooms. It is shown by the orthographic projections where

the two solutions are superimposed. In this case, the two directrices (the first one semicircular and the second a semi-elliptic) use the side length of the room as the diameter or axis. Also, the semi-circular arc with the same diameter of the other is set on the long side. Guarini's drawing shows some ambiguities. In the plan, the intersection between the groins of the first solution does not appear. In the vertical section the lining which render the two arcs suggests that it is a cloister vault instead of a groin one (Fig. 14).
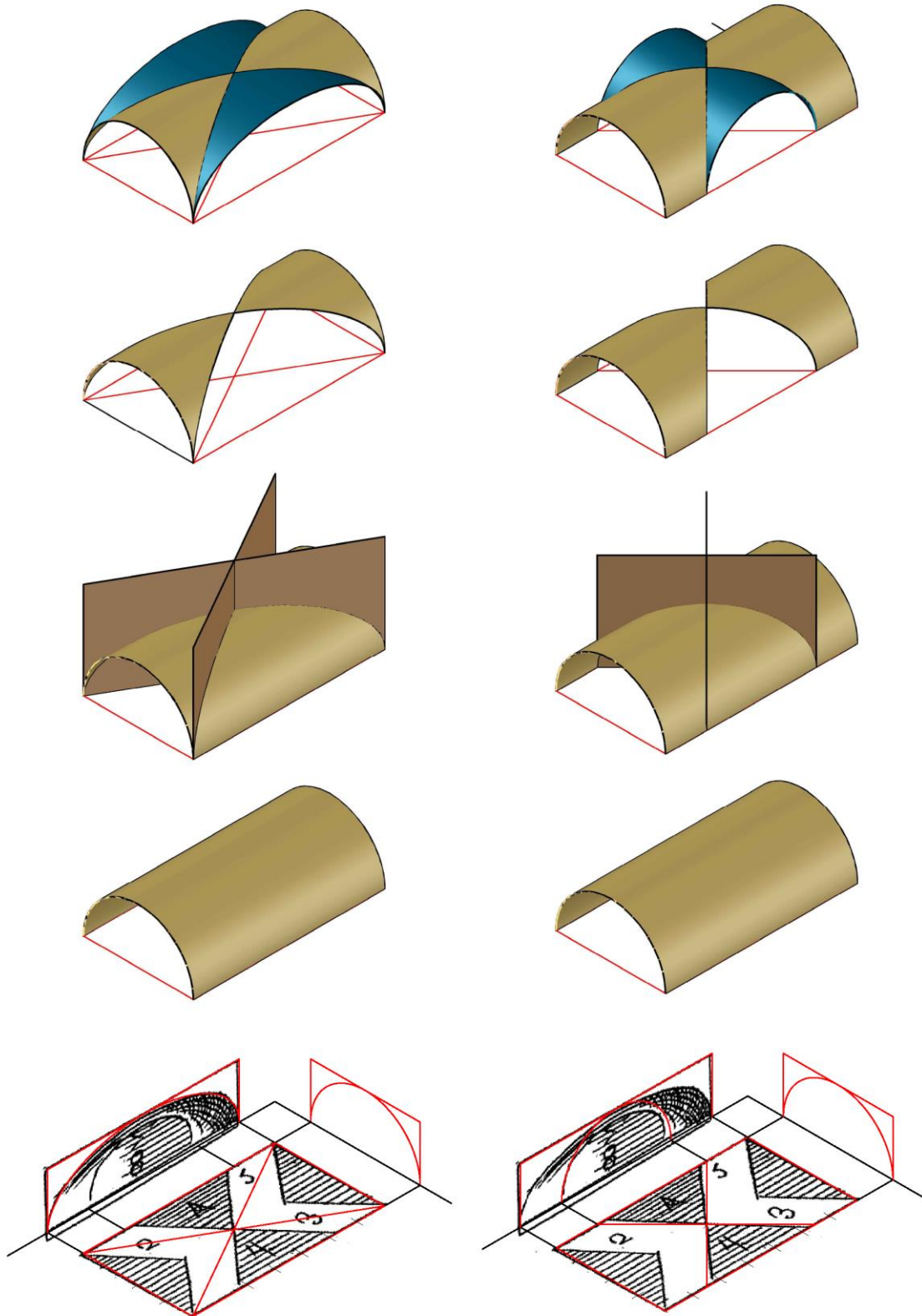

Fig. 14. Guarini, cross vaults. (Source: Guarini, 1737, plate XIX. Modelling: author).

Lunette vaults are widely discussed in the treatise and drawn in plate XX, surely because they are the most widespread covering system of that time.

The first two examples are based on late-Renaissance lunette vault models.

In the first case, the rectangular plan is covered by a cloister vault that is cut with vertical planes parallel to the intersection between the coves. The rampant groins, inserted for closing the cuts are equal in number for each side and have different spans of the perimeter arches on each of the walls. Therefore, the arches are semicircular along two sides of the room and semi-elliptical along the others. The main vault on a rectangular plan has a 3:4 ratio of the sides and semi-elliptical directrices of the two semi-cylinders that shape the coves of the cloister vault. The rampant groins, having a triangular plan of different base along the two sides of the room originate from circular arches along the walls. This model could refer to the drawing of the honor hall in the Guarinian design of Castello di Govone, present in another plate of the *Architettura Civile*. The reconstructive model shows an adequate overlapping with Guarini's projections, except for the lateral groins that are not rampants in the original drawing (Fig.15).

In the second example a lunette vault covers a rectangular plan with side ratio of about 1:2. It is a lunettes barrel vault with cloister heads. The lunettes, defined by cuts parallel to the intersections between the barrel and the coves, have the same shape and are in a different number on the sides. The compensation is necessary for the regularity of the system. It is obtained by setting a different distance between the lunettes along the sides. The Guarinian vertical section of the main vault is not geometrically consistent with the plan: the profile of the section is elliptical rather than flat in the central part and circular at the ends (Fig. 15). The above-mentioned vault may refer to a drawing attributed to Francesco Lanfranchi: the cross-section of the design for the central body of Castello di Racconigi dating back around 1665 (Dardanello 2006: 435). Guarini succeeded him in the task of castle's transformation from 1677.
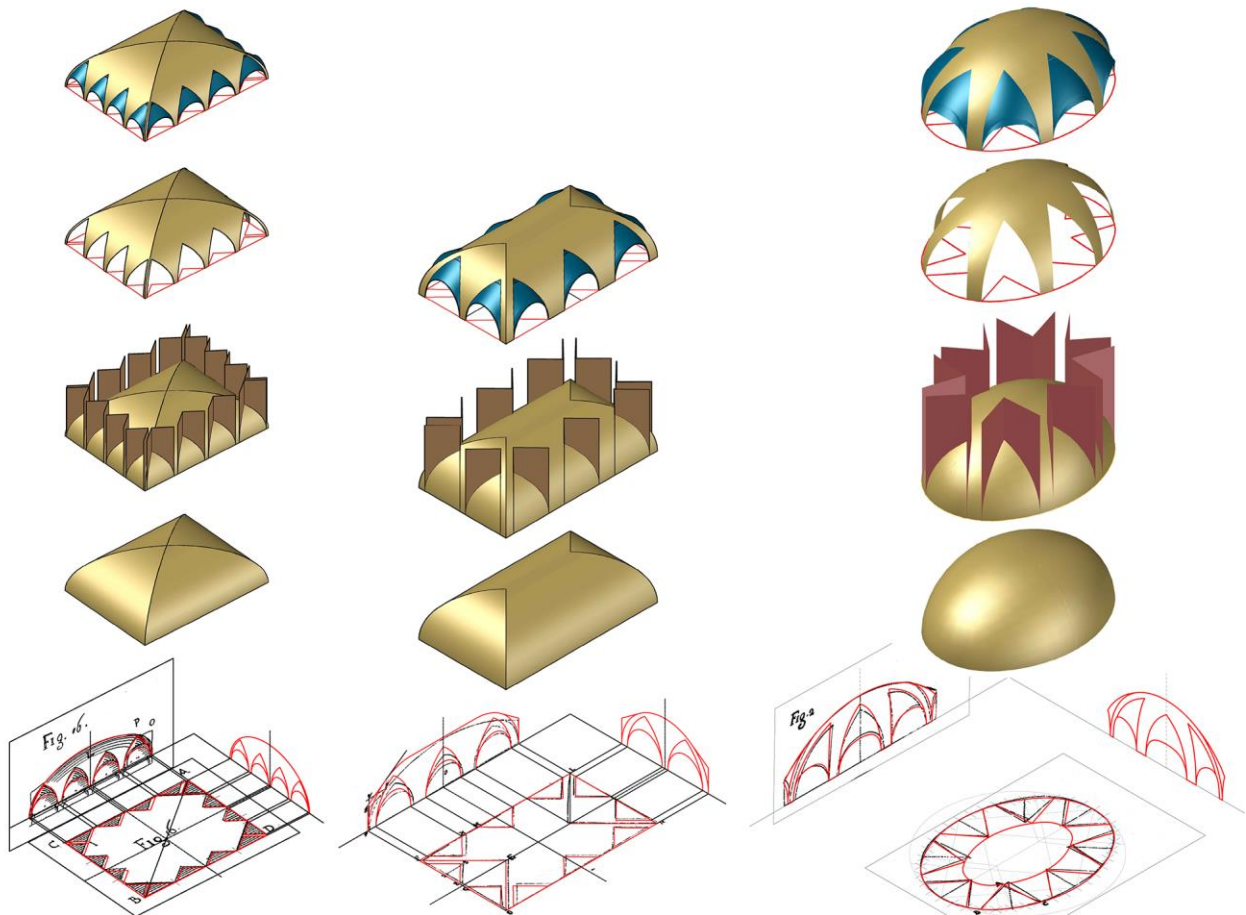

Fig. 15. Guarini, lunette vaults. (Source: Guarini, 1737, plate XX. Modelling: author).

The third case witnesses the passage from the late-Renaissance to the Baroque style, through Guarini's invention of a lunette vault set on the ovate plan. It recalls, for the geometric layout, the vault of the atrium of Palazzo Carignano, even if the author does not explicitly refer to it. In the original drawing the axes of the oval plan have a ratio 4:3, while the vertical section reveals that the main surface is generated by rotation of the oval around the major axis: the resulting rise is equal to the minor semi-axis. According to the text, the horizontal projections of the groins have equal sides, and the vertex is tangent to an inner oval, offset of the main one. Guarini also shows a variant with different sides in plan for the diagonal groins. In the reconstructive model, the rotating oval that shapes the main surface is cut with vertical planes extruded from the projection of the groins. These cuts involve groin vertices higher than the drawing by Guarini. The rampant groins, which originate from round arches, are modeled by parallel arcs with variable curvature along an oblique trajectory (Fig. 15).

Also the *a fascie* vaults (marked by arches) play a significant role in the *Architettura civile* and in Baroque architecture, just think to several vaulted halls designed by Filippo Juvarra. They are flanked by the *a fascie piane* vaults (marked by beams), which develop as three-dimensional or flat coverings. Both types are used for dividing the space to be covered in smaller surfaces.

The plan projection of the *a fascie* vault in plate XX is similar to the vault designed by Guarini for one of the five halls of the first noble floor of the Castello di Racconigi. Indeed, he affirms that in the plate he is showing an example among the many vaults designed for Racconigi (Guarini 1737: 189). The reconstructive model of the vault starts from four *fascie a squarcio* (i.e. conical arches) that delimit the space of covering. A sort of sail is set on the four arches and on short portions of the perimeter walls. It is not an elliptical or ovaloid surface cut by vertical planes, as Guarini taught describing the sail vaults (Guarini 1737: 186-187): it must be geometrically built through parallel curved sections. Moreover, in Guarini's drawing, the angular vaults are suitable for different hypotheses: the *a tromba* vaults, above described, confirm the correctness of both the projections, while the mid-cloister or mid-groin vaults, the more plausible ones from the formal point of view, implicate some author's omissions.

Indeed, in the case of the mid-cross vault, the intersection between the groins and the vertical projection of one of them are lacking, in the case of the mid-cloister vault, the intersection between the coves (Fig. 16).
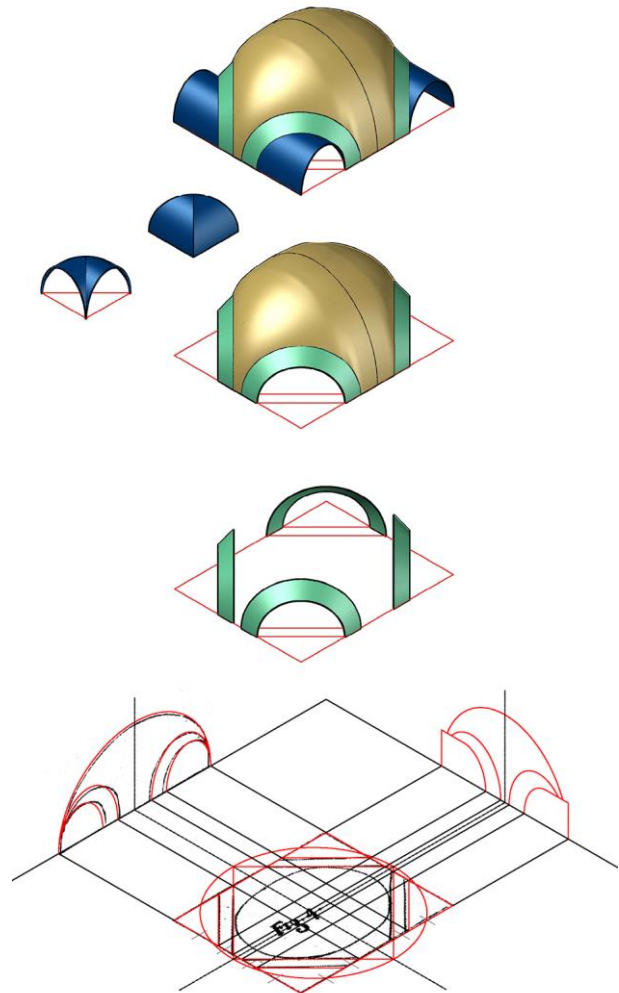

Fig. 16. Guarini, *a fascie* vault. (Source: Guarini, 1737, plate XX. Modelling: author).

However, while the design drawing of the plan for Racconigi is quite similar to that of the treatise, the drawing of the cross-section reveals a different spatial conception. Indeed, Guarini drew the roof of the room as covered by linear beams that support almost flat surfaces. Therefore, the *a fascie* vault offered as a model in the treatise becomes in the design practice an *a fascie piane* vault.

Moreover, the *a fascie piane* vaults are developed as a sort of three-dimensional frameworks of absolute formal innovation, applied by Guarini in the design of the vault of Palazzo Madama (Zangirolami 2015: 130-136) or, simply, as flat roof.

7. CONCLUSIONS

The parallel reading of the *Architettura Civile* and the *Modo di misurare le fabriche*, in light to the theoretical contribution of the *Euclides adauctus*, about the theme of the geometry of the vaults, offers an overview of the whole production cycle of vaulted systems that goes from the ideation, to the design, to the construction, the calculation, and the verification of the quantities of materials during construction.

In the two treatises there is a further confirmation of Guarini's vision according to which architecture is dependent to mathematics and therefore must be traced back to its geometric matrices (Tavassi La Greca 1968: 452-453).

The *Architettura Civile* states a real vocabulary of elements strictly derived from geometric primitives usefully cut by planes. Their combination originates a lot of composed vaults. Passing from the typological patterns to the design models the geometric criterion gives way to the proportional one and opens to numerous inventions that feature the vaulted systems in Baroque architecture.

In the *Modo di misurare le fabriche*, the link with architecture is less stringent than in *Architettura civile*. Even if the treatise aims at being a subsidy to the architect's operative activity, the discourse, as stated by Luigi Vagnetti "is kept on a theoretical plane of absolute abstraction, in which the scientific and speculative mentality of the author is decidedly overwhelming" (Vagnetti 1970: 508). Nevertheless, the scope of possible shapes broadens, presenting some novelties with respect to the *Architettura Civile* inspired to the real cases surveyed in the Baroque city.

Graphical analyses and reconstructive digital models allowed to highlight the role of geometry in Guarini's theorization and the logic of shapes' composition at the basis of his inventions, and constitute a knowledge base to recognize and interpret the geometric structure of the vaults in Baroque built heritage.

REFERENCES

Bianchini, Carlo. 2008. *La scienza della rappresentazione nella concezione di Guarino Guarini*. Roma: Gangemi. ISBN 8849214456

Boetti, Marco. 2006. "La geometria delle strutture voltate ad archi intrecciati". In Dardanello, Giuseppe, Klaiber, Susan, Millon, Henry A., ed. *Guarino Guarini*. Torino: Allemandi, p. 35-42. ISBN 978-88-422-1471-7

Dardanello, Giuseppe. 2006. "Le idee di Guarini per il palazzo con cupola di Racconigi". In Dardanello, Giuseppe, Klaiber, Susan, Millon, Henry A., ed. *Guarino Guarini*. Torino: Allemandi, p. 425-440. ISBN 978-88-422-1471-7

Guarini, Guarino. 1671. *Euclides adauctus et methodicus, matematicaque universalis*. Torino: Typis Bartolomaei Zapatae.

Guarini, Guarino. 1674. *Modo di misurare le fabriche*. Torino: Per gl'Eredi Gianelli.

Guarini, Guarino. 1686. *Dissegni d'architettura civile et ecclesiastica, inventati et delineati dal Padre D. Guarino Guarini*. Torino: Domenico Paulino.

Guarini, Guarino. 1737. *Architettura Civile*. Torino: Appresso Gianfrancesco Mairesse.

Kruft, Hanno-Walter. 1999. *Storia delle teorie architettoniche da Vitruvio al Settecento*. Roma: Laterza. 1999. ISBN 978-88-420-5804-5

Lange, Augusta. 1970. "Disegni e documenti di Guarino Guarini: catalogo dei disegni manoscritti". In *Guarino Guarini e l'internazionalità del Barocco, atti del convegno internazionale promosso dall'Accademia delle Scienze di Torino*, tomo I. Torino: Accademia delle Scienze, p. 92-346.

Millon, Henry A. 1970. "La geometria nel linguaggio architettonico di Guarini". In *Guarino Guarini e l'internazionalità del Barocco, atti del convegno internazionale promosso dall'Accademia delle Scienze di Torino*, tomo I. Torino: Accademia delle Scienze, p. 35-60.

Piccoli, Edoardo. 2006. "Disegni di Guarini per le volte di edifici civili". In Dardanello, Giuseppe, Klaiber, Susan, Millon, Henry A., ed. *Guarino Guarini*. Torino: Allemandi, p. 43-49. ISBN 978-88-422-1471-7

Spallone, Roberta. 2016. "'On the vaults and various modes of making them'. Interpretative digital models of the XIX and XX plates in Guarini's Architettura Civile, between designs and buildings" / "'Delle volte, e vari modi di farle'. Modelli digitali interpretativi delle lastre XIX e XX nell'Architettura Civile di Guarini, fra progetti e realizzazioni". In Bertocci, Stefano, Bini, Marco, ed. *The Reasons of Drawing. Thought Shape and Model in the Complexity Management / Le Ragioni del Disegno. Pensiero Forma e Modello nella Gestione della Complessità*. Roma: Gangemi, pp. 1275-1282. ISBN 978-88-492-3295-0

Spallone, Roberta. 2017. "Matrici geometriche e saperi costruttivi". In Spallone, Roberta, Vitali, Marco. *Star-shaped and Planterian Vaults in the Baroque Atria of Turin / Volte stellari e Planteriane negli atri barocchi in Torino*. p. 85-148. Ariccia: Aracne. ISBN 978-88-255-0472-9

Spallone, Roberta. 2019a. "Geometry, Arithmetic, Architecture. Calculation Methods for Vault Surfaces in the Modo di Misurare le Fabriche by Guarini". In Cocchiarella, Luigi, ed. *ICGG 2018—Proceedings of the 18th International Conference on Geometry and Graphics*. Cham, Switzerland: Birkhäuser-Springer, p. 2108-2119. ISBN 978-3-319-95587-2. https://doi.org/10.1007/978-3-319-95588-9_188

Spallone, Roberta. 2019b. "Lunette Vaults in Guarini's Work. Digital Models between Architettura Civile and Modo di Misurare le Fabriche / Volte a lunette nell'opera di Guarini. Modelli digitali fra Architettura Civile e Modo di misurare le fabriche". *Disegno*, no. 4, p. 91-102. ISSN 2533-2899

Zangirolami, Diego. 2015. "Immaginazione e potere: ricostruzioni dei progetti di Guarino Guarini per Palazzo Madama e Racconigi, mai realizzati". *Palazzo Madama 2014-2015*, pp. 130-136.

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