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A set-membership approach to direct data-driven control design

By

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Mohammad Abuabiah

Turin, June 7, 2019

This thesis is wholeheartedly dedicated to my beloved wife, parents, brother and sisters, who have been my source of inspiration and gave me strength when I thought of giving up, who continually provide their moral, spiritual and emotional support.

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Abstract

Significant research efforts have been devoted in recent years to the problem of designing a control system under the assumption that a mathematical model for the plant is not available. In particular, several interesting results have been obtained through the direct data-driven controller (DDDC) design approach, that is the direct design of the controller from a set of input-output experimental data characterizing the plant behaviour.

The DDDC design approach has a wide representation capability for designing the controller for different dynamical systems and this framework of the DDDC approach is also recently supported by a well worked out research and some industrial applications. Despite the advances of the DDDC field, designing of such a controller without the availability of a mathematical model is still in its immature state, due to many open problems of DDDC design theory. One of the most important problem in the DDDC approach is that, in practice, the available experimental data is always imperfect, as it is affected by measurement noise. Therefore, this thesis focuses on the development of a novel non-iterative direct data-driven technique to deal with linear-time-invariant (LTI) controller design, such that the controller is directly identified from a collected experimental input/output data corrupted by bounded additive noise. Based on the assumption of corrupting bounded noise, the design problem is formulated then in the framework of set-membership (SM) identification theory.

In this work, we propose two original non-iterative direct data-driven techniques to deal with a linear-time-invariant (LTI) controller design, such that the con-

troller is directly identified from input/output data without plant identification step:

1. **Fixed structure controllers**: in this approach, we formulate the problem of designing a fixed controller in order to match the behaviour of a given reference model, in terms of an equivalent set-membership errors-in-variables problem and we define the feasible controller parameter set. Then, we design the controller parameters by applying recent results in the field of set-membership errors-in-variables identification.

2. **Nonparametric controllers**: we present a novel non-iterative approach to direct data-driven nonparametric controller design. In this approach, the DDDC problem is formulated in the robust Reproducing kernel Hilbert space (RKHS) framework. First, by assuming that the available input-output data are corrupted by bounded noise, we formulate the problem of designing a controller in order to match the behaviour of an assigned reference model. Then, the controller is designed by means of a non-parametric approach, inspired by recent results in the field of RKHS approach.

Moreover, in this work, we present an original approach to design, in a systematic way, the reference model M to be able to meet performance specifications. In the proposed method, the desired performance specifications of the closed-loop system are translated into a model reference design paradigm in the framework of the DDDC approach. The design of a suitable reference model M is carried out based on H_∞ control design approach by using a suitable fictitious plant. Then, stability conditions both for stable minimum-phase plant and stable non-minimum phase plant are discussed and analyzed to guarantee the internal stability of the designed closed-loop system.

Finally, the obtained design algorithm is applied to different electronic test bench networks to show the effectiveness of the proposed approach.

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Chapter 1

Introduction

In general, controller design is the scientific discipline that employs methods from mathematics and engineering in order to force dynamical systems to behave in a desired fashion. One way to classify control design techniques is to separate them into indirect techniques, in which the controller is designed on the basis of the available mathematical model of the dynamical system to be controlled, and direct techniques, which do not explicitly use the model of the plant, but rely on experimental data in order to directly design the controller. The process of building a mathematical model of a dynamic system from experimental data is called system identification. To cope with the direct technique, one possibility is to use what is called a direct data-driven control (DDDC) approach. The philosophy of the DDDC design approach is to identify the controller directly from input/output experimental data without plant identification step.

The common need for accurate and efficient control of today's industrial applications is driving the system identification field to face the constant challenge of providing better models of physical phenomena. However, systems encountered in real-world applications are often complex and an accurate model of the plant to be controlled usually is not available. Therefore, this raises the need for DDDC approach, where DDDC approaches do not rely on plant model identification since available input-output data experimentally collected from the plant are directly used to design the controller. The control specifications are usually given in this context in terms of a desired closed-loop reference model; then, the controller parameters are computed by formulating the problem in terms of model matching design.

The DDDC design approach has a wide representation capability for designing the controller for different dynamical systems and this framework is also recently supported by a well worked out research and some industrial applications. Despite the advances of the DDDC field, identification of such a controller without the availability of a mathematical model of the plant is still in its immature state, due to many open problems of DDDC design theory. One of the most important problem in the DDDC approach is that, in practice, the available experimental data is always imperfect, as it is affected by measurement noise. Therefore, this thesis focuses on the development of a novel non-iterative direct data-driven technique to deal with linear-time-invariant (LTI) controller design, such that the controller is directly identified from collected experimental input/output data corrupted by bounded additive noise. Based on the assumption of corrupting bounded noise, the design problem is formulated then in the framework of set-membership (SM) identification theory.

In set-membership identification, the assumption that the noise signal is bounded is less restrictive than a traditional statistical assumption, as in practice all signals are bounded and the bounds can often be roughly derived from the specifications of the measurement equipment. Therefore, handling bounded uncertainty models in DDDC design is more natural and often easier than dealing with a probability density function that is used in general in the field of DDDC framework by using the basic probabilistic identification methods. This means in brief, no statistical information about the noise and/or the disturbance is assumed to be a-priori available when SM theory is used. Therefore, this research has been directed both towards understanding the fundamental properties of the SM approach itself and its limitations as well as towards developing computationally efficient and less resource demanding for designing a controller using the set membership algorithm. However, further research related to both the set membership identification algorithm and to its use in DDDC design approach is required in order to achieve its wider use in controller design and exploit all the benefits that it may offer. Thus, this thesis is aimed at giving a contribution to these research efforts by treating three topics related to set membership identification and its use in controller design, as discussed briefly in the next section.

1.1 Outline and contribution

This thesis is organized as follows. In Chapter 2 we introduce the concept of set membership identification, discuss its main contribution with respect to the more popular probabilistic identification approach. The main contributions of the thesis are given in Chapters 3, Chapter 4 and Chapter 5.

In particular, in Chapter 2 we briefly overview the system identification and set membership identification theory in general. Then Set-Membership Errors-In-Variables (SM-EIV) identification problem will be discussed in details for both LTI single-input-single-output(SISO) and LTI multi-input-multi-output(MIMO) systems. The contribution and the main results of this chapter is based on the work of Cerone et al. (see e.g., [36], [39], [34] and [35]).

In Chapter 3, we propose a novel non-iterative direct data-driven technique to deal with linear-time-invariant (LTI) controller design, such that the controller is directly identified from input/output data without plant identification step. First, we formulate the problem of designing a controller in order to match the behaviour of a given reference model, in terms of an equivalent set-membership errors-in-variables problem and we define the feasible controller parameter set. Then, we design the controller parameters by applying the results from Chapter 2 in the field of set-membership errors-in-variables identification. The main distinctive features of the proposed approach with respect to those already available in the literature are as follows: (i) the noise corrupting the data is assumed to be bounded and no statistical information is assumed to be a-priori available; (ii) in contrast to existing approaches where an iterative procedure is exploited, the set-membership approach leads to a non-iterative algorithm to design the controller; (iii) differently from existing approaches, the controller transfer function does not need to depend linearly on the parameters to be tuned; (iv) the proposed strategy is applicable to deal with both diagonal and non-diagonal multivariable reference models. Finally, the effectiveness of the presented technique is shown by means of both simulation examples and experimental results.

In Chapter 4, we present an original approach to design, in a systematic way, the **reference model** M to be able to meet performance specifications. In the proposed method, the desired performance specifications of the closed-loop system are translated into a model reference design paradigm in the framework of the DDDC approach. The design of a suitable reference model M is carried out based on H_∞ control design approach by using a suitable fictitious plant. Then, stability conditions both for stable minimum-phase plant and stable non-minimum phase (NMP) plant are discussed and analyzed to guarantee internal stability of the designed closed-loop system. In particular, by assuming that available input-output data are corrupted by bounded noise, we formulate the problem of designing a controller in order to match the behaviour of the designed reference model M in terms of an equivalent set-membership errors-in-variables (SM-EIV) identification problem discussed in Chapter 3. Then, a two-stage procedure, to detect the presence of NMP zeros in the plant, is proposed for the design of the controller. The main distinctive features of the proposed approach with respect to those already available in the literature are as follows: (i) the reference model M is designed such that the closed-loop system fulfills performance specifications; (ii) no a-priori information on the NMP zeros location is needed; (iii) the proposed strategy guarantees stability without the need of additional constraints on the problem formulation when the plant is stable and possibly NMP system.

Finally, in Chapter 5, we present a novel non-iterative approach to direct data-driven **nonparametric** controller design. The approach is inspired by the method described in Chapter 3, where a novel set-membership based direct data-driven controller design technique is presented. By exploiting the results given in [30], where an original kernel-based set-membership nonparametric approach for LTI identification is proposed, DDDC problem is then formulated in the robust Reproducing kernel Hilbert space (RKHS) framework. First, by assuming that the available input-output data are corrupted by bounded noise, we formulate the problem of designing a controller in order to match the behaviour of an assigned reference model. Then, the controller is designed by means of a non-parametric approach, inspired by results in [30].

1.2 Publications

The material presented in this thesis is based on the following works:

1. Set-membership errors-in-variables identification (Chapter 2)

    - V. Cerone, D. Regruto, **M. Abuabiah**, Set-membership identification of a dry-clutch transmission model, proc. of IEEE Conference on Control Technology and Applications (CCTA). August-2017 (pp. 1159-1164). [42]
    - D. Regruto, **M. Abuabiah**, V. Razza. A set-membership approach to robust control design of a dry clutch transmission system. Under-review.

2. Direct data driven control design for LTI systems (Chapter 3)

    - V. Cerone, D. Regruto, **M. Abuabiah**, Direct data-driven control design through set-membership errors-in-variables identification techniques, IEEE American Control Conference (ACC), May-2017 (pp. 388-393). [41]

3. Reference model design (Chapter 4)

    - V. Cerone, D. Regruto, **M. Abuabiah**, A set-membership approach to Direct Data-Driven Control design for non-minimum phase plants, proc. of 56th IEEE Conference on Decision and Control (CDC). December-2017 (pp. 1284-1290). [26]
    - V. Cerone, D. Regruto, **M. Abuabiah**, V. Razza. An H_∞ method to design the reference model in Direct Data-Driven Control approach. Under-review.

4. Nonparametric approach to direct data-driven control design (Chapter 5)

    - V. Cerone, D. Regruto, **M. Abuabiah**, E. Fadda, A kernel-based nonparametric approach to direct data-driven control of LTI systems, proc. of 18th IFAC Conference on Symposium on System Identification (SYSID). July-2018 (pp. 1026-1031). [27]

Chapter 2

Set-membership identification

2.1 Introduction

The process of going from observed data to a mathematical model is fundamental in physics, biology and in many fields of engineering. In the control area this process has been termed as *System Identification* and the objective is then to find a dynamical model to describe the behavior of real-word processes from observed input/output signals. Therefore, estimation theory is concerned with the problem of evaluation some unknown variables depending on given data obtained usually from measurements on a real process.

System identification exploits experimental data measurements to construct mathematical models of systems. Two main classes of problem are typical considered: *Black box modeling*, when no information on the physical application is prior available; *Gray box modeling*, when some physical insights are available. Among black box models, there are familiar models such as ARX, ARMAX, and ANN (Artificial Neural Networks). Such models are established by means of a set of equations, e.g., difference equations (DEs), partial differential equations (PDEs) or ordinary differential equations (ODEs).

The area of system identification has its roots in standard statistical techniques such as Least Squares, gradient correction, and Maximum Likelihood. By now, the area is well mature with established and well understood techniques see e.g., [96], [80], [95] and [138]. In addition, there have been many modern methods for system identification in recent years based on the neural networks, genetic algorithm, fuzzy logic, swarm intelligence optimization algorithms, auxiliary model identification algorithm, multi-innovation identification algorithm etc. (see e.g., [129, 169, 44, 86, 56, 55]). Although system identification technology has been developed for several decades and has achieved many research results, there are still many problems needing to be further discussed and studied. One of the main problems is that the available data are always known with some uncertainty and the identified model can not be equivalent to the measured system completely. To deal with this problem an alternative approach called *set-membership* identification, which computes models that reproduce the observed data within a certain given error bound, has been introduced and studied in the last decades.

Set membership (SM) or *Unknown-But-Bounded* UBB error description has been pioneered in the late 1960-1970s [167, 14, 98]. In this approach, uncertainty is described by means of an additive noise which is known only to have given bounds. The motivation for this approach is that in many practical cases the UBB error description is more realistic and less demanding than the statistical description [101]. Recently, SM has gained renewed attention due to its connection with robust control theory (see, e.g, [62, 99, 45, 87]).

This chapter is organized as follows. In Section 2.2 we give a general introduction to set-membership identification. The concept and the main properties of SM EIV identification are discussed in Section 2.3. In particular, the specifics of SM identification for LTI SISO systems are addressed in Subsection 2.3.1 and for MIMO systems in Subsection 2.3.2. The contribution and the main results of Section 2.3 is based on the work of Cerone and co-workers (see e.g., [36], [39], [34] and [35]).

2.2 On set-membership identification

Set-membership theory was born at the end of the Sixties and was applied to problems of state estimation of dynamical systems [167, 14, 98]. In the Eighties, this approach deserved interest due to the development of robust control theory; in fact, by giving hard bounds on the uncertainty, this theory provides models that are useful in the robust control context [62, 99, 45, 87]). Moreover, several successful practical applications of SM identification have been reported (see e.g. [42, 38, 102, 104, 103]). The main research topics related to SM identification include, but are not limited to the characterization and description of the feasible parameter set and its efficient online refinement and updating, computationally efficient algorithms for finding the optimal or almost optimal estimate and experiment design in the context of SM identification.

The aim of the estimation problem is to obtain a dynamic model of the system from noisy input-output measurements. Depending on the hypothesis on the noise, it is possible to distinguish between a statistical and a deterministic approach. In the classical approach (statistical estimation), uncertainty is described in terms of confidence intervals (soft bounds). On the contrary, in the deterministic approach, a feasible set of all admissible solutions is found (hard bounds). In this case, such a set contains all the feasible solutions of the problem, thus providing an evaluation of the uncertainty associated with the estimation problem. For this reason, this approach is usually called *set-membership*. Moreover, while the statistical estimation deals with the average case, the deterministic theory usually considers the worst-case, that is the estimate that shows the best performance in a worst-case setting [24].

The main difference between the classical (statistical) estimation and the SM (deterministic) one lies on the fact that in statistical estimation the noise is represented as a stochastic process (usually a filtered white noise), while in set-membership estimation the noise is supposed to be unknown but bounded, i.e. the only knowledge about noise consists in its bounds evaluated in a given norm, e.g.,

$$|\eta(t)| \leq \Delta\eta, \ \forall t, \tag{2.1}$$

where the error term η is unknown, but bounded by some given positive number $\Delta\eta$. This means that the signal $\eta(t)$ is considered unknown, but bounded (UBB), which is the reason why the set membership identification approach is often called UBB in the literature (see e.g., [101, 105]).

In the set-membership framework, all parameters vector θ, that are consistent with the assumption on the model structure define as,

$$y(t) = g(\phi(t), \theta) + \eta(t) \tag{2.2}$$

and the priori information on the error ($\eta(t)$) and the collected measurement, belong to the so-called *feasible parameter set* (FPS) $\mathcal{D}_\theta$:

$$\mathcal{D}_\theta = \{\theta \in \mathbb{R}^{n_\theta} : |y(t) - g(\phi(t), \theta)| \leq \Delta\eta, \ \forall t = 1, ..., N\} \tag{2.3}$$

where N denotes the number of measurements, $y(t)$ is the measured output and n_θ is the number of element entries in the parameter vector that need to be estimated from the data. The function g represents the dynamics of the underlying system as it describes the dependence of the current plant output on the past inputs and outputs through the regressor $\phi(t)$. Then, for each parameter θ_j , with $j = 1, ..., n_\theta$, deterministic uncertainty bounds $\underline{\theta}_j$ and $\bar{\theta}_j$ are evaluated by looking for the minimum and maximum value of θ over the FPS $\mathcal{D}_\theta$, i.e.

$$\underline{\theta}_j = \min_{\theta \in \mathcal{D}_\theta} \theta_j, \quad \overline{\theta}_j = \max_{\theta \in \mathcal{D}_\theta} \theta_j \tag{2.4}$$

Problem (2.4), is in general, a hard non-convex optimization problem, which falls into the class of semi-algebraic optimization problems, widely studied in recent years. More specifically, it has been shown that, at least in principle, the global optimum of a constrained semi-algebraic program can be approximated arbitrarily well by exploiting either the moment-based-approach [90] or the sum-of-squares-based decomposition approach proposed in [47] and [111]. Methodologies proposed in [90], [47], [111] allow the user to construct a sequence of convex linear matrix inequality (LMI) problems, guaranteed to converge to the global optimum of the original non-convex polynomial problem as the order of relaxation goes to infinity (see the book [92] and the references therein for details). However, direct application of such methods to large-scale identification problems (i.e, a large number of parameters to be estimated and/or a large set of experimental input-output data) might lead to intractable LMI problems due to the required memory storage and/or computational time. To overcome this limitation, ad hoc approaches have been proposed in ([34], [36], [35], [37]), aimed to reduce the computational complexity by exploiting some structural features of the polynomial optimization problems arising from the context of system identification. In particular, it is possible to show that problem (2.4) enjoys the same sparsity structure of the problem considered in ([34], [36], [35], [37]), thus computationally effective implementation can be applied to solve DDDC problems with several hundreds of input-output data.

Furthermore, when the dynamic system to be identified is linear, the relation between the measured plant inputs and outputs can be the following,

$$y(t) = g(\phi(t), \theta) + \eta(t) = \phi(t)^T \theta + \eta(t) \tag{2.5}$$

In this case, the description of the FPS $\mathcal{D}_\theta$ in (2.3) becomes:

$$\mathcal{D}_\theta = \{\theta \in \mathbb{R}^{n_\theta} : |y(t) - \phi(t)^T \theta| \leq \Delta\eta,\ \forall t = 1, ..., N\} \tag{2.6}$$

Each inequality in (2.6) gives rise to an infinitely long strip (i.e. a hyper-slab) in the space of the vector θ. The intersection of a finite number of hyper-slabs gives a polytope. Therefore, the FPS set $\mathcal{D}_\theta$ is also a polytope according to (2.6), and more precisely it is an intersection of two polytopes. Although a polytope can be exactly described, for example by means of its vertexes, an exact description of the polytope could be critical from the memory requirement point of view in particular if there is a lot of available measurements (i.e. if N is large). In addition, updating the FPS recursively (i.e. with each new measurement) in this form is not easy. Therefore, many of the research has been directed in the last decades at developing algorithms for approximating the complex polytopic FPS by geometrically less complex sets. In particular, approximation of the FPS by a fixed complexity polytope (see e.g. [19, 114, 156]), an ellipsoid (see e.g. [13, 54, 59]), a box (see e.g. [100, 106, 143, 25]), a parallelotope (see e.g. [158, 48]) and a zonotope (see e.g. [18, 43]) has been considered in order to make the use of the FPS for the controller design easier and to facilitate computationally efficient recursive update of FPS according to [144].

2.3 Set-membership EIV identification

The field of identification and process-parameter estimation has developed rapidly during the past decades. Many contributions can be found in the literature addressing the problem of linear system identification, both in time and in frequency domain (see, e.g., [94, 118]). Estimation theory is concerned with the problem of evaluating some unknown variables depending on given data on the assumption that only the output signal is corrupted by noise, while the input signal is supposed to be exactly known. However, in many practical problems, input and output data sequences are experimentally collected and, as a consequence, the assumption of noise-free input is not realistic in such situations [115]. Representations, where both the input and the output signals are corrupted by noise, are referred to as errors-in-variables (EIV) models. Obviously, EIV identification is recognized to be a more difficult problem with respect to linear system identification where only the output measurements are affected by noise [12].

A number of approaches for parameter estimation of errors-in-variables methods in system identification has been presented in the survey paper proposed by Söderström [133]. According to the survey, the problem of parameter estimation EIV models is dealt with in [49, 135, 137, 155] through instrumental variable, in [63, 97, 136, 173, 174] through bias-compensation, while Frisch scheme has been used in [11, 128, 134]. Moreover, total least-squares approach used in [64, 88, 154], frequency domain approach in [116, 130, 131], and finally maximum likelihood method was proposed in [58, 117, 166].

In the framework of set-membership identification, the problem of EIV has been presented by Veres and Norton in [157] where it is shown that the exact feasible parameter set (FPS) for dynamic EIV models is described by nonlinear bounds, whose shape may become fairly complex when the number of data increases. As a consequence, parameter bounds cannot be easily computed and the use of either polytopic or ellipsoidal outer approximation is suggested. As far as EIV for static models is considered, an exact mathematical description of the feasible parameter set is provided by Cerone in [28] where relevant topological features, such as convexity and connectedness, are also discussed. Results from [28] are then applied to EIV identification of linear dynamic systems in [29] where an outer approximation of the true nonconvex feasible parameter set is obtained as a union of a number of polytopes. Such an outer-bounding set is then used to compute bounds on the parameters of the system to be identified through the solution to a finite number of linear programming (LP) problems. Actually, parameter bounds obtained in [29] are not tight and their degree of conservativeness is, in general, not easy to quantify [115]. Therefore, a new technique called LMI-based EIV relaxation has been introduced in [36], where relaxation techniques based on linear matrix inequalities has been employed to evaluate parameter bounds by means of convex optimization. The last approach will be reviewed in the next two subsections for both SISO and MIMO LTI systems for self-consistency of the thesis, in order to be used later for DDDC approach which is the main core of this research.

2.3.1 SM EIV identification for SISO LTI systems

Set-membership error-in-variables identification for SISO LTI system described in [115, 36, 32–34] is briefly reviewed in this section for completeness and self-consistency of the report. Then, a simulated example is discussed in order to show the effectiveness of the presented identification scheme.

Problem Formulation

Let us consider the linear single-input-single-output (SISO) system depicted in Fig. 2.1 with noise-corrupted input and output measurements.

The noise-free input is denoted by $x(t)$ and the undisturbed output by $w(t)$. We assume that the signals are corrupted by additive measurement noise $\epsilon(t)$ and $\eta(t)$. The available signals are in discrete time and of the form

$$u(t) = x(t) + \epsilon(t) \tag{2.7}$$

$$y(t) = w(t) + \eta(t) \tag{2.8}$$

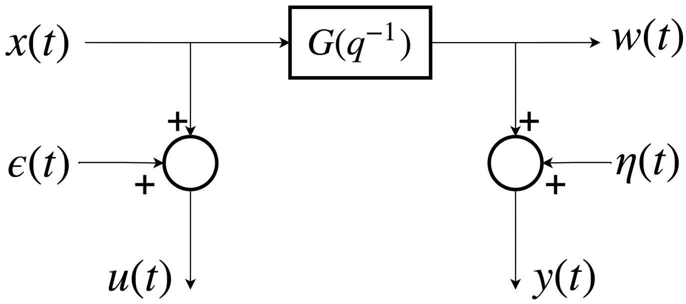

Figure 2.1 Errors-in-variables setup for linear dynamic systems.

According to the set-membership characterization, the noise samples $\epsilon(t)$ and $\eta(t)$ are assumed to range within given bounds $\Delta\epsilon$ and $\Delta\eta$ respectively, that is

$$|\epsilon(t)| \leq \Delta\epsilon \tag{2.9}$$

$$|\eta(t)| \leq \Delta\eta \tag{2.10}$$

The plant $G(q^{-1})$ is discrete time LTI system which transforms $x(t)$ into the noise-free output $w(t)$ according to the difference equation

$$A(q^{-1})w(t) = B(q^{-1})x(t) \tag{2.11}$$

where $A(.)$ and $B(.)$ are polynomials in the backward shift operator q^{-1} of the form

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + ... + a_{n_a} q^{n_a} \tag{2.12}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + ... + b_{n_b} q^{n_b} \tag{2.13}$$

The general problem is to determine the plant G characteristics, such as the system transfer function is given in the following form

$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} \tag{2.14}$$

The unknown parameter vector $\theta \in \mathbb{R}^p$ to be identified is defined as

$$\theta = [a_1 \ ... \ a_{n_a} \ b_0 \ b_1 \ ... \ b_{n_b}]^T \tag{2.15}$$

where $n_a + n_b + 1 = p$, while the Feasible Parameter Set (FPS) $\mathcal{D}_\theta$ is defined as

$$\begin{aligned}\mathcal{D}_\theta = &\{\theta \in \mathbb{R}^p : A(q^{-1})(y(t) - \eta(t)) = B(q^{-1})(u(t) - \epsilon(t)),\\ &|\epsilon(t)| \leq \Delta\epsilon,\ |\eta(t)| \leq \Delta\eta;\ \forall t = 1, \ldots, N\}\end{aligned} \tag{2.16}$$

where N is the length of data sequences. Equation (2.16) provides an exact description of the set of all possible values of the unknown parameter θ consistent with measured data, error bounds and assumed model structure. In this section we address the problem of evaluating the so-called *Parameter Uncertainty Intervals* PUI, defined as:

$$PUI_j = [\underline{\theta}_j\ \bar{\theta}_j],\ \forall j = 1, \ldots, n_p \tag{2.17}$$

where,

$$\underline{\theta}_j = \min_{\theta \in \mathcal{D}_\theta} \theta_j, \tag{2.18}$$

$$\overline{\theta}_j = \max_{\theta \in \mathcal{D}_\theta} \theta_j \tag{2.19}$$

As discussed in section 2.2, problems (2.18) and (2.19), are in general, hard non-convex optimization problems, which fall into the class of semi-algebraic optimization problems. Therefore, standard nonlinear optimization tools (gradient method, Newton method, etc.) cannot be used since they can trap in local minima which may result arbitrary far from the global one. Thus, the PUI_js obtained using such tools are not guaranteed to contain the true unknown parameter, which is a key requirement of any set-membership identification method. One possible solution to overcome this problem is to relax (2.18) and (2.19) to convex problems to obtain a lower and upper bounds of $\underline{\theta}_j$ and $\overline{\theta}_j$ receptively. The relaxation technique presented in [29], which consists in the application of the results for static EIV problems derived in [28], provides an outer approximation of the FPS $\mathcal{D}_\theta$. While, in [34], and [36] an LMI relaxation technique is used to compute the PUIs, and will be reviewed briefly in the next subsection.

Parameter bounds computation for SISO systems

The dynamic (LMI) EIV relaxation procedure described in [34], [36] is briefly reviewed in this section for completeness and self-consistency of the chapter.

The main idea in [34] is to compute the approximate parameter uncertainty interval PUI_j through LMI relaxation techniques. Problems (2.18) and (2.19) can be rewritten as constrained polynomial optimization problems as follows

$$\begin{cases} \underline{\theta}_j = \min\limits_{\theta,\epsilon,\eta} \theta_j \\ \textbf{s.t.} \\ y(t) = -\sum\limits_{i=1}^{n_a}(y(t-i)-\eta(t-i))a_i + \sum\limits_{j=0}^{n_b}(u(t-j)-\epsilon(t-j))b_j + \eta(t) \\ |\epsilon(t)| \le \Delta\epsilon,\ |\eta(t)| \le \Delta\eta;\ \forall t = 1,...,N \end{cases} \tag{2.20}$$

$$\begin{cases} \overline{\theta}_j = \max\limits_{\theta,\epsilon,\eta} \theta_j \\ \textbf{s.t.} \\ y(t) = \sum\limits_{i=1}^{n_a}(y(t-i)-\eta(t-i))a_i + \sum\limits_{j=0}^{n_b}(u(t-j)-\epsilon(t-j))b_j + \eta(t) \\ |\epsilon(t)| \le \Delta\epsilon,\ |\eta(t)| \le \Delta\eta;\ \forall t = 1,...,N \end{cases} \tag{2.21}$$

where, $\eta = [\eta(1),...,\eta(N)]^T$ and $\epsilon = [\epsilon(1),...,\epsilon(N)]^T$.

The computation of $\underline{\theta}_j$ and $\bar{\theta}_j$ is then reduced to a minimization or maximization problem over $p+2N$ optimization variables. The feasible set is called now *Semialgebraic* set, defined by N bilinear polynomial equality constraints and $4N$ linear inequalities.

Since (2.20) and (2.21) are semialgebraic optimization problems, they can be relaxed through direct implementation of the LMI-relaxation technique based on the theory of moments and proposed by Lasserre in [90]. Such a procedure is based on the idea of relaxing a polynomial optimization problem by a hierarchy sequence of *SemiDefinite Programming* (SDP) problems with increasing dimension, whose optima are guaranteed to converge monotonically to the global optimum of the original polynomial problem. Although the method is guaranteed to converge as far as the length of the number of successive SDP problems (relaxation order) goes to infinity, exact global optima can be obtained in practice with a reasonably low relaxation order (see [36] for details).

Remark 2.3.1

- Due to the high computational burden and memory storage requirement, the use of such an LMI-relaxation technique to relax (2.20) and (2.21) is limited, in practice, to identify problems with a small number N of measurements (roughly not greater than 10), that is certainly an unusual and unfavourable condition in the identification framework. A possible way to overcome this restriction is to relax (2.20) and (2.21) through the procedure presented in [91] in the spirit of [160]. Such an approach exploits the sparsity in the original polynomial problems to formulate a sparse version of the Lasserre's relaxation [90], in order to extend the applicability of such a methodology to medium and large scale problems (see e.g., [36], [34]).

- An efficient implementation of LMI relaxation for polynomial optimization problems with structured sparsity has been developed in the Matlab package *SparsePOP* [159] which exploits the LMI solver *SeDuMi* [142] or *Mosek* [110] to solve SDP problems in polynomial time. It must be noted that, in general, such a relaxation method does not guarantee convergence to the global optimum of the original constrained polynomial problem. However, it is shown in [91] that if the structure of the original polynomial problem satisfies suitable assumptions on the sparsity structure, this LMI relaxation method provides a solution that converges to the global optimum of the original polynomial optimization problem, as the relaxation order δ goes to infinity. In practice, a good solution can be found with a relaxation order $\delta \leq 3$ with few hundred of the data sample ($N \leq 1000$).

2.3.2 SM EIV identification for MIMO LTI systems

In this section, we briefly review the problem of set membership identification of multiple-input-multiple-output (MIMO) linear models when both input and output measurements are affected by bounded additive noise. This section is based on the work of Cerone & co-workers in [39] and [40].

Most of the works available in the literature for SM-EIV approach deal with single-input-single-output (SISO) linear models, while only a few papers address the problem of identification of MIMO linear models in the presence of bounded errors [40]. In particular, the identification of MIMO systems affected by bounded equation error is addressed in the paper [163] by means of an interval analysis-based approach. An output-error model structure is considered, instead, in [119], under the assumption that a bound on the energy of the output measurement error is known. Zaiser and co-workers focus on the problem of computing parameter bounds for MIMO state-space model ([172]) and for MIMO ARX models ([171]), by assuming that both the input and the output sequences are corrupted by additive noise (errors-in-variables) bounded in the l_∞ norm.

In [40], the authors assume that the order of the system is a-priori known, and an algorithm for computing tight parameter uncertainty intervals (PUI) is proposed, by taking explicitly into account the correlation between the uncertainty variables affecting the regressor. Then, they address the problem of computing the PUIs for MIMO linear models, with both input and output measurements corrupted by bounded noise. Finally, they consider a general description, in transfer function form, that allows the user to consider possible a-priori knowledge on the structure of each entry of the matrix transfer function. To this aim, we briefly review this method in the following subsections.

Problem Statement

Let us consider the multi-input multi-output (MIMO) linear-time-invariant (LTI) system depicted in Figure 2.1, where $\boldsymbol{x}(t)$ is the n_x dimensional input and

$\boldsymbol{w}(t)$ is the n_w dimensional output. The MIMO linear system to be identified is modeled by a discrete time system that transforms $\boldsymbol{x}(t)$ into the noise-free output $\boldsymbol{w}(t)$ according to the following input-output equation

$$\boldsymbol{w}(t) = \boldsymbol{G}(q^{-1})\boldsymbol{x}(t), \tag{2.22}$$

where, $\boldsymbol{x}(t) = [x_1(t) \quad x_2(t) \quad ... \quad x_{n_x}(t)]^T \in \mathbb{R}^{n_x}$ and $\boldsymbol{w}(t) = [w_1(t) \quad w_2(t) \quad ... \quad w_{n_w}(t)]^T \in \mathbb{R}^{n_w}$ are the samples at time instant t of the multi-variable input and output respectively $\boldsymbol{G}(q^{-1})$ is the system matrix transfer function. The entry of $\boldsymbol{G}(q^{-1})$ relating the j-th input to the i-th output, is described by

$$G_{ij}(q^{-1}) = \frac{\sum_{k=0}^{m_{ij}} b_k^{(ij)} q^{-k}}{1 + \sum_{h=1}^{n_{ij}} a_h^{(ij)} q^{-h}} \tag{2.23}$$

where $a_h^{ij} \in \mathbb{R}, \ (h = 1, ..., n_{ij})$ and $b_k^{ij} \in \mathbb{R}, \ (k = 0, ..., m_{ij})$ are the unknown parameters to be estimated. The i-th output of the system can be described as

$$w_i(t) = z_{i1}(t) + z_{i2}(t) + ... + z_{in_x}(t) \tag{2.24}$$

where z_{ij} is the contribution of the j-th input to the i-th output, i.e (for more details see [40])

$$z_{ij}(t) = G_{ij}(q^{-1})x_j(t) \tag{2.25}$$

Let us call z_{ij} the ij-th *partial output*. On the basis of equation (2.25) we can relate $z_{ij}(t)$ and $x_j(t)$ through the following difference equation

$$\sum_{h=0}^{n_{ij}} a_h^{ij} z_{ij}(t-h) = \sum_{k=0}^{m_{ij}} b_k^{ij} x_j(t-k) \tag{2.26}$$

We assume that the signals are corrupted by additive measurement noise $\epsilon(t)$ and $\eta(t)$. The available signals are in discrete time and of the form

$$\boldsymbol{u}(t) = \boldsymbol{x}(t) + \boldsymbol{\epsilon}(t) \tag{2.27}$$

$$\boldsymbol{y}(t) = \boldsymbol{w}(t) + \boldsymbol{\eta}(t) \tag{2.28}$$

where the scalar noise variables $\epsilon_j(t)$ and $\eta_i(t)$, affecting the generic input $x_j(t)$ and the generic output $w_i(t)$ respectively, are assumed to range within given bounds $\Delta\epsilon$ and $\Delta\eta$, given by

$$|\epsilon_j(t)| \leq \Delta\epsilon,\ \forall t = 1, ..., N \tag{2.29}$$

$$|\eta_i(t)| \leq \Delta\eta,\ \forall t = 1, ..., N \tag{2.30}$$

The unknown parameter vector $\boldsymbol{\theta} \in \mathbb{R}^p$ to be identified is

$$\boldsymbol{\theta} = [\boldsymbol{\theta}_{11}\ ...\ \boldsymbol{\theta}_{1n_x}\ \boldsymbol{\theta}_{n_w1}\ ...\ \boldsymbol{\theta}_{n_wn_x}]^T \tag{2.31}$$

where,

$$\boldsymbol{\theta}_{ij} = [a_0^{ij}\ ...\ a_{n_{ij}}^{ij}\ b_0^{ij}\ ...\ b_{m_{ij}}^{ij}] \tag{2.32}$$

and $p = \sum_{i=1}^{n_w}\sum_{j=1}^{n_x}(m_{ij} + n_{ij} + 1)$. The feasible parameter set (FPS) $\mathcal{D}_\theta$ is

$$\begin{aligned}\mathcal{D}_\theta = \Big\{ & \theta \in \mathbb{R}^p : \sum_{h=0}^{n_{ij}} a_h^{(ij)} z_{ij}(\tau_{ij} - h) = \sum_{k=0}^{m_{ij}} b_k^{(ij)}(u_j(\tau_{ij} - k) - \epsilon_j(\tau_{ij} - k)), \\ & z_{i1}(t) + z_{i2}(t) \ldots + z_{in_x}(t) = (y_i(t) - \eta_i(t)),\ \tau_{ij} = n_{ij} + 1, \ldots, N \\ & i = 1, \ldots, n_w,\ j = 1, \ldots, n_x,\ \mid \epsilon_j(t) \mid \leq \Delta\epsilon,\ \mid \eta_i(t) \mid \leq \Delta\eta;\ t = 1, \ldots, N \Big\}\end{aligned} \tag{2.33}$$

where N is the length of the data sequences.

Equation (2.33), as in the SISO case, provides an implicit exact description of the set of all possible values of the unknown parameter $\boldsymbol{\theta}$ consistent with measured data, error bounds and assumed model structure. Therefore, the problem of evaluating the parameter uncertainty intervals PUI_r can be defined as

$$PUI_r = \left[\underline{\theta}^{(r)},\ \overline{\theta}^{(r)}\right] \quad \text{for } r = 1, \ldots, p \tag{2.34}$$

where $\theta^{(r)}$ is the r-th element of the array θ, while

$$\underline{\theta}^{(r)} = \min_{\theta \in \mathcal{D}_\theta} \theta^{(r)}, \tag{2.35}$$

$$\overline{\theta}^{(r)} = \max_{\theta \in \mathcal{D}_\theta} \theta^{(r)}. \tag{2.36}$$

Remark 2.3.2

The formulation proposed in this subsection is quite general since it allows the user to take into account possible a-priori information on the structure of the MIMO system to be identified, i.e., the order of the numerator (m_{ij}) and denominator (n_{ij}) of each single transfer function of the MIMO system G_{ij} .

Parameter bounds computation for MIMO systems

The key idea in [40] and [39] is that the system parameters and the partial (unmeasurable) output signals z_{ij} can be simultaneously estimated through the solution of the following optimization problem

$$\begin{cases} \min\limits_{\theta,z,\eta,\xi} J(\theta) \\ \text{s.t.} \\ \sum\limits_{h=0}^{n_{ij}} a_h^{(ij)} z_{ij}(\tau_{ij}-h) = \\ = \sum\limits_{k=0}^{m_{ij}} b_k^{(ij)} (u_j(\tau_{ij}-k) - \epsilon_j(\tau_{ij}-k)), \\ \tau_{ij} = n_{ij}+1,\dots,N \\ z_{i1}(t)+z_{i2}(t)\dots+z_{in_x}(t) = (y_i(t)-\eta_i(t)), \\ i=1,\dots,n_w,\ j=1,\dots,n_x \\ \mid \epsilon_j(t) \mid \leq \Delta\epsilon, \quad \mid \eta_i(t) \mid \leq \Delta\eta;\ t=1,\dots,N \end{cases} \tag{2.37}$$

where also the samples of the unmeasurable partial output signals z_{ij} appears as decision variables of problem (2.37) together with the system parameters $\boldsymbol{\theta}$ to be estimated. The functional $J(\theta)$ to be minimized is set to $J(\theta) = \theta^{(r)}$ for the computation of $\underline{\theta}^{(r)}$ and to $J(\theta) = -\theta^{(r)}$ when the computation of $\overline{\theta}^{(r)}$ is of interest (for more details see e.g., [40]). Thus, the computation of the PUI_r requires the solution to the constrained optimization problem of equation (2.37) by using the same method that has been discussed in the previous sections.

Chapter 3

Direct data driven control (DDDC) design of LTI systems

3.1 Introduction

In many control applications, trying to write a mathematical model of the plant is considered a hard task, requiring efforts and time to the process and control engineers. This problem is overcome by applying what is so-called *direct data-driven control* (DDDC) design. The DDDC methods allow tuning a controller, belonging to a given class, without the need of an identified model of the system since available input-output data experimentally collected from the plant are directly used to design the controller.

The automatic control design is the scientific discipline that employs methods from mathematics and engineering in order to force dynamical systems to behave in a desired fashion. One way to classify control design techniques is to separate them into *indirect* techniques, in which the controller is designed on the basis of the available mathematical model of the dynamical system to be controlled, and *direct* techniques, which do not explicitly use the model of the plant, but rely on experimental data in order to directly design the controller. In the direct approaches, system identification and controller design are very strongly related and controller design can be seen as a system identification procedure that is aimed at identifying the controller rather than the plant model. To cope with the direct technique, one possibility is to use the DDDC approach.

Although the studies on DDDC are still at in the embryonic stage, they have attracted plenty of attention within the control theory community and this framework is also recently supported by a well worked out research and some industrial applications. Despite the advances of the DDDC field, identification of such a controller without the availability of a mathematical model is still having open problems that need to be solved. One of the most important problem in the DDDC approach is that, in practice, the available experimental data is always imperfect, as it is affected by measurement noise. Therefore, this chapter focuses on the development of a novel non-iterative direct data-driven technique to deal with linear-time-invariant (LTI) controller design, such that the controller is directly identified from a collected experimental input/output data corrupted by bounded additive noise. Based on the assumption of corrupting bounded noise, the design problem is formulated in the framework of set-membership (SM) identification theory.

In this Chapter, we propose a novel non-iterative direct data-driven technique to deal with linear-time-invariant (LTI) controller design, such that the controller is directly identified from input/output data without plant identification step. First, we formulate the problem of designing a controller in order to match the behaviour of a given reference model, in terms of an equivalent set-membership errors-in-variables problem and we define the feasible controller parameter set. Then, we design the controller parameters by applying the results from Chapter 2 in the field of set-membership errors-in-variables identification. The main distinctive features of the proposed approach with respect to those already available in the literature are as follows: (i) the noise corrupting the data is assumed to be bounded and no statistical information is assumed to be a-priori available; (ii) in contrast to existing approaches where an iterative procedure is exploited, the set-membership approach leads to a non-iterative algorithm to design the controller; (iii) differently from existing approaches, the controller transfer function does not need to depend linearly on the parameters to be tuned; (iv) the proposed strategy is applicable to deal with both diagonal and non-diagonal multivariable reference models. Finally, the effectiveness of the presented technique is shown by means of both simulation examples and experimental results. Concluding remarks end the chapter.

This chapter is organized as follows. In Section 3.2 we review the existing direct control design approaches. In section 3.3 we describe the control design problem that we address in Section 3.2 and present the novel direct design approach based on Set-Membership (SM) identification method in Section 3.4. The effectiveness of the presented method is shown in Section 3.5 and 3.6 by means of both simulation examples and experimental results respectively. Concluding remarks end the chapter.

3.2 Existing DDDC design approaches

In this section, a summary of existing direct data-driven design approaches is discussed by focusing on the recently available control theories about DDDC. The state of the art of the existing DDDC methods is presented with appropriate classifications.

Significant research efforts have been devoted to the direct data-driven control (DDDC) theory in recent years, where experimental data are directly used to design the controller. The DDDC approach is of particular interest in real-world applications where an accurate model of the plant to be controlled is not available.

DDDC approaches do not rely on plant model identification since available input-output data experimentally collected from the plant are directly used to design the controller. The control specifications are usually given in this context in terms of a desired closed-loop reference model; then, the controller parameters are computed by formulating the problem in terms of model matching design.

Wang and Hou ([73]) proposed a brief survey on the existing problems and challenges inherent in Model-Based Control (MBC) theory, and some important issues in the analysis and design of Data-Driven Control (DDC) methods were reviewed and addressed. According to them, there are over 10 kinds of different

DDC methods. Sorted according to the type of **data usage**, these methods can be summarized as three classes (See Figure 3.1):

- DDC based on on-line data ([7, 8, 77, 122, 149, 139, 75, 162]).
- DDC based on off-line data ([2, 69, 70, 83, 84, 107, 125, 124]).
- DDC based on both off-line and on-line data (hybrid DDC) ([17, 6, 16, 46, 50, 127]).

On the other hand, if sorted by method of **controller structure design**, they can be divided into two classes (See Figure 3.2):

- DDC methods with pre-specified fixed controller structures.
- DDC methods with unknown controller structures.

Several contributions have been proposed in the field of Off-line data based DDC. Guardabassi in [66] proposed a virtual reference feedback tuning (VRFT) method. It is a one-shot direct data-driven method where the data-based procedure proposed to design the controller is based on the notion of the virtual reference signal. Unlike the standard VRFT for SISO systems, the authors in [60] make use of a variance weighting to achieve a consistent controller estimate with a single set of input-output data. Extensions and improvements of this method have been presented in some papers (see, e.g., [22, 23, 125, 168, 20]). An alternative data-based approach for controller design, called Iterative feedback tuning (IFT), has been proposed by Hjalmarsson 1994 [72]. IFT is a data-driven control DDC scheme involving iterative optimization of a fixed structure controller, whose parameters are tuned according to an estimated gradient of a control performance criterion. In a more recent paper [70], the standard IFT approach has been extended to the case of multivariable linear time-invariant systems. Furthermore, the Iterative Correlation-based Tuning method (ICbT), proposed [82], is a data-driven control method in which the controller parameters are tuned iteratively to decorrelate the closed-loop output error, between the designed and achieved closed-loop system, from an external reference signal. Therefore, at each iteration, in general, several experiments are needed for the gradient estimation.

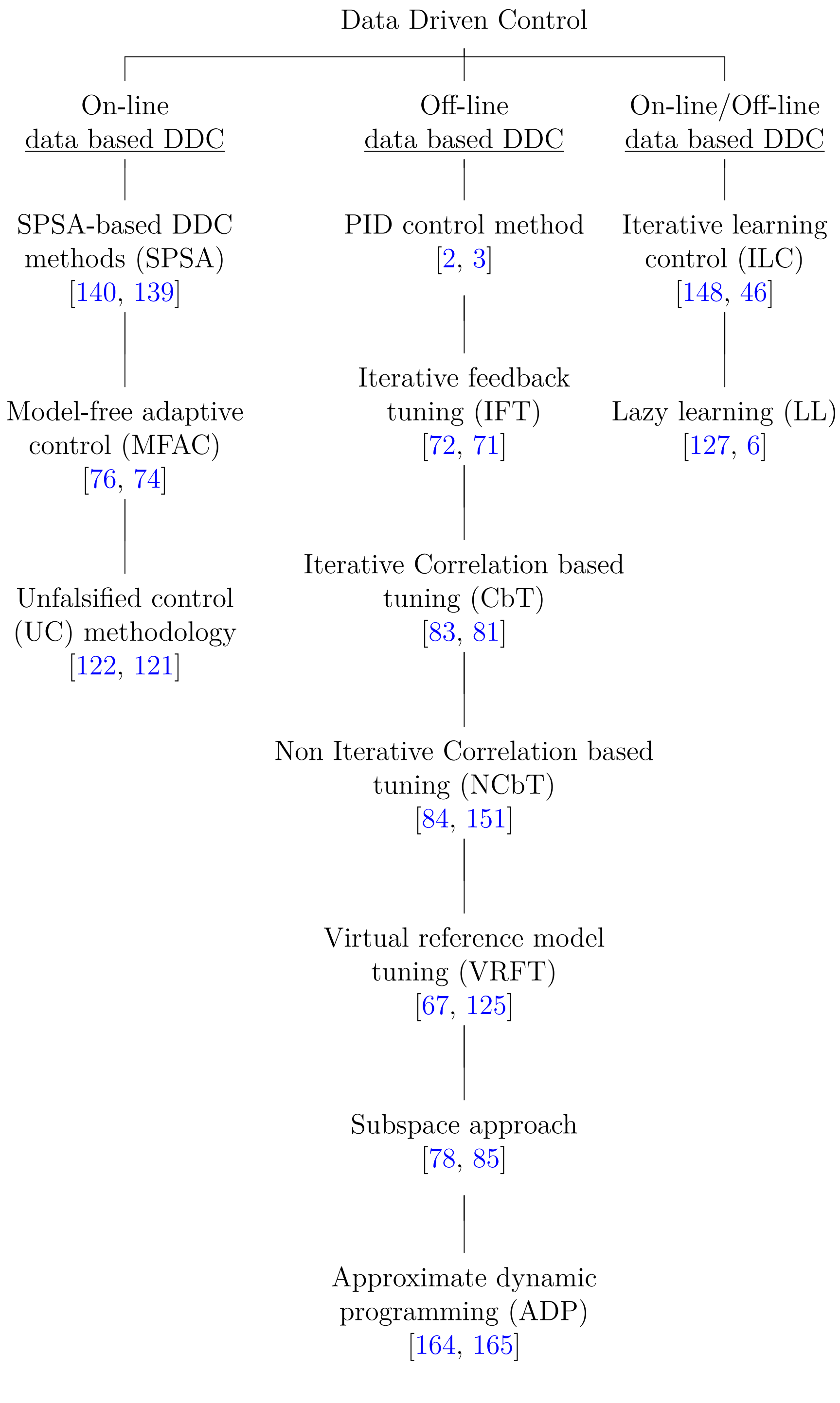

Figure 3.1 Hierarchy map for DDC sorted according to the type of data usage.

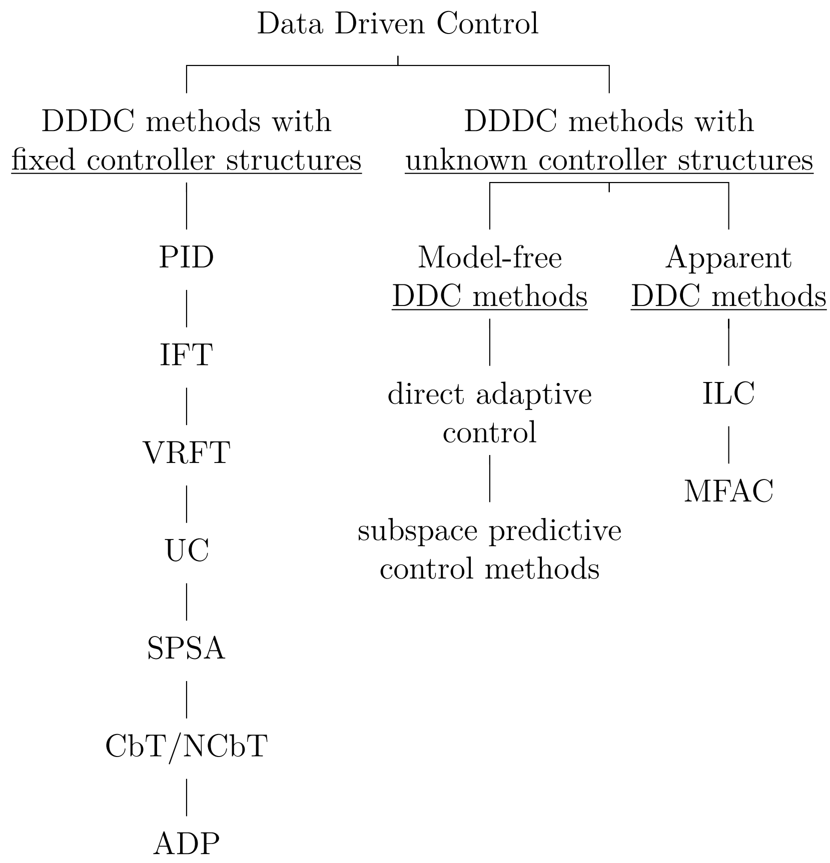

Figure 3.2 Hierarchy map for DDC sorted according to the controller structure design.

Direct design of the controller when the collected experimental data are affected by noise is one of the main challenges in the context of DDDC methods. In the context of the VRFT method, the problem has been addressed by means of an extended instrumental variable (IV) approach by Formentin [60]. An alternative non-iterative approach called Non-iterative Correlation based Tuning (NCbT) method has been proposed in the work of Karimi [84]. This controller tuning approach leads to the formulation of a controller identification problem where the input is affected by noise while the output is noiseless. Few other solutions have been proposed to cope with the effect of measurement noise (see, e.g., the comparison proposed in the paper [153]).

In this chapter, we focus only on DDC based on off-line data, in particulate on DDC methods with pre-specified fixed controller structures. To this aim, we will review briefly the basic idea of VRFT and NCbT methods in the following two-subsections for self-consistency of this thesis, and in order to be used later for a comparison with the presented novel DDDC approach based on SM-EIV identification method.

3.2.1 Virtual reference feedback tuning (VRFT)

VRFT was proposed by Guardabassi and Savaresi in 2000 [67]. It is a one-shot direct data-driven method that can be used to select the controller parameter for the LTI system. VRFT formulates the controller tuning problem as a controller parameter identification problem via introducing virtual reference signal.

Basic Idea: In [22] the authors supposes that a controller $K(q^{-1},\theta)$ is in a closed-loop system whose transfer function is given by $M(q^{-1})$; where M denotes the reference model, i.e. the desired closed loop transfer function, and q^{-1} is the standard backward shift operator. Then, if the closed-loop system is fed by any reference signal $r(t)$, its output equals $M(q^{-1})r(t)$, as shown in Figure 3.3. Hence, a necessary condition for the closed-loop system to have the same transfer function as the reference model is that the output of the two systems is the same for a given signal $\overline{r}(t)$, where $\overline{r}(t)$ is called a *virtual reference* signal.

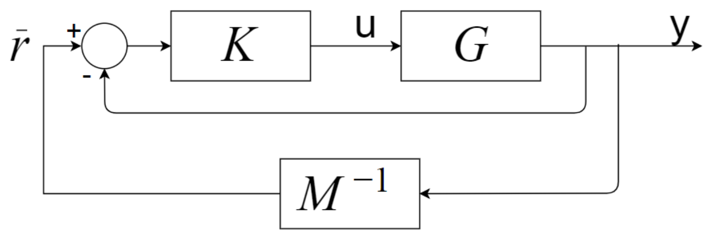

Figure 3.3 The construction of the virtual reference.

Standard model reference design methods try to impose such a necessary condition by first selecting a reference $\overline{r}(t)$ and then by choosing $K(q^{-1},\theta)$ such that the condition is satisfied. However, for a general selection of $\overline{r}(t)$, the above task is difficult to accomplish if a model of the plant is not available.

Remark 3.2.1

In VRFT method, the controller $K(q^{-1},\theta)$ is linearly parameterized in θ, such as

$$K(q^{-1},\theta) = \beta(q^{-1})^T \rho,\ \rho \in \mathbb{R}^n \tag{3.1}$$

where $\beta(q^{-1})$ is a vector of basis functions of q^{-1}.

VRFT procedure: The above idea can be implemented by the following 3-step algorithm (where filtering of data through a user-chosen filter $L(q^{-1})$ is also considered); which represents the bulk of the VRFT method.

Given a set of measured I/O data {u(t); y(t)}$_{t=1;\ldots;N}$, do the following:

1. Calculate:

- a virtual reference $\overline{r}(t)$ such that $y(t) = M(q^{-1})\overline{r}(t)$, and
- the corresponding tracking error e(t)=$\overline{r}(t)$−y(t) (we assume $M(q^{-1}) \neq 1$, otherwise e(t) = 0);

2. Filter the signals e(t) and u(t) with a suitable Filter $L(q^{-1})$:

$$e_L(t) = L(q^{-1})e(t), \quad u_L(t) = L(q^{-1})u(t) \tag{3.2}$$

3. Select the controller parameter vector, say $\hat{\theta}_N$, that minimizes the following criterion:

$$J_{VR}^N(\theta) = \frac{1}{N}\sum_{t=1}^{N}(u_L(t) - K(q^{-1},\theta)e_L(t))^2 \tag{3.3}$$

Note that when $K(q^{-1},\theta) = \beta^T(q^{-1})\theta$ the previous equation can be:

$$J_{VR}^N(\theta) = \frac{1}{N}\sum_{t=1}^{N}(u_L(t) - \varphi_L^T(t)\theta)^2 \tag{3.4}$$

$$\varphi_L(t) = \beta(q^{-1})e_L(t) \tag{3.5}$$

and the parameter vector $\hat{\theta}_N$ is given by:

$$\hat{\theta}_N = \left[\sum_{t=1}^{N} \varphi_L(t)\varphi_L(t)^T\right]^{-1} \sum_{t=1}^{N} \varphi_L(t)u_L(t) \tag{3.6}$$

In other words, the complete VRFT algorithm can be summarized as follow (including noisy data):

1. Set $L(q^{-1}) = (1 - M(q^{-1}))M(q^{-1})W(q^{-1})U(q^{-1})^{-1}$, where $U(q^{-1})$ is such that $|U(e^{jw})|^2 = \Phi_u(w)$, and $W(q^{-1}) = 1$.
2. Compute $u_L(t) \quad as: \quad u_L(t) = L(q^{-1})u(t)$
3. compute $\widetilde{\varphi}_L(t) \quad as: \quad \widetilde{\varphi}_L(t) = \beta(q^{-1})L(q^{-1})(M(q^{-1})^{-1} - 1)y(t)$
4. Identify a high-order model $\hat{G}(q^{-1})$ from $u(t)_{t=1,..,N}$ to $y(t)_{t=1,..,N}$
5. Compute $\zeta(t) \quad as: \quad \zeta(t) = \beta(q^{-1})L(q^{-1})(M(q^{-1})^{-1} - 1)\hat{G}(q^{-1})u(t)$
6. Compute the parameter vector of the controller as $\hat{\theta}_N$

$$\hat{\theta}_N = \left[\sum_{t=1}^{N} \zeta(t)\widetilde{\varphi}_L(t)^T\right]^{-1} \sum_{t=1}^{N} \zeta(t)u_L(t)$$

Remark 3.2.2

When step number (4) is used, strictly speaking, we can no longer claim that the method is fully direct since $\hat{G}(q^{-1})$ has to be estimated. However, recently in [60], this problem has been solved by using what is called the Extended-Instrumental Variable $\zeta(t)$, given by

$$\zeta(t) = \begin{bmatrix} u(t+l) \\ . \\ . \\ . \\ u(t-l) \end{bmatrix}, \tag{3.7}$$

where l is a sufficiently large integer, and define the decorrelation cost function as

$$J_d^N(\rho) = (r - R_\rho)^T \hat{W}^{-1}(r - R_\rho) \tag{3.8}$$

where,

$$R = \frac{1}{N}\sum_{t=1}^{N} \zeta_L(t) \otimes \phi_L(t), \tag{3.9}$$

$$r = \frac{1}{N}\sum_{t=1}^{N} \zeta_L(t) \otimes u_L(t), \tag{3.10}$$

$$\phi_L(t) = [e_L^T(t) \otimes I \; ... \; e_L^T(t-n) \otimes I]^T \tag{3.11}$$

where, I is the identity matrix, and for SISO system $I = 1$, and $\hat{W}$ is a suitable user-chosen filter.

3.2.2 Non-iterative Correlation based Tuning (NCbT)

Non-iterative Correlation based Tuning (NCbT) approach is proposed by Van Heusden and Karimi et al. in [84, 151]. This controller tuning approach leads to an identification problem where the input is affected by noise but not the output as in standard identification problems.

Consider the unknown LTI SISO plant $G(q^{-1})$. The objective is to design a linear, fixed controller $K(\theta, q^{-1})$ with parameters θ such that the closed-loop approximates the reference model $M(q^{-1})$. This can be achieved by minimizing the following model- reference criterion:

$$J(\theta) = \left\| M(q^{-1})r - \frac{K(\theta, q^{-1})G(q^{-1})}{1 + K(\theta, q^{-1})G(q^{-1})} r \right\|^2 \tag{3.12}$$

where r is the reference signal. Note that the objective is to design a fixed controller and $J(\theta) = 0$ cannot generally be achieved. The model reference criterion is nonconvex with respect to the controller parameters θ. An approximation that is convex for linearly parameterized controllers can be defined using the reference model $M(q^{-1})$ as shown below. The notation is shortened by dropping q^{-1} for simplicity. M can be represented as follows:

$$M = \frac{K^*G}{1 + K^*G} \tag{3.13}$$

where K^* is the ideal controller, which is defined indirectly by G and M:

$$K^* = \frac{M}{G(1 - M)} \tag{3.14}$$

The controller K^* exists if $M \neq 1$. The unknown ideal controller will only be used for analysis:

$$J(\theta) = \left\| \frac{K^*G - K(\theta)G}{(1 + K^*G)(1 + K(\theta)G)} r \right\|^2 \tag{3.15}$$

Replacing $(1 + K(\theta)G)$ with $(1 + K^*G)$, the following approximation can be derived:

$$\hat{J}(\theta) = \left\| \frac{K^*G - K(\theta)G}{(1 + K^*G)^2} r \right\|^2 = \left\| (1 - M)Mr - K(\theta)(1 - M)^2 Gr \right\|^2 \tag{3.16}$$

If the controller is linearly parameterized, then $\hat{J}(\theta)$ is convex.

Considering the case where there is measurement noise on the plant output, namely $y(k) = G(q^{-1})u(k) + v(k)$, where $u(k)$ is the plant input, $v(k)$ is the measurement noise and $G(q^{-1})$ is stable, the optimal solution can be found by minimizing the norm of the following error:

$$\begin{aligned}\epsilon_K(\theta) &= M(1-M)r - K\theta(1-M)^2 y && (3.17)\\ &= M(1-M)r - K(\theta)(1-M)^2 Gr - K(\theta)(1-M)^2 v && (3.18)\end{aligned}$$

The diagram of (3.18) is shown in Figure 3.4.a. This diagram can be redrawn as in Figure 3.4.b in order to clearly show the nature of the identification problem. In Figure 3.4.b, the unknown signals are $y_c^*(k), v(k)$ and $\tilde{y}_c$. The known signals are $r(k)$, $y_c = (1-M)^2 y$, and s(k). They are given by the following:

$$s(k) = (1-M)^2 GK^* r(k) = M(1-M)r(k) \tag{3.19}$$

The problem of tuning the controller parameter has become a parameter identification problem. Here the system to be identified is K^* and the model to be identified is $K(\theta)$. The main difference between the controller tuning and the standard identification is that the input is affected by noise while the output is noiseless. The correlation approach is applied to address the effects of noise on input (see [150]).

In the schemes of Figure 3.4, $G(q^{-1})$ is assumed to be stable. For unstable $G(q^{-1})$, an initial stabilizing controller is needed to perform the experiment [150].

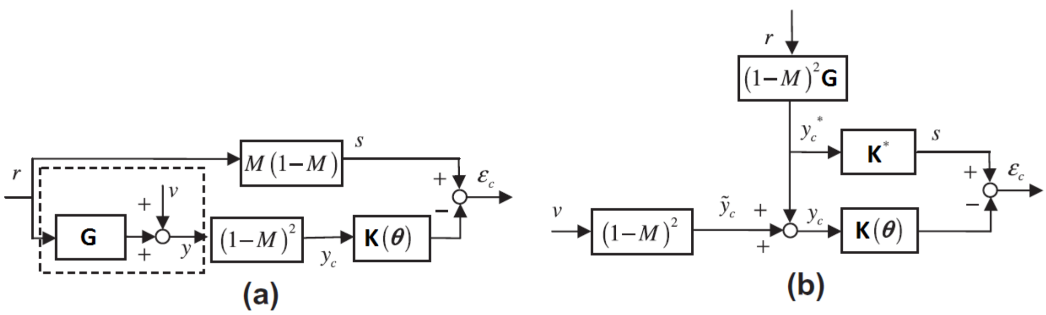

Figure 3.4 Noniterative data-driven model reference control.

3.3 Problem statement

In this section, we describe the control design problem that we address in Section 3.2, such that the direct-data driven control system will be designed on the basis of a set-membership approach for both SISO and MIMO systems.

3.3.1 SISO systems

Let us consider the discrete-time linear-time invariant (LTI) single-input-single-output (SISO) feedback control scheme depicted in Fig. 3.5, where q^{-1} denotes the standard backward shift operator, $G(q^{-1})$ is a stable and minimum-phase (MP) plant transfer function, $K(\rho, q^{-1})$ is the controller transfer function, ρ is the vector of controller parameters, and $M(q^{-1})$ is the transfer function of a suitable given reference model describing the desired behavior of the controlled plant.

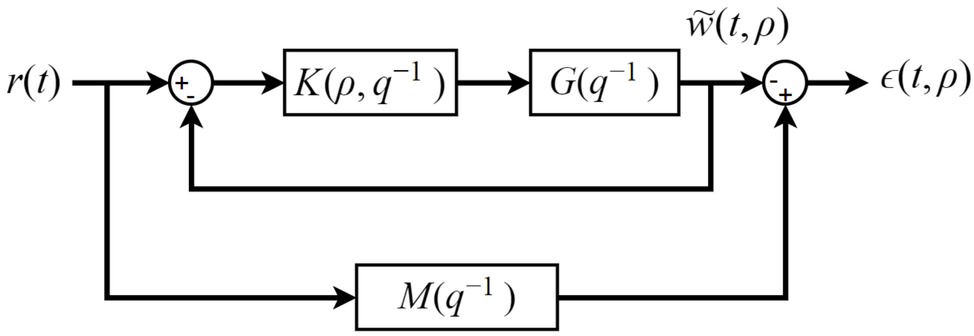

Figure 3.5 Feedback control system to be designed compared with the reference model $M(q^{-1})$.

The objective of the contribution is to propose an algorithm to design the transfer function of the LTI controller $K(\rho, q^{-1})$ such that the closed-loop transfer function $T_{\tilde{w}r}(q^{-1})$ given by

$$T_{\tilde{w}r}(q^{-1}) = \frac{K(\rho, q^{-1})G(q^{-1})}{1 + K(\rho, q^{-1})G(q^{-1})} \tag{3.20}$$

matches, as close as possible, in some sense, $M(q^{-1})$. This is pursued under the assumption that the plant transfer function $G(q^{-1})$ is unknown, and only a set of input-output data collected by performing suitable experiments on the plant is available.

Let us now introduce the following definitions.

Definition 3.3.1: Model matching error transfer function

The model matching error transfer function $E(\rho, q^{-1})$ is defined as the difference between the reference model and the achieved closed-loop transfer function, i.e.

$$E(\rho, q^{-1}) = M(q^{-1}) - \frac{G(q^{-1})K(\rho, q^{-1})}{1 + G(q^{-1})K(\rho, q^{-1})} \tag{3.21}$$

Definition 3.3.2: Output matching error

The output matching error $\epsilon(\rho, t)$ is defined as the signal obtained by multiplying both sides of equation (3.21) by a reference signal $r(t)$, i.e.

$$\epsilon(t, \rho) = M(q^{-1})r(t) - \frac{G(q^{-1})K(\rho, q^{-1})r(t)}{1 + G(q^{-1})K(\rho, q^{-1})} \tag{3.22}$$

To simplify notation, in the rest of the chapter we drop the backward shift operator q^{-1} from equations and corresponding block diagrams.

Since the output matching error $\epsilon(t, \rho)$ in equation (3.22) still depends on the unknown plant G, we derive an alternative way of designing the controller $K(\rho)$. For this purpose, we introduce the following result.

Result 3.3.1

The following three conditions are equivalent

$$\textbf{(i)} \quad E(\rho) = 0 \tag{3.23}$$

$$\textbf{(ii)} \quad \epsilon(t, \rho) = 0, \quad \forall r(t) \tag{3.24}$$

$$\textbf{(iii)} \quad M(1 - M)^{-1} r(t) = K(\rho)w(t), \ \forall r(t) \tag{3.25}$$

Proof. Equivalence between (ii) and (iii) in Result 3.3.1 can be proved by noting that from equation (3.22) $\epsilon(t,\rho) = 0$ is equivalent to

$$M(1-M)^{-1}r(t) = \; GK(\rho)r(t). \tag{3.26}$$

therefore, condition (iii) is now obtained thanks to the fact that $GKr(t) = KGr(t) = Kw(t)$, where $w(t)$ is the plant output sequence obtained by applying the signal $r(t)$ to the plant input (see the block diagram description of the output matching error in Fig. 3.6). Equivalence between (i) and (ii) is based on the trivial fact that the output of a system is identically zero for all the possible inputs if and only if the system transfer function is identically zero.

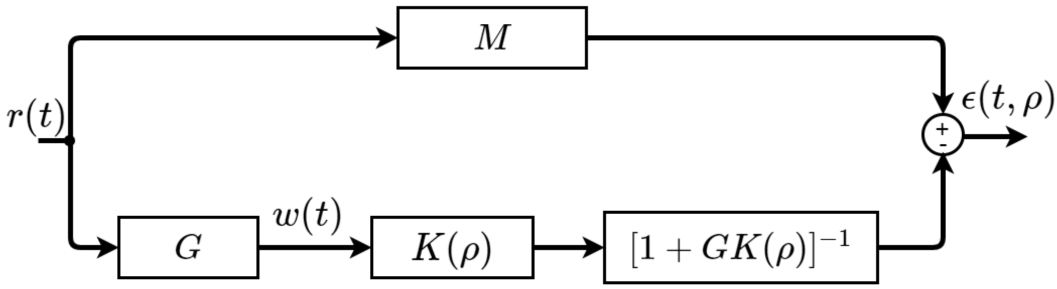

Figure 3.6 A block diagram description of the output matching error $\epsilon(t,\rho)$.

□

Remark 3.3.1

Result 3.3.1 plays a crucial role here since it suggests a way for turning the condition on the model matching error $E = 0$, which depends on the unknown plant transfer function G, into a condition on the output matching error $\epsilon(t) = 0$ which, on the contrary, depends only on the output sequence $w(t)$ collected by applying the signal $r(t)$ to the plant (see equation (3.25)).

Remark 3.3.2

Condition (ii) considered in Result 3.3.1 can be approximated in practice by the condition $\epsilon(\rho,t) = 0$ for a reference signal $r(t)$ which is persistently exciting in the sense that its spectrum is rich enough to properly excite the dynamics of both M and G. For more details see [41].

Remark 3.3.3

It is worth noting that in a number of previous papers on the subject (see, e.g., .[152], [170] and [60]) at this stage the following approximation is introduced,

$$1 - M \cong [1 + GK(\rho)]^{-1}. \tag{3.27}$$

The meaning of such an approximation is that the actual sensitivity of the controlled system to be obtained with the designed controller will be equal to the ideal sensitivity $(1-M)$, a condition which, in turn, implies perfect matching of the input-output reference model. Such an assumption, which was also made in our preliminary conference paper ([41]), is no more required in this work. In fact, by exploiting the set-membership technique presented in the next sections, the controller $K(\rho)$ in equation (3.25) can be directly designed from the I/O collected data without assuming that the final controlled system transfer function will perfectly match the desired reference model (a condition which cannot be guaranteed in the general case).

3.3.2 MIMO systems

In this section, we consider the discrete-time linear-time invariant (LTI) feedback control scheme shown in Fig. 3.5 as a multi-input/multi-output (MIMO) system where, $\boldsymbol{G}$, $\boldsymbol{K}(\rho)$ and $\boldsymbol{M}$ are matrix whose elements are discrete time transfer functions of a stable and MP plant, a controller and a given reference model respectively. The dimension of $\boldsymbol{G}$, $\boldsymbol{K}(\rho)$ and $\boldsymbol{M}$ are $n \times m$, $m \times n$ and $n \times m$ respectively. In this work we assume that the plant to be controlled is a square system in the sense that the input and the output vectors, $\boldsymbol{u}(t)$ and $\boldsymbol{w}(t)$

respectively, have the same dimension n. In this chapter G_{ij}, M_{ij} and k_{ij} denotes the (i,j)-element of $\boldsymbol{G}$, $\boldsymbol{M}$ and $\boldsymbol{K}$ matrices respectively.

Now we introduce a working assumption which is exploited in the derivation of the following result.

Assumption 1. *We assume to perform n separate experiments where the i-th experiment is performed by applying a multiple reference signal $\boldsymbol{r}^{[j]}$ defined as a column vector of length n which has a scalar signal $s(t)$ at j-th row and zero in all the other rows. Similarly, the output $\boldsymbol{w}^{[i]}$ collected by applying the reference signal $\boldsymbol{r}^{[i]}$, is a column vector whose elements are the n outputs of the MIMO system, given by*

$$\boldsymbol{w}^{[i]}(t) = \boldsymbol{G}\boldsymbol{r}^{[i]}(t), \quad \forall i = 1,...,n \tag{3.28}$$

Thanks to this assumption, we introduce the following result.

Result 3.3.2

The following three conditions are equivalent

$$\textbf{(i)} \quad \boldsymbol{E}(\boldsymbol{\rho}) = \boldsymbol{M} - \boldsymbol{G}\boldsymbol{K}(\boldsymbol{\rho})(\boldsymbol{I} + \boldsymbol{G}\boldsymbol{K}(\rho))^{-1} = 0 \tag{3.29}$$

$$\textbf{(ii)} \quad \boldsymbol{\epsilon}^{[i]}(\boldsymbol{\rho},t) = 0, \quad \forall r^{[i]}(t), \quad \forall i = 1,...,n \tag{3.30}$$

$$\textbf{(iii)} \quad \boldsymbol{M}(\boldsymbol{I} - \boldsymbol{M})^{-1}\boldsymbol{r}^{[j]}(t) = \sum_{i=1}^{n} k_{ij}(\rho)\boldsymbol{w}^{[i]}(t) \tag{3.31}$$

where, $\boldsymbol{I}_{n\times n}$ is a square identity matrix, and the zero output matching error signal condition $\boldsymbol{\epsilon}^{[i]}(\boldsymbol{\rho},t) = 0$ for MIMO systems depends on the output sequence $\boldsymbol{w}^{[i]}$ collected by applying to the plant the reference signal $\boldsymbol{r}^{[i]}$ $\forall i,j = 1,..,n$.

Proof. Equivalence between (ii) and (iii) in Result 3.3.2 can be proved by noting that the following two equations are equivalent,

$$\textbf{(a)} \quad \boldsymbol{\epsilon}^{[i]}(\boldsymbol{\rho},t) = 0, \quad \forall \boldsymbol{r}^{[i]}(t), \quad \forall i = 1,...,n \tag{3.32}$$

$$\textbf{(b)} \quad \boldsymbol{M}(\boldsymbol{I} - \boldsymbol{M})^{-1}\boldsymbol{r}^{[i]}(t) = \boldsymbol{G}\boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{r}^{[i]}(t), \quad \forall i = 1,...,n. \tag{3.33}$$

By applying Lemma 1 reported in Appendix A to (3.33) we get

$$\boldsymbol{M}(\boldsymbol{I}-\boldsymbol{M})^{-1}\boldsymbol{r}^{[i]}(t)=\sum_{i=1}^{n}k_{ij}\boldsymbol{G}\boldsymbol{r}^{[i]}(t). \tag{3.34}$$

Condition (iii) is finally obtained from (3.34) thanks to equation (3.28) in Assumption 1. Equivalence between (i) and (ii) is based on the trivial fact that the output of a system is identically zero for all the possible inputs if and only if the system transfer function is identically zero. □

Now we are in the position to state, in general terms, the problem to be solved in this chapter.

Problem 1. ***[SM-DDDC problem]***
The problem addressed in this chapter is to find the controller transfer function $K(q^{-1})$ *such that equation* (3.25) *(or* (3.31) *for MIMO system) is satisfied, where* $w(t)$ *is the output plant signals obtained by applying to the plant input a suitable (i.e., with frequency spectrum rich enough) signal* $r(t)$.

3.4 A set-membership approach to DDDC design

In this section, a set-membership based algorithm to DDDC design is proposed for both SISO and MIMO systems.

3.4.1 SM-DDDC for SISO LTI systems

Here we assume that a set of N input-output plant data are collected experimentally by applying a suitable (persistently exciting) signal $r(t)$ to the plant. The output measurements $y(t)$ are assumed to be corrupted by bounded additive noise according to (see also Fig. 3.7)

$$y(t)=w(t)+\eta(t) \tag{3.35}$$

where

$$w(t) = Gr(t) \tag{3.36}$$

is the noiseless output of the system, while the noise $\eta(t)$ is assumed to range within a given bound $\Delta\eta$ as follows

$$|\eta(t)| \leq \Delta\eta \tag{3.37}$$

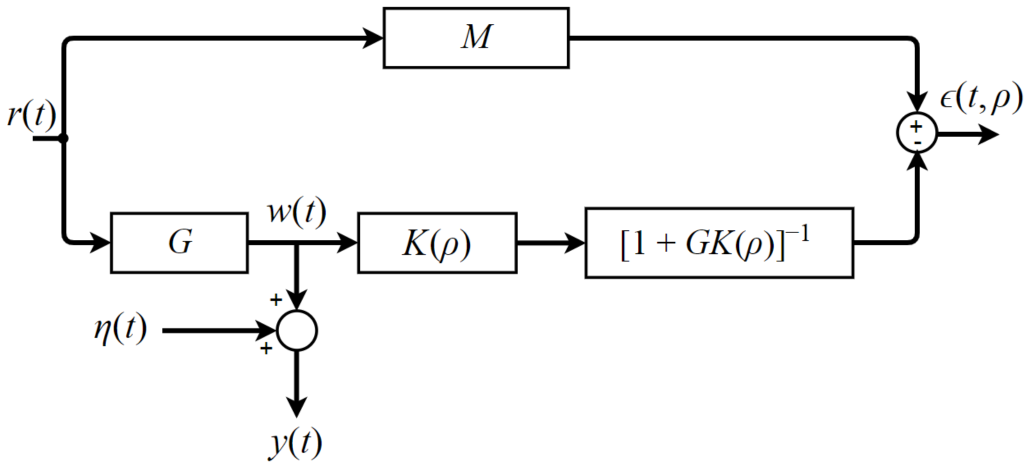

Figure 3.7 A block diagram description of the output matching error $\epsilon(t,\rho)$ when the collected output data is corrupted by additive bounded noise $\eta(t)$.

Substitution of equation (3.35) into (3.25), leads to the following equation

$$M(1-M)^{-1}r(t) = K(\rho)[y(t)-\eta(t)] \tag{3.38}$$

which depends on the uncertain variable $\eta(t)$, due to the presence of the output measurement noise.

Now, let us introduce the *Feasible Controller Set* defined as follows.

Definition 3.4.1: Feasible Controller Set for SISO systems

The Feasible controller set for SISO systems is defined as the set of all the controllers belonging to the following given class $\mathcal{K}_{(n_a,n_b)}$

$$\begin{cases} K(\rho) \in \mathcal{K}_{(n_a,n_b)} = \{K(\rho) : K(\rho) = \\ = \dfrac{b_0 + b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n_b} q^{-n_b}}{1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_{n_a} q^{-n_a}}, \\ \rho = [a_1 \; \ldots.. \; a_{na} \; b_0 \; \ldots.. \; b_{nb}]^T, \rho \in \mathbb{R}^{n_\rho}\} \\ \text{where, } n_\rho = n_a + n_b + 1 \end{cases} \tag{3.39}$$

such that the equation

$$\epsilon(t, \rho, \eta) = 0 \tag{3.40}$$

is fulfilled for at least one noise sequence $\eta(t)$ satisfying the bound $|\eta(t)| \leq \Delta\eta, \; \forall t = 1, \ldots, N$.
The Feasible controller set can then be described as

$$\begin{cases} \mathcal{D}_{\mathcal{K}} = \{K(\rho) \in \mathcal{K}_{(n_a,n_b)} : \\ M(1-M)^{-1} r(t) - K(\rho)[y(t) - \eta(t)] = 0 \\ |\eta(t)| \leq \Delta\eta, \; \forall t = 1, \ldots, N\} \end{cases} \tag{3.41}$$

It is worth noting that, the controller class $\mathcal{K}_{(n_a,n_b)}$ is general in the context of linear control, since it includes all the linear time-invariant (LTI) controller of order at most $\max(n_a, n_b)$. It is worth noting that, the he controller orders n_a and n_b are a-priori given information.

The importance of the *feasible controller set* comes from the fact that $\mathcal{D}_{\mathcal{K}}$ is the set of all the controllers of order less or equal than $\max(n_a, n_b)$ that are consistent with the assumption that the output matching error is identically zero for at least one of the possible feasible noise sequences. In turn, this implies that no controller in the considered class $\mathcal{K}_{(n_a,n_b)}$ is able to solve the problem if and only if the feasible controller set $\mathcal{D}_{\mathcal{K}}$ is empty. If that is the case, we need to enrich the model class by considering higher values for n_a and/or n_b (i.e., higher controller order). On the contrary, in the case the feasible set $\mathcal{D}_{\mathcal{K}}$ is not empty, we can state that certainly there exists at least one controller in the class $\mathcal{K}_{(n_a,n_b)}$ which satisfactorily solves the considered model matching problem.

Since the controller class considered here is parametrized by ρ, we can conveniently replace the set $\mathcal{D}_{\mathcal{K}}$ with the *Feasible controller parameter (FCP) set* defined as follows.

Definition 3.4.2: FCP Set for SISO systems

The feasible controller parameter set is the set of all the controller parameters ρ such that $K(\rho) \in \mathcal{D}_{\mathcal{K}}$.

Since equation (3.41) not only depends on ρ but also on the uncertain variable $\eta(t)$ we consider the following set.

Definition 3.4.3: Extended Feasible Controller Parameter Set for SISO systems

The extended feasible controller parameter set $\mathcal{D}_\rho$ is the set of all the controller parameters ρ and noise sequences $\eta(t), t = 1, ..., N$, such that $K(\rho) \in \mathcal{D}_{\mathcal{K}}$.

Based on the definition of the *extended feasible controller parameter (EFCP) set* we can formulate the following result.

Result 3.4.1: Structure of the EFCP Set for SISO systems

The *extended feasible controller parameter set* can be written in the following form

$$\begin{cases} \mathcal{D}_\rho = \{(\rho,\eta) \in \mathbb{R}^{n_\rho+N} : L(t) + \sum_{i=1}^{n_a} a_i L(t-i) = \\ = \sum_{j=0}^{n_b} b_j (y(t-j) - \eta(t-j)), \ |\eta_t| \leq \Delta\eta, \\ \forall t = \max(n_a, n_b) + 1, ..., N\} \end{cases} \tag{3.42}$$

where

$$L(t) = M(1-M)^{-1} r(t) \tag{3.43}$$

Proof. From definition 3.4.3, $\mathcal{D}_\rho$ is the set of parameter $\rho \in \mathbb{R}^n_\rho$ such that the following condition holds

$$M(1-M)^{-1}r(t) = K(\rho)[y(t)-\eta(t)] \tag{3.44}$$

which, in turn, is equivalent to

$$L(t) = K(\rho)[y(t)-\eta(t)] \tag{3.45}$$

where $L(t)$ is defined as in (3.43). The statement of *Result 3.4.1* finally follows from the fact that

$$K(\rho) = \frac{b_0 + b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n_b} q^{-n_b}}{1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_{n_a} q^{-n_a}} \tag{3.46}$$

3.4.2 SM-DDDC for MIMO LTI systems

In this Section, a set-membership algorithm to DDDC design is proposed to deal with the case of MIMO LTI stable, MP plants.

According to *Assumption 1*, we assume that a set of N input-output plant data are collected for n open-loop experiments by applying a suitable (i.e. persistently exciting) reference signal $\boldsymbol{r}^{[i]}(t)$ to the plant $\boldsymbol{G}$. The output measurements $\boldsymbol{y}^{[i]}(t)$ are assumed to be corrupted by bounded additive noise according to

$$\boldsymbol{y}^{[i]}(t) = \boldsymbol{w}^{[i]}(t) + \boldsymbol{\eta}^{[i]}(t), \quad \forall i = 1,..,n \tag{3.47}$$

where the noise variables $\boldsymbol{\eta}^{[i]}(t)$ acting on the noiseless output $\boldsymbol{w}^{[i]}(t)$ are assumed to range within given bound $\Delta\eta^{[i]}$, that is

$$|\boldsymbol{\eta}^{[i]}(t)| \leq \Delta\eta^{[i]} \tag{3.48}$$

Substitution of equation (3.47) into (3.31), leads to the following equation

$$\boldsymbol{M}(\boldsymbol{I}-\boldsymbol{M})^{-1}\boldsymbol{r}^{[j]}(t) = \sum_{i=1}^{n} k_{ij}[\boldsymbol{y}^{[i]}(t) - \boldsymbol{\eta}^{[i]}(t)] \quad \forall j = 1,...,n \tag{3.49}$$

where, due to the presence of the noise on the output measurements, the controller design depends on the uncertain variables $\boldsymbol{\eta}^{[i]}(t)$ as in the SISO case.

Now, let us introduce the definition of *Feasible Controller Set* for MIMO systems.

Definition 3.4.4: Feasible Controller Set for MIMO system

The Feasible controller set can then be described as

$$\begin{cases} \mathcal{D}_{\mathcal{K}} = \{\boldsymbol{K}(\rho) \in \mathcal{K}_{(n_a^{[ij]}, n_b^{[ij]})} : \\ \boldsymbol{M}(\boldsymbol{I}-\boldsymbol{M})^{-1}\boldsymbol{r}^{[j]}(t) = \sum_{i=1}^{n} k_{ij}(\rho)[\boldsymbol{y}^{[i]}(t) - \boldsymbol{\eta}^{[i]}(t)] \\ |\boldsymbol{\eta}^{[i]}(t)| \leq \Delta\eta^{[i]}, \ \forall t = 1, ..., N, \forall i, j = 1, ..., n\} \end{cases} \tag{3.50}$$

Such that, the *feasible controller set* for MIMO system is the set of all the controllers belonging to the following given class $\mathcal{K}_{(n_a^{[ij]}, n_b^{[ij]})}$

$$\begin{cases} \boldsymbol{K}(\rho) \in \mathcal{K}_{(n_a^{[ij]}, n_b^{[ij]})} = \{\boldsymbol{K}(\boldsymbol{\rho}) : \boldsymbol{K}(\boldsymbol{\rho}) \text{ is a square matrix;} \\ \text{where the generic entry of such a matrix is described as,} \\ k_{ij}(\rho_{ij}) = \dfrac{b_0^{[ij]} + b_1^{[ij]} q^{-1} + b_2^{[ij]} q^{-2} + \ldots + b_{n_b^{[ij]}}^{[ij]} q^{-n_b^{[ij]}}}{1 + a_1^{[ij]} q^{-1} + a_2^{[ij]} q^{-2} + \ldots + a_{n_a^{[ij]}}^{[ij]} q^{-n_a^{[ij]}}}, \\ \boldsymbol{\rho} = [\rho_{11} \ \ldots \ \rho_{1n} \ldots \ \rho_{n1} \ \ldots \ \rho_{nn}]^T, \\ \rho_{ij} = [a_1^{[ij]} \ \ldots.. \ a_{na^{[ij]}}^{[ij]} \ b_0^{[ij]} \ \ldots.. \ b_{nb^{[ij]}}^{[ij]}]^T, \boldsymbol{\rho} \in \mathbb{R}^{n_\rho} \\ \forall i, j = 1, ..., n\}, \text{ where, } n_\rho = \sum_{i=1}^{n} \sum_{j=1}^{n} n_a^{[ij]} + n_b^{[ij]} + 1 \end{cases} \tag{3.51}$$

such that the equation

$$\boldsymbol{\epsilon}^{[i]}(\boldsymbol{\rho}, t, \boldsymbol{\eta}^{[i]}) = 0 \tag{3.52}$$

is fulfilled for a noise sequence $\boldsymbol{\eta}^{[i]}(t)$ satisfying the bound $|\boldsymbol{\eta}^{[i]}(t)| \leq \Delta\eta^{[i]}$, $\forall t = 1, \ldots, N$, and $\forall i, j = 1, ..., n$.

Since the controller class considered here is parametrized by $\boldsymbol{\rho}$, we can conveniently replace $\mathcal{D}_{\mathcal{K}}$ with the *feasible controller parameter (FCP) set*

Definition 3.4.5: FCP Set for MIMO system

The feasible controller parameter set $\mathcal{D}_\rho$ is the set of all the controller parameter vectors $\boldsymbol{\rho}$ such that $\boldsymbol{K}(\rho) \in \mathcal{D}_\mathcal{K}$.

Since equation (3.41) not only depends on $\boldsymbol{\rho}$ but also on the uncertain variable $\boldsymbol{\eta}^{[i]}(t)$ we consider the following set.

Definition 3.4.6: EFCP Set for MIMO systems

The extended feasible controller parameter (EFCP) set $\mathcal{D}_\rho$ is the set of all the controller parameter vectors $\boldsymbol{\rho}$ and noise sequences $\boldsymbol{\eta}^{[i]}(t), t = 1, ..., N$ such that $\boldsymbol{K}(\boldsymbol{\rho}) \in \mathcal{D}_\mathcal{K}$.

Based on the definition of the EFCP set we can formulate the following result.

Result 3.4.2: Structure of the EFCP Set for MIMO system

The extended feasible controller parameter set can be written in the equivalent form, by introducing some additional variables $Z_{ij}$and Q_{ih}, called partial outputs (see appendix 3.B and [39] for more details)

$$\begin{cases} \mathcal{D}_\rho = \{(\boldsymbol{\rho}, Z_{ij}, Q_{ih}, \boldsymbol{\eta}^{[i]}) \in \mathbb{R}^{n_\rho + 2i*N(j+1)}: \\ \boldsymbol{L}(t)^{[j]} = \boldsymbol{M}(\boldsymbol{I}-\boldsymbol{M})^{-1}\boldsymbol{r}^{[j]} = [l_1(t), l_2(t), ..., l_n(t)]^T \\ \sum_{i=1}^{n} Z_{ij}(t) = l_j(t), \quad Z_{ij}(t) + \sum_{f=1}^{n_a^{[ij]}} a_f^{[ij]} Z_{ij}(t-f) = \\ = \sum_{f=0}^{n_b^{[ij]}} b_f^{[ij]} (y_j^{[i]}(t-f) - \eta_j^{[i]}(t-f)), \\ \sum_{i=1}^{n} Q_{ih}(t) = l_h(t), \quad Q_{ih}(t) + \sum_{f=1}^{n_a^{[ij]}} a_f^{[ij]} Q_{ih}(t-f) = \\ = \sum_{f=0}^{n_b^{[ij]}} b_f^{[ij]} (y_h^{[i]}(t-f) - \eta_h^{[i]}(t-f)), \\ |\boldsymbol{\eta}^{[i]}(t)| \le \Delta\eta^{[i]}, \forall t = max(n_a^{[ij]}, n_b^{[ij]}) + 1, ..., N, \\ \forall i, j, h = 1, ..., n, h \neq j\} \end{cases} \tag{3.53}$$

Proof: See Appendix 3.B.

Remark 3.4.1

In many real-world applications the off-diagonal transfer functions of the controller matrix $\boldsymbol{K}$ are tuned in such a way that the interactions between the outputs are decoupled, and therefore $\boldsymbol{G}$ is assumed to be diagonalizable by output feedback (see e.g., [108]). Thanks to Result 3.4.2, this decoupling can be satisfied by choosing a diagonal reference model $\boldsymbol{M}$, and imposing the *off-diagonal constraints* represented in equations (3.84) and (3.85). This decoupling is guaranteed since $l_h(t)$ will be equal to zero by definition of $\boldsymbol{L}(t)$ in equation (3.53) (see Appendix 3.C). For more details about controller decoupling in linear multivariable systems, see e.g., [132], [108] and [161].

By comparing the structure of the *feasible controller parameter set* highlighted in *Result 3.4.2* with the results presented in [39], it turns out that the problem of direct data-driven controller design considered in this chapter is equivalent to the set-membership errors-in-variables identification problem considered, e.g., in [36] and [31] (see Chapter 2), in the case where the output noise is identically zero, and only the input data are corrupted by noise. From equations (3.80) and (3.81) we see that the problem of achieving a specific performance is to estimate the diagonal transfer functions in matrix $\boldsymbol{K}(\boldsymbol{\rho})$ which maps uncertain inputs signals to a noiseless output sequence $l_j(t), \forall j = 1, ..., n$. On the other side, equations (3.84) and (3.85) deal with the problem of defining the interactions between the outputs in order to estimate the off-diagonal transfer functions in matrix $\boldsymbol{K}(\boldsymbol{\rho})$, which maps the uncertain input signals to a noiseless output sequence $l_h(t)$, $\forall h \neq j, h, j = 1, ..., n$.

3.4.3 Central estimate

Given the set $\mathcal{D}_\rho$ of all the feasible controller parameters $\boldsymbol{\rho}$, we pick a specific single value in the set to design the controller $\boldsymbol{K}$ to be actually implemented in the feedback control system of Fig. 3.5. According to the set-membership identification theory, we select a single point in $\mathcal{D}_\rho$ by looking for the value of the parameter $\boldsymbol{\rho}$ that minimizes the distance in the ℓ_∞ norm from the farthest point in the feasible controller parameter set, i.e.

$$\rho^c \doteq \arg \min_{\rho \in \mathbb{R}^{n_\rho}} \max_{(\rho_\nu, \eta) \in \mathcal{D}_\rho} \|\rho_\nu - \rho\|_\infty. \tag{3.54}$$

The estimate ρ^c computed by solving (3.54) is the so-called ℓ_p-*Chebyshev center* of $\mathcal{D}_\rho$, also called *central estimate* in the set-membership literature. The central estimate is the center of the minimum-volume-box outer-bounding $\mathcal{D}_\rho$ and can be computed by exploiting the convex relaxation approach proposed by [36]. In particular, for each single component ρ_x of the parameter vector $\boldsymbol{\rho}$ the central estimate ρ_x^c is given by

$$\rho_x^c = \frac{\overline{\rho}_x^c + \underline{\rho}_x^c}{2} \tag{3.55}$$

where

$$\underline{\rho}_x = \underset{\rho, \eta \in \mathcal{D}_\rho}{min} \rho_x, \quad \overline{\rho}_x = \underset{\rho, \eta \in \mathcal{D}_\rho}{max} \rho_x \tag{3.56}$$

Controller parameter bounds $\underline{\rho}_x$ and $\overline{\rho}_x$ implicitly define the *Controller parameter uncertainty intervals* (CPUIs) given by

$$CPUI_{\rho_x} = [\underline{\rho}_x, \ \overline{\rho}_x]. \tag{3.57}$$

Problem (3.56), is in general, a hard non-convex optimization problem, which falls into the class of semi-algebraic optimization problems, widely studied in the recent years. For more details about a possible solution to this problem, the reader is advised to see Section 2.2 and 2.3.

3.5 Simulation examples

In this section the effectiveness of the presented approach is shown by means of simulation examples.

3.5.1 Example 1. DDDC design for SISO MP system

In this example, the proposed approach is employed to tune a SISO controller to be compared with the standard NCbT introduced in [152] and the VRFT approach (see e.g., [60]). The plant considered in this example has the following transfer function

$$G(q^{-1}) = \frac{0.09516q^{-1} + 0.02q^{-2} + 0.05q^{-3} - 0.04q^{-4}}{1 - 0.9048q^{-1} + 0.2q^{-2} - 0.5q^{-3} + 0.4q^{-4}}$$

while the assigned reference model is given by

$$M_1(q^{-1}) = \frac{0.4q^{-1}}{1 - 0.6q^{-1}}$$

As to the SM-EIV approach, a random signal with unity amplitude is used as input, r(t), to the system. The output of the plant is disturbed by a bounded random noise, with uniform distribution in the range $[-\Delta\eta, +\Delta\eta]$, such that the *signal-to-noise ratio* of the open-loop experiment, given by

$$SNR_w = 10log\frac{\sum_{t=1}^{N} w_t^2}{\sum_{t=1}^{N} \eta_t^2},$$

is about $20dB$. Results are given for 100 experimental data. While, for VRFT and NCbT, a PRBS signal of 255 samples with unity amplitude is used as input to the system. Four periods of this signal are used to design the controller, $N = 1020$. The periodic output is disturbed by a zero-mean white noise such that the signal-to-noise ratio is about $20dB$.

The general LTI controller structure in (3.39) for the SM-EIV approach is considered here, where $n_a = n_b = n$ and the controller order $n = 4$ is selected by trial starting from $n = 1$ and by increasing n until the feasible set is not empty. The final selected controller structure is as follows

$$K(\rho^c, q^{-1}) = \frac{b_0 + b_1q^{-1} + b_2q^{-2} + b_3q^{-3} + b_4q^{-4}}{1 + a_1q^{-1} + a_2q^{-2} + a_3q^{-3} + a_4q^{-4}}$$

The controller structure for NCbT and VRFT is chosen as follows,

$$K(\rho, q^{-1}) = \frac{b_0 + b_1q^{-1} + b_2q^{-2} + b_3q^{-3} + b_4q^{-4}}{1 - q^{-1}}$$

Note that, the controller structure for VRFT and NCbT is chosen to be linearly parameterized with a fixed known denominator according to [60] and [84].

Central estimates for the parameters of the SM-EIV controller have been computed exploiting the approach proposed in the chapter. For the NCbT and the VRFT method, the length of the instrumental variable l is found to be 15 by trial-and-error. Fig. 3.8 and 3.9 display the comparison between the reference model and the obtained closed-loop system in terms of magnitude frequency and step responses respectively for SM-EIV, VRFT and NCbT methods.

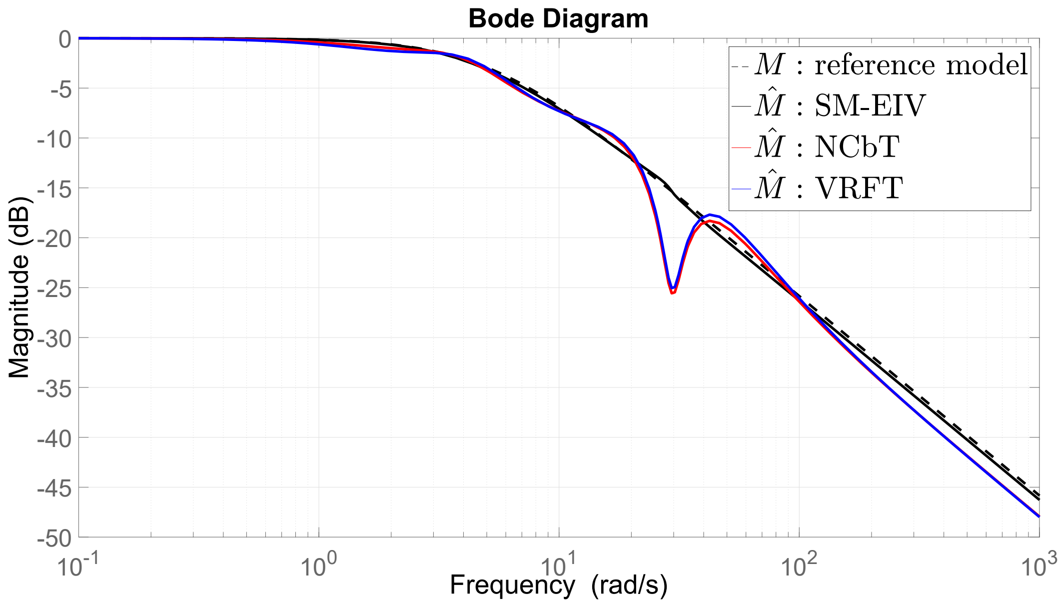

Figure 3.8 Comparison of frequency responses: designed feedback control system with SM-EIV (black solid-line), NCbT method (red-line), VRFT method (blue-line) and reference model (black dashed-line).

From the comparison, we see that the controlled system obtained with the proposed method performs better than the NCbT and VRFT methods both in the frequency and in the time domain.

3.5.2 Example 2. DDDC design for non-symmetric M

In this example, we present a comparison between the SM-EIV approach proposed in this chapter, the VRFT method proposed in [60] and the method proposed in [61]. The plant transfer function used for generating the data is given by

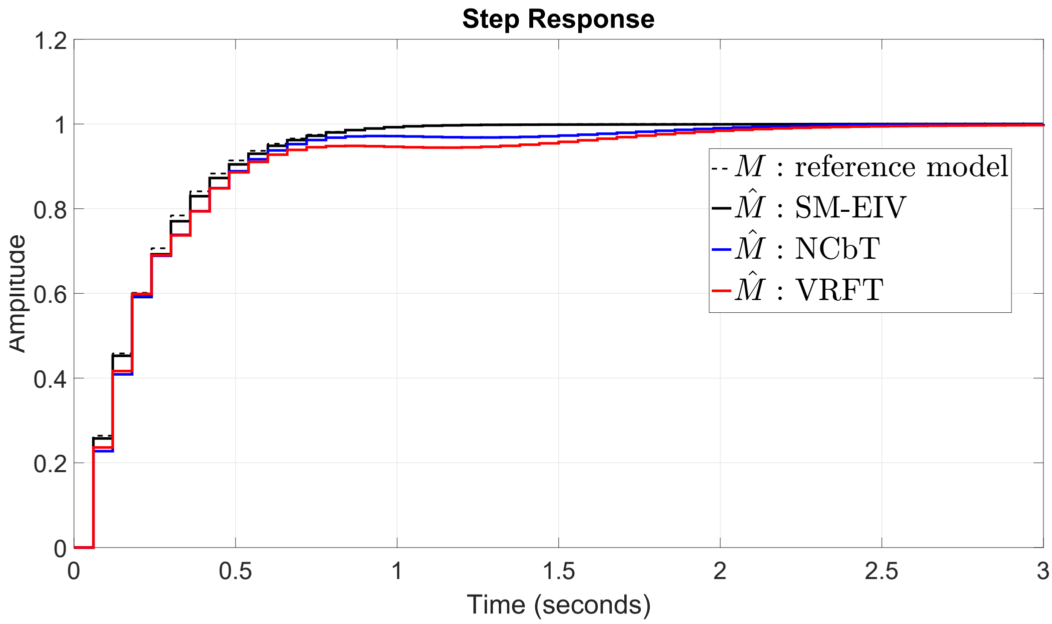

Figure 3.9 Comparison of step responses: designed feedback control system with SM-EIV (black solid-line), NCbT method(red-line), VRFT method(blue-line) and reference model (black dashed-line).

$$\boldsymbol{G}(q^{-1}) = \begin{bmatrix} \frac{0.09516q^{-1}}{(1-0.9048q^{-1})} & \frac{0.03807q^{-1}}{(1-0.9048q^{-1})} \\ \frac{-0.02974q^{-1}}{(1-0.9048q^{-1})} & \frac{0.04758q^{-1}}{(1-0.9048q^{-1})} \end{bmatrix}$$

while the assigned non-symmetric ($M_{ii} \neq M_{jj}$) reference model $\boldsymbol{M_2}$ is considered for the same plant, given by

$$\boldsymbol{M_2}(q^{-1}) = \begin{bmatrix} \frac{0.1148q^{-1}-0.0942q^{-2}}{1-1.79q^{-1}+0.8106q^{-2}} & 0 \\ 0 & \frac{0.4q^{-1}}{1-0.6q^{-1}} \end{bmatrix}$$

The input signal for the SM-EIV method can be defined as $\boldsymbol{r}^{[1]}(t) = [s(t),0]^T$ and $\boldsymbol{r}^{[2]}(t) = [0,s(t)]^T$, where $s(t)$ is a random signal uniformly distributed in the range $[-1,+1]$ with length $N = 100$. The input signal for the VRFT method is $u(t) = [A_1(t),A_2(t)]$, where $A_i(t), i = 1,2$ is a random signal with length $N = 5000$. Finally, the input signals for the method proposed in [61] are two random input sequences u_1 and u_2 with length $N = 5000$ such that, the sequence u_1 is used to feed the first channel, then the same input is switched to the second channel and u_2 is used to separately feed the two channels, analogously to what has been done for u_1. Note that, for the SM-EIV we need n experiments and for the VRFT we need one experiment while for the method proposed in [61] we need $n \times n$ experiments, where in this example $n = 2$.

To establish a fair comparison with the method of [60] and [61] two different tests have been carried out, one with bounded noise uniformly distributed in the range $[-\Delta\eta, +\Delta\eta]$ and another one with stochastic zero-mean white noise, such that both systems are characterized by $SNRw \cong 25dB$. Nonetheless, it should be noticed that the quality of the step response of all the previous methods is the same for both tests (stochastic and bounded noise) in terms of overshoot, performance in channel decoupling, settling time and overall shape.

The controller order for the SM-EIV method is selected by trial starting from $n = 1$ and by increasing n until the feasible set is not empty. Since the feasible set is not empty for $n = 1$, we select the following structure for the controller

$$\boldsymbol{K}(q^{-1}) = \begin{bmatrix} \dfrac{b_0^{[11]} + b_1^{[11]} q^{-1}}{1 + a_1^{[11]} q^{-1}} & \dfrac{b_0^{[12]} + b_1^{[12]} q^{-1}}{1 + a_1^{[12]} q^{-1}} \\ \dfrac{b_0^{[21]} + b_1^{[21]} q^{-1}}{1 + a_1^{[21]} q^{-1}} & \dfrac{b_0^{[22]} + b_1^{[22]} q^{-1}}{1 + a_1^{[22]} q^{-1}} \end{bmatrix}$$

While, the controller structure for the VRFT and for the approach proposed in [61] is chosen to be linearly parameterized with a fixed known denominator, as follows

$$\boldsymbol{K}(q^{-1}) = \begin{bmatrix} \dfrac{b_0^{[11]} + b_1^{[11]} q^{-1}}{1 - q^{-1}} & \dfrac{b_0^{[12]} + b_1^{[12]} q^{-1}}{1 - q^{-1}} \\ \dfrac{b_0^{[21]} + b_1^{[21]} q^{-1}}{1 - q^{-1}} & \dfrac{b_0^{[22]} + b_1^{[22]} q^{-1}}{1 - q^{-1}} \end{bmatrix}$$

A closed-loop noiseless experiment with the controller given by the proposed approach and the one returned by MIMO VRFT and the one proposed in [61] is illustrated in Figure 3.10. As can be seen from Fig. 3.10, the proposed approach (SM-EIV) provides a perfect decoupling, ensuring that the closed-loop system is diagonalized, thanks to the *off-diagonal constraints* represented in equations (3.84) and (3.85). On the contrary, the VRFT method for MIMO system and the method proposed by [61] doesn't guarantee the decoupling when the reference model is not symmetric.

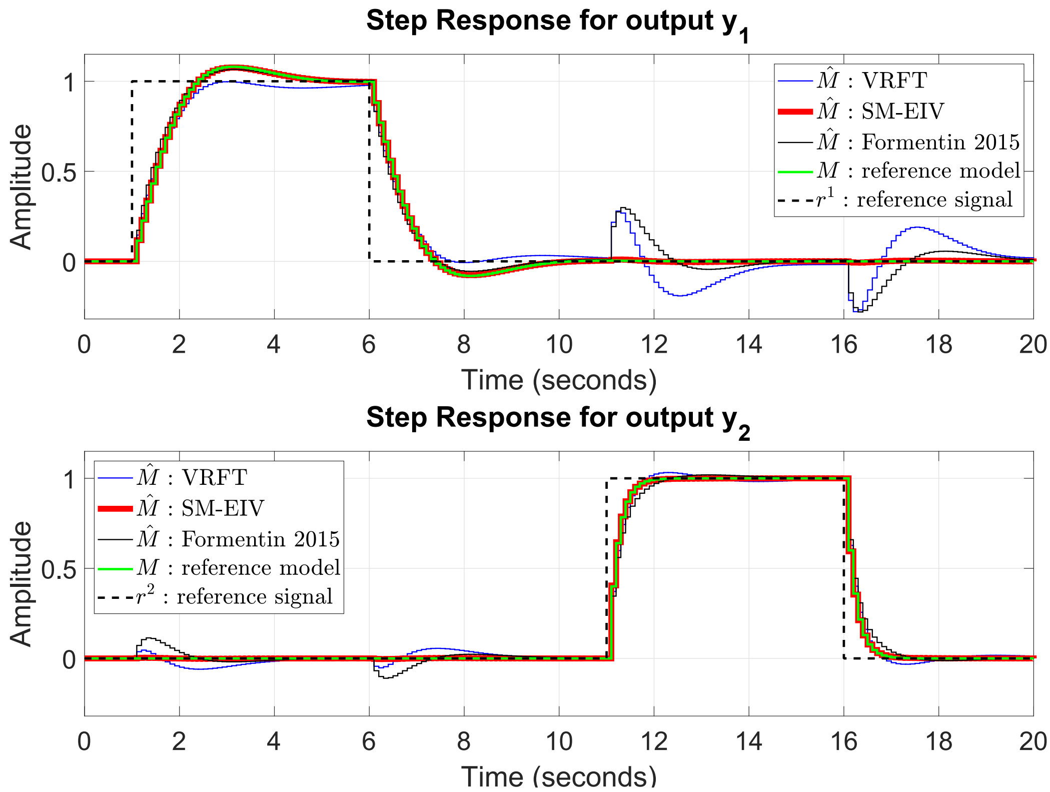

Figure 3.10 Step responses: designed feedback control system with the SM-EIV approach (red-line), the VRFT method (blue-line), the method proposed in [61] (black solid-line), reference model (green-line) and reference signals (black dashed-line). Notice that, red-line and green-line are perfectly overlapped.

3.6 Experimental results

The algorithm presented in Section 3.4.2, has also been tested on the experimental input output data collected on a test bench MIMO electronic filter with 2 inputs and 2 outputs taken from [40]. Fig. 3.11 shows the experimental setup used to collect the data.

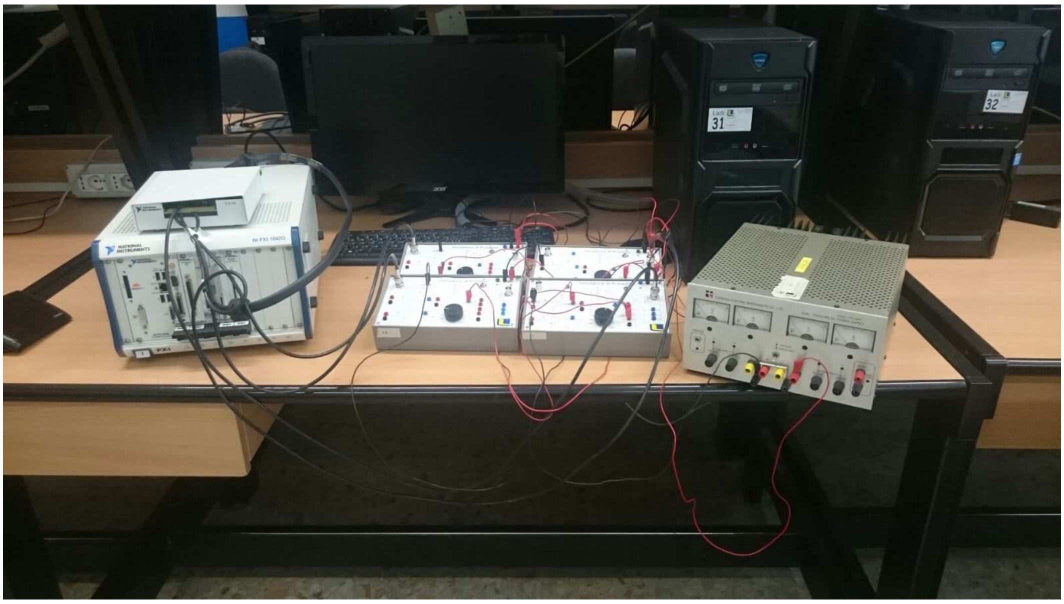

Figure 3.11 The experimental MIMO system used as test bench

The system structure is reported in the block-diagram depicted in Fig. 3.12, for the details on the transfer function, see [40].

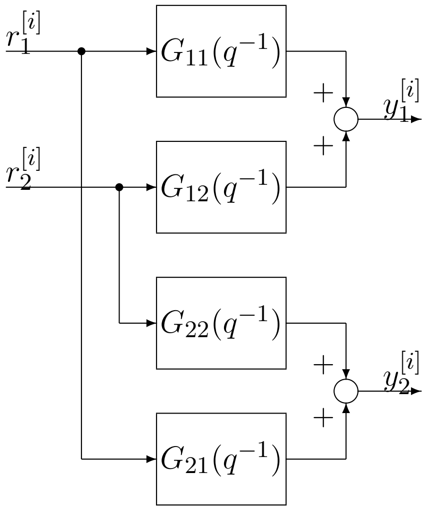

Figure 3.12 Block-diagram description of the MIMO circuit considered in the experimental test bench section.

We point out that, in this example, we do not assume any a-priori information on the plant G.

In order to achieve reasonable tracking and decoupling between $y_1(t)$ and $y_2(t)$, we have chosen the following discrete-time reference model

$$\boldsymbol{M_3}(q^{-1}) = \begin{bmatrix} \frac{0.0008671q^{-1}}{1-q^{-1}} & 0 \\ 0 & \frac{0.0008671q^{-1}}{1-q^{-1}} \end{bmatrix}$$

The system has been excited with two input signals, $\boldsymbol{r}^{[1]}(t) = [s(t), 0]^T$ and $\boldsymbol{r}^{[2]}(t) = [0, s(t)]^T$, where $s(t)$ is a random sequence of 200 samples, uniformly distributed within the range $[-1, +1]$V. A National Instruments PXI equipped with a NI–6221 DAQ board has been used to generate the input signal $s(t)$ and to collect the signals $\boldsymbol{r}^{[1]}(t)$, $\boldsymbol{r}^{[2]}(t)$, $\boldsymbol{y}^{[1]}(t)$ and $\boldsymbol{y}^{[2]}(t)$ at a sample rate of 4kHz. The upper bound on the measurement errors is derived from the precision of the measurement equipment which is given by $\Delta\eta = 0.003$V. The software Sparse-Pop and MOSEK have been used to solve the underlying optimization problems.

We have also designed a controller according to the VRFT method proposed by [60]. For the VRFT approach, the input signal is $u(t) = [A_1(t), A_2(t)]^T$, where $A_i(t), i = 1, 2$ is a random signal uniformly distributed in the range $[-1, +1]V$, with length $N = 5000$.

The controller order for the SM-EIV method is selected by trial starting from $n = 1$ and by increasing n until the feasible set is not empty. Since the feasible set is not empty for $n = 1$, we select the following structure for the controller

$$\boldsymbol{K}(q^{-1}) = \begin{bmatrix} \frac{b_0^{[11]} + b_1^{[11]}q^{-1}}{1 + a_1^{[11]}q^{-1}} & \frac{b_0^{[12]} + b_1^{[12]}q^{-1}}{1 + a_1^{[12]}q^{-1}} \\ \frac{b_0^{[21]} + b_1^{[21]}q^{-1}}{1 + a_1^{[21]}q^{-1}} & \frac{b_0^{[22]} + b_1^{[22]}q^{-1}}{1 + a_1^{[22]}q^{-1}} \end{bmatrix}$$

While, the controller structure for the VRFT is chosen to be linearly parameterized with a fixed known denominator, as follows

$$\boldsymbol{K}(q^{-1}) = \begin{bmatrix} \dfrac{b_0^{[11]} + b_1^{[11]} q^{-1}}{1 - q^{-1}} & \dfrac{b_0^{[12]} + b_1^{[12]} q^{-1}}{1 - q^{-1}} \\ \dfrac{b_0^{[21]} + b_1^{[21]} q^{-1}}{1 - q^{-1}} & \dfrac{b_0^{[22]} + b_1^{[22]} q^{-1}}{1 - q^{-1}} \end{bmatrix}$$

The final controller parameters for both the SM-EIV (ρ^c_{SM-EIV}) and the VRFT (ρ_{VRFT}) methods are reported in Table 3.1. As far as the parameters value of $a_1^{[ij]}$, $\forall i, j = 1, 2$ are concerned in the SM-EIV technique, the computed central estimated for these parameters are equal to -1.

Table 3.1 Controller parameters for the SM-EIV ($\boldsymbol{\rho}^c_{SM-EIV}$) and the VRFT ($\boldsymbol{\rho}_{VRFT}$) method

	Parameter	$\boldsymbol{\rho}^c_{SM-EIV}$	$\boldsymbol{\rho}_{VRFT}$
k_{11}	$b_0^{[11]}$	$+0.0075370$	$+0.006414$
	$b_1^{[11]}$	-0.0066700	-0.005581
k_{12}	$b_0^{[12]}$	-0.0007793	$+0.007304$
	$b_1^{[12]}$	$+0.0007432$	-0.007258
k_{21}	$b_0^{[21]}$	-0.0065240	-0.0003617
	$b_1^{[21]}$	$+0.0056700$	-0.0003326
k_{22}	$b_0^{[22]}$	$+0.0073020$	$+0.002341$
	$b_1^{[22]}$	-0.0064450	-0.001604

A comparison between the reference model and the obtained closed-loop system for the SM-EIV method (by setting the value of the parameter to the central estimate $\boldsymbol{\rho}^c_{SM-EIV}$) and the VRFT method in terms of the step response is presented in Fig. 3.13, from which we see that the output of the controlled response for the SM-EIV approach perfectly overlaps the reference model, while the results for the VRFT technique show the effects of a coupling between the outputs.

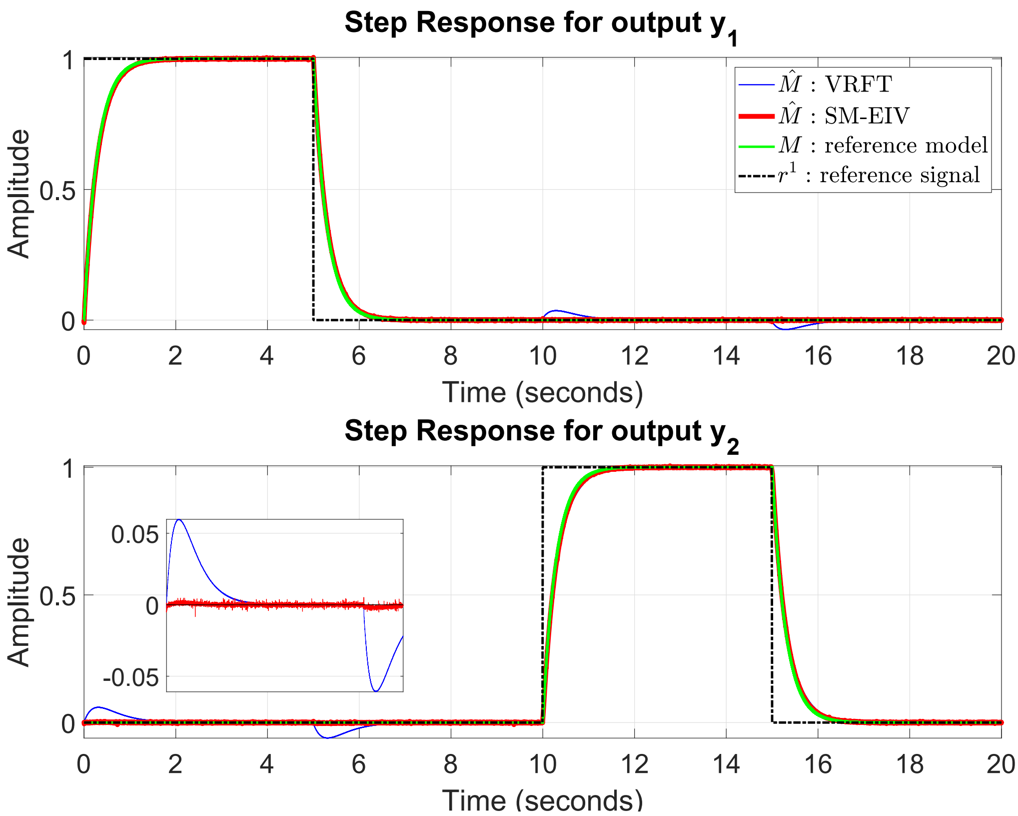

Figure 3.13 Step responses: designed feedback control system with the SM-EIV approach (red-line), the VRFT method (blue-line), reference model (green-line) and reference signals (black dashed-line). Notice that, red-line and green-line are perfectly overlapped.

3.7 Discussion and conclusion

In this chapter, we have proposed an original approach to the problem of designing linear controllers directly from a set of input-output data, experimentally collected on the plant to be controlled. Assuming that the output measurements are corrupted by bounded noise, the controller design problem is formulated as a peculiar input-error set-membership identification problem, solved by adapting results on errors-in-variables identification available from the literature and summarized in Chapter 2. In particular, we formulate the problem of designing a controller in order to match the behaviour of a given reference model, in terms of an equivalent set-membership errors-in-variables problem and we define the feasible controller parameter set. Then, we design the controller parameters by applying the results from Chapter 2 in the field of set-membership errors-in-variables identification.

The main advantages of the proposed method, with respect to previously proposed non-iterative direct data-driven design algorithms, are that (i) the noise corrupting the data is assumed to be bounded and no statistical information is assumed to be a-priori available; (ii) in contrast to Iterative feedback tuning (IFT) and Iterative Correlation-based Tuning (ICbT) approaches where an iterative procedure is exploited, the set-membership approach leads to a non-iterative algorithm to design the controller; (iii) differently from the standard Virtual Reference Feedback Tuning (VRFT) and Non-iterative Correlation based Tuning (NCbT) approaches, the controller transfer function does not need to depend linearly on the parameters to be tuned; (iv) the proposed strategy is applicable to deal with both diagonal and non-diagonal multivariable reference models.

Finally, the effectiveness of the proposed approach is shown by means of different simulation examples and through the application to a laboratory test bench.

Appendix 3.A Lemma 1

The following equivalence holds for the MIMO control system considered in Section 3.4.2,

$$\boldsymbol{G}\boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{r}^{[j]}(t) = \sum_{i=1}^{n} k_{ij}\boldsymbol{G}\boldsymbol{r}^{[i]}(t), \quad \forall j = 1, ..., n \tag{3.58}$$

Proof of Lemma 1

We sketch here a proof of Lemma 1 for the case where $n = 2$. A general detailed proof can be found in [170] and [61].

Let us consider here the 2x2 MIMO plant and the controller respectively described by the following matrix transfer functions

$$\boldsymbol{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{3.59}$$

and

$$\boldsymbol{K}(\boldsymbol{\rho}) = \begin{bmatrix} k_{11}(\rho_{11}) & k_{12}(\rho_{12}) \\ k_{21}(\rho_{21}) & k_{22}(\rho_{22}) \end{bmatrix} \tag{3.60}$$

By selecting the reference vector signal as follows (according to *Assumption 1*),

$$\boldsymbol{r}^{[1]}(t) = \begin{bmatrix} s(t) \\ 0 \end{bmatrix} \quad , \quad \boldsymbol{r}^{[2]}(t) = \begin{bmatrix} 0 \\ s(t) \end{bmatrix} \tag{3.61}$$

we obtain the following expression for the left hand side of equation (3.58) (case $j = 1$):

$$\begin{aligned} \boldsymbol{G}\boldsymbol{K}(\rho)\boldsymbol{r}^{[1]}(t) &= \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} k_{11}(\rho_{11}) & k_{12}(\rho_{12}) \\ k_{21}(\rho_{21}) & k_{22}(\rho_{22}) \end{bmatrix} \begin{bmatrix} s(t) \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} G_{11}k_{11}s(t) + G_{12}k_{21}s(t) \\ G_{21}k_{11}s(t) + G_{22}k_{21}s(t) \end{bmatrix} \end{aligned} \tag{3.62}$$

As far as the right-hand side of (3.58) is concerned, we get the following equation for $j = 1$:

$$\begin{aligned} &\sum_{i=1}^{2} k_{i1}\boldsymbol{G}\boldsymbol{r}^{[i]}(t) = \\ &= k_{11} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} s(t) \\ 0 \end{bmatrix} + k_{21} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} 0 \\ s(t) \end{bmatrix} \\ &= \begin{bmatrix} G_{11}k_{11}s(t) + G_{12}k_{21}s(t) \\ G_{21}k_{11}s(t) + G_{22}k_{21}s(t) \end{bmatrix} \end{aligned} \tag{3.63}$$

By comparing equations (3.62) and (3.63) the proof of Lemma 1 is obtained for $j = 1$. As far as $j = 2$ is concerned, the following equation is obtained for the left-hand side of (3.58)

$$\begin{aligned} \boldsymbol{G}\boldsymbol{K}(\rho)\boldsymbol{r}^{[2]}(t) &= \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} k_{11}(\rho_{11}) & k_{12}(\rho_{12}) \\ k_{21}(\rho_{21}) & k_{22}(\rho_{22}) \end{bmatrix} \begin{bmatrix} 0 \\ s(t) \end{bmatrix} \\ &= \begin{bmatrix} G_{11}k_{12}s(t) + G_{12}k_{22}s(t) \\ G_{21}k_{12}s(t) + G_{22}k_{22}s(t) \end{bmatrix} \end{aligned} \tag{3.64}$$

while the right-hand side is given by

$$\begin{aligned} &\sum_{i=1}^{2} k_{i2}\boldsymbol{G}\boldsymbol{r}^{[i]}(t) = \\ &= k_{12} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} s(t) \\ 0 \end{bmatrix} + k_{22} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} 0 \\ s(t) \end{bmatrix} \\ &= \begin{bmatrix} G_{11}k_{12}s(t) + G_{12}k_{22}s(t) \\ G_{21}k_{12}s(t) + G_{22}k_{22}s(t) \end{bmatrix} \end{aligned} \tag{3.65}$$

By comparing equations (3.64) and (3.65) the proof of Lemma 1 is also obtained for $j = 2$.

Appendix 3.B Proof of Result 3.4.2

Consider the discrete-time linear-time invariant (LTI) multi-input multi-output (MIMO) feedback control scheme depicted in Fig. 3.5, where

$$\boldsymbol{G}(q^{-1}) = \begin{bmatrix} G_{11}(q^{-1}) & . \; . \; . & G_{1n}(q^{-1}) \\ . & . & . \\ . & . & . \\ . & . & . \\ G_{n1}(q^{-1}) & . \; . \; . & G_{nn}(q^{-1}) \end{bmatrix} \tag{3.66}$$

$$\boldsymbol{M}(q^{-1}) = \begin{bmatrix} M_{11}(q^{-1}) & . & . & . & M_{1n}(q^{-1}) \\ . & & . & & . \\ . & & & . & . \\ . & & & . & . \\ M_{n1}(q^{-1}) & . & . & . & M_{nn}(q^{-1}) \end{bmatrix} \tag{3.67}$$

$$\boldsymbol{K}(\boldsymbol{\rho}, q^{-1}) = \begin{bmatrix} k_{11}(\rho_{11}, q^{-1}) & . & . & k_{1n}(\rho_{11n}, q^{-1}) \\ . & . & & . \\ . & & . & . \\ . & & . & . \\ k_{n1}(\rho_{n1}, q^{-1}) & . & . & k_{nn}(\rho_{nn}, q^{-1}) \end{bmatrix} \tag{3.68}$$

From Fig. 3.5, we can derive the following equations

$$\boldsymbol{w}(t) = \boldsymbol{G}(q^{-1})\boldsymbol{u}(t) \tag{3.69}$$

$$\boldsymbol{u}(t) = \boldsymbol{K}(\boldsymbol{\rho}, q^{-1})e(t) \tag{3.70}$$

$$\boldsymbol{e}(t) = \boldsymbol{r}(t) - \boldsymbol{w}(t) \tag{3.71}$$

$$\boldsymbol{w}(t) = \boldsymbol{M}(q^{-1})\boldsymbol{r}(t) \tag{3.72}$$

by substituting (3.71) into (3.70)

$$\boldsymbol{u}(t) = \boldsymbol{K}(\boldsymbol{\rho}, q^{-1})[\boldsymbol{r}(t) - \boldsymbol{w}(t)] \tag{3.73}$$

and, by substituting (3.73) into (3.69) we obtain

$$\boldsymbol{w}(t) = [\boldsymbol{I} + \boldsymbol{G}(q^{-1})\boldsymbol{K}(\boldsymbol{\rho}, q^{-1})]^{-1}\boldsymbol{G}(q^{-1})\boldsymbol{K}(\boldsymbol{\rho}, q^{-1})\boldsymbol{r}(t) \tag{3.74}$$

finally, by substituting (3.74) into (3.72)

$$\boldsymbol{M}(q^{-1})[\boldsymbol{I} - \boldsymbol{M}(q^{-1})]^{-1}\boldsymbol{r}(t) = \boldsymbol{G}(q^{-1})\boldsymbol{K}(\boldsymbol{\rho}, q^{-1})\boldsymbol{r}(t) \tag{3.75}$$

Under *Assumption 1*, the following result follows

$$\boldsymbol{L}^{[j]}(t) = \boldsymbol{G}(q^{-1})\boldsymbol{K}(\boldsymbol{\rho}, q^{-1})\boldsymbol{r}^{[j]}(t) \tag{3.76}$$

where,

$$\begin{aligned}\boldsymbol{L}^{[j]}(t) &= \boldsymbol{M}(q^{-1})[\boldsymbol{I}-\boldsymbol{M}(q^{-1})]^{-1}\boldsymbol{r}^{[j]}(t)\\ &= [l_1(t),...,l_n(t)]^T \quad \forall j = 1,...,n\end{aligned} \tag{3.77}$$

Equation (3.76) can be written as

$$\boldsymbol{L}^{[j]}(t) = \begin{bmatrix} G_{11}(q^{-1}) & . & . & . & G_{1n}(q^{-1}) \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ G_{n1}(q^{-1}) & . & . & . & G_{nn}(q^{-1}) \end{bmatrix} \begin{bmatrix} k_{11}(\rho_{11},q^{-1}) & . & . & . & k_{1n}(\rho_{11n},q^{-1}) \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ k_{n1}(\rho_{n1},q^{-1}) & . & . & . & k_{nn}(\rho_{nn},q^{-1}) \end{bmatrix} \boldsymbol{r}^{[j]}$$

$$\boldsymbol{L}^{[j]}(t) = \begin{bmatrix} \sum_i^n G_{1i}(q^{-1})k_{i1}(\rho_{i1},q^{-1}) & . & . & . & \sum_i^n G_{1i}(q^{-1})k_{in}(\rho_{in},q^{-1}) \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ \sum_i^n G_{ni}(q^{-1})k_{i1}(\rho_{i1},q^{-1}) & . & . & . & \sum_i^n G_{ni}(q^{-1})k_{in}(\rho_{in},q^{-1}) \end{bmatrix} \boldsymbol{r}^{[j]} \tag{3.78}$$

Case 1. By considering only the $j-th$ output for the $j-th$ open-loop experiment, $\forall j = 1,...,n$, we have

$$l_j(t) = \sum_{i=1}^{n} G_{ji}(q^{-1})k_{ij}(\rho_{ij},q^{-1})r^{[j]}(t) \tag{3.79}$$

now, substitution of equations (3.58) and (3.47) into (3.79), leads to the following equation

$$l_j(t) = \sum_{i=1}^{n} k_{ij}(\rho_{ij},q^{-1})[y_j^{[i]}(t) - \eta_j^{[i]}(t)] \tag{3.80}$$

where, $y_j^{[i]}$ and $\eta_j^{[i]}(t)$ are the output and the additive noise j for the $i-th$ open-loop experiment respectively. Now, by introducing the partial output $Z(t)$ (see [39] for more details) the following equivalent condition is derived

$$l_j(t) = \sum_{i=1}^{n} Z_{ij}(t) \tag{3.81}$$

where,

$$Z_{ij}(t) = k_{ij}(\rho_{ij})[y_j^{[i]}(t) - \eta_j^{[i]}(t)] \tag{3.82}$$

Case 2. By considering now only the $h-th$ output for the $j-th$ open-loop experiment, $\forall h \neq j, h, j = 1, ..., n$, we have

$$l_h(t) = \sum_{i=1}^{n} G_{hi}(q^{-1}) k_{ij}(\rho_{ij}, q^{-1}) r^{[j]}(t) \tag{3.83}$$

now, substitution equations (3.58) and (3.47) into (3.83), leads to

$$l_h(t) = \sum_{i=1}^{n} k_{ij}(\rho_{ij}, q^{-1})[y_h^{[j]}(t) - \eta_h^{[j]}(t)] \tag{3.84}$$

which is equivalent to:

$$l_h(t) = \sum_{i=1}^{n} Q_{ih}(t) \tag{3.85}$$

where,

$$Q_{ih}(t) = k_{ij}(\rho_{ij})[y_h^{[i]}(t) - \eta_h^{[i]}(t)] \tag{3.86}$$

Appendix 3.C Decoupling constraints

In order to guarantee decoupling controllers for MIMO systems, the reference model should be diagonal; i.e., $M_{ij} = 0, \forall i \neq j$. Therefore, equations (3.80) and (3.84) can be recomputed for the new reference model respectively as follows

$$\sum_{i=1}^{n} k_{ij}(\rho_{ij}, q^{-1})[y_j^{[i]}(t) - \eta_j^{[i]}(t)] = M_{jj}(1 - M_{jj})^{-1} s(t)$$
$$\forall j = 1, ...n \tag{3.87}$$

$$\sum_{i=1}^{n} k_{ij}(\rho_{ij}, q^{-1})[y_h^{[i]}(t) - \eta_h^{[i]}(t)] = 0$$
$$\forall j \neq h, j, h = 1, ...n \tag{3.88}$$

where (3.87) and (3.88) are important constraints in the *feasible controller parameter set* when decoupling controllers is concerned.

Chapter 4

Reference model design

4.1 Introduction

The model reference control design paradigm has been around since at least the 1960s. This paradigm has caught more attention within the Adaptive Control framework, probably because it lends itself naturally to the automatic adjustment of the controller parameters, see e.g., [79], [89] and [109]. In the model reference design framework, the designer is asked to create a transfer function whose behaviour is the one expected from the closed-loop system. This target transfer function is called in general the *Reference Model* [10].

The control design formulation in the Adaptive Control framework has been also called, the *Model Matching Control*. Several contributions have been proposed to solve the model matching control problem by means of tools such as Riccati equations, Linear Matrix Inequalities (LMI's), Bilinear Matrix Inequalities (BMI's), etc. [65]. Provided, of course, that the process model G is known, which is not the case in this chapter.

The model matching control framework has also been applied to the direct data-driven control (DDDC) theory, where experimental data are directly used to design the controller with no resort to the plant identification. For example, Guardabassi and Savaresi in 2000 [67] proposed a virtual reference feedback tuning (VRFT) method. It is a one-shot direct data-driven method where the data-based procedure proposed for designing an LTI controller is based on the concept of a virtual reference signal. An alternative non-iterative approach called Non-iterative Correlation based Tuning (NCbT) method has been proposed in the work of [84]. This controller tuning approach leads to the formulation of a controller identification problem where the input is affected by noise while the output is noiseless. Recently, a non-iterative approach has been proposed in [41], [42], where a novel set-membership based direct data-driven controller (SM-DDDC) design technique is presented and discussed in details in Chapter 3.

In VRFT, NCbT and SM-DDDC, the user is looking for an algorithm to find the controller K that provides a desired input-output relationship to be matched with a given reference model (see Section 3.2.1, 3.2.2 and 3.4). The problem of deriving the reference model for LTI control problems in the framework of direct data-driven control is discussed in papers [10] and [51] for the single-input single-output (SISO) case and the multi-input multi-output (MIMO) case respectively. Although those contributions discuss some important guidelines on the design of an appropriate reference model, they do not provide a systematic approach to select a reference model which accounts for a set of given performance specifications. To the best of the authors' knowledge, there are no works in the literature that systematically address such a problem in the framework of DDDC.

In this chapter, we present an original approach to design, in a systematic way, the reference model M to be able to meet performance specifications. Through the proposed method, the desired performance specifications of the closed-loop system are translated into a model reference design paradigm in the framework of the DDDC approach. The designing of a suitable reference model M is done based on H_∞ control design problem by using a suitable fictitious plant. Then, stability conditions both for stable minimum-phase plant and stable non-minimum phase plant are discussed and analyzed to guarantee the stability of the designed closed-loop system. The main distinctive features of the proposed approach with respect to those already available in the literature are as follows: (i) the reference model M is designed such that the closed-loop system is able to fulfil quantitative performance specifications; (ii) no a-priori information on the NMP zeros location is needed.

The chapter is organized as follows. Fundamental facts on model-reference direct data-driven control design are discussed in sections 4.2. Section 4.3 is devoted to the formulation of the problem, while Section 4.4 presents an H_∞ method to design the reference model M for DDDC approach. Section 4.5 presents the proposed approach to deal with non-minimum phase (NMP) systems. The effectiveness of the presented method is demonstrated in Section 4.6 and 4.7 by means of both simulation examples and experimental results respectively. Concluding remarks end the chapter.

4.2 Basics on model reference DDDC design

In this section, we are briefly reviewing some fundamental facts on model-reference direct data-driven control design approach.

Let us consider the discrete-time linear-time invariant (LTI) single-input single-output (SISO) feedback control scheme depicted in Fig. 4.1, where q^{-1} denotes the standard backward shift operator, $G(q^{-1})$ is a stable plant transfer function, $K(\rho, q^{-1})$ is the controller transfer function, ρ is the vector of controller parameters, and $M(q^{-1})$ is the transfer function of a given suitable reference model that describes the desired behavior of the controlled plant.

The objective of the model reference direct data-driven control approach is to design the transfer function of an LTI controller $K(\rho, q^{-1})$ such that the closed loop transfer function $T_{\tilde{w}r}(q^{-1})$ given by

$$T_{\tilde{w}r}(\rho, q^{-1}) \triangleq \frac{K(\rho, q^{-1})G(q^{-1})}{1 + K(\rho, q^{-1})G(q^{-1})} \tag{4.1}$$

matches, as close as possible, the reference model $M(q^{-1})$, under the assumption that the plant transfer function $G(q^{-1})$ is unknown, and only a set of input-output data, collected by performing suitable experiments on the

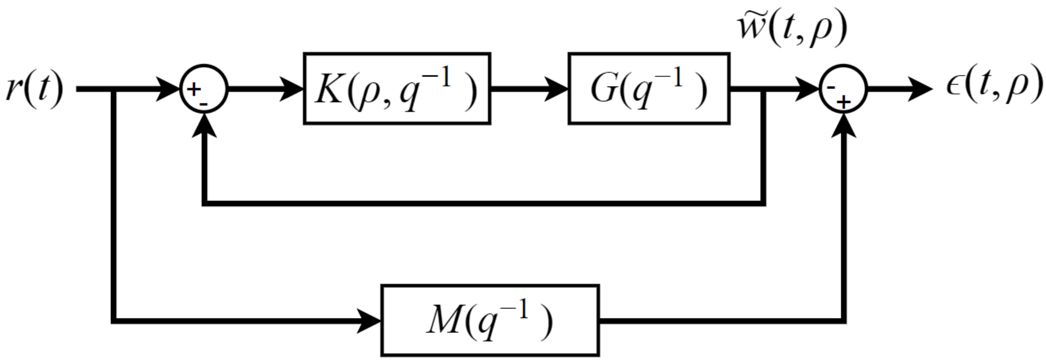

Figure 4.1 Feedback control system to be designed compared with the reference model $M(q^{-1})$.

plant, are available.

More precisely, the objective is to design a controller $K(\rho, q^{-1})$, such that the designed closed-loop resembles the reference model $M(q^{-1})$. This can be achieved, for example, by minimizing the p-norm of the difference between the reference model and the achieved closed-loop system, such as,

$$J(\rho) = \left\| M(q^{-1}) - T_{\tilde{w}r}(\rho, q^{-1}) \right\|_p \tag{4.2}$$

where $p = 1, 2, \infty$.

The controller that exactly solves the model matching problem, in this context is called the *ideal controller* and is defined as follows:

$$K^* \triangleq \frac{M}{G(1-M)}. \tag{4.3}$$

It is worth noting that, in general, the ideal controller K^* is not guaranteed to be physically realizable. Therefore, the objective of any data-driven control approach is, roughly speaking, to estimate, from a set of noisy experimental data, the physically realizable controller K which better approximate K^*, without relying on the knowledge of the plant transfer function G.

To simplify notation, in the rest of the chapter we drop the backward shift operator q^{-1} from equations and corresponding block diagrams.

Remark 4.2.1

We highlight that one of the main ingredients of any model reference based DDDC approach is the reference model M, which describes the desired input-output behavior of the feedback system. To the best of the author's knowledge, there is no method in the literature that systematically addresses the selection of a suitable reference model for the DDDC approach, to account for a set of quantitative performance specifications.

Remark 4.2.2

The problem of properly selecting the reference model M, discussed in the next section, is a common issue of all the DDDC approaches that use the model-matching reference model technique, e.g., VRFT, NCbT, and SM-DDDC tuning approaches. However, in this work we will refer to a specific approach, the SM-DDDC, in order to better present our results.

4.3 Problem formulation

In this section, we formulate the problem of designing the reference model M in the context of H_∞ optimal control to be used in DDDC system design, such that this reference model is able to meet quantitative performances specification on both reference tracking and disturbance rejection.

Let us consider the Single Input Single Output (SISO) feedback control system depicted in Fig. 4.2. In such a structure, K and G are the plant and the controller transfer functions respectively. The reference signal is $r(t)$, $y(t)$ is the controlled output. Additive output (d_p) and sensor (d_s) disturbances are also considered.

The loop, sensitivity, and complementary sensitivity transfer functions are defined, respectively, as

$$L \triangleq KG, \quad (4.4)$$

$$S \triangleq \frac{1}{1+L}, \quad (4.5)$$

$$T \triangleq \frac{L}{1+L}. \quad (4.6)$$

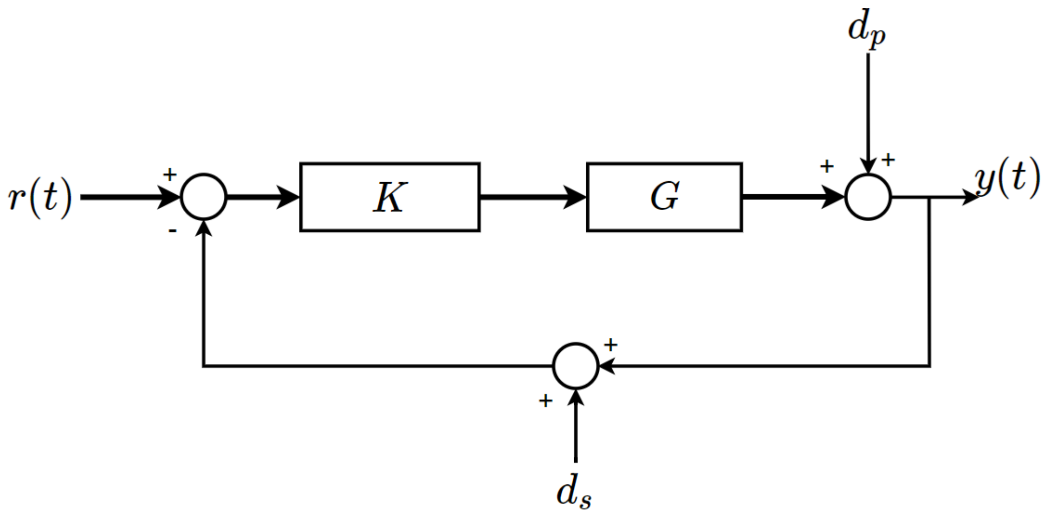

Figure 4.2 General SISO feedback control system.

Common classes of quantitative performance requirements for the control system in Fig. 4.2 accounts for (i) steady-state response to polynomial reference inputs, (ii) steady-state response to polynomial disturbance d_p, (iii) steady-state response to measurement disturbances d_s, (iv) transient step response requirements on overshoot ($\hat{s}$), rise time (t_r) and settling time (t_s).

As is well known, in the context of H_∞ control (see e.g, [15] and [113] for details), such performance requirements can be translated into frequency domain constraints on a weighted H_∞ norm of the sensitivity (S) and complementary sensitivity (T) functions, such as,

$$\|W_T(j\omega)T(j\omega)\|_\infty \leq 1, \tag{4.7}$$

$$\|W_S(j\omega)S(j\omega)\|_\infty \leq 1. \tag{4.8}$$

In the context of DDDC control, the plant transfer function G is not assumed to be known, and a data based controller is directly designed in order to make the input-output behavior of the controlled system to match a given reference model M. Thus the following problem is formulated.

Problem 2. ***[Reference model design in DDDC method]***
The problem addressed in this chapter is to design the reference model M (when the plant transfer function G is unknown) such that the following conditions are satisfied,

$$\|W_T(j\omega)M(j\omega)\|_\infty \leq 1, \tag{4.9}$$

$$\|W_S(j\omega)(1-M)(j\omega)\|_\infty \leq 1. \tag{4.10}$$

4.4 An H_∞ method to design the reference model for DDDC approaches

In this section, we propose an approach based on H_∞ control techniques for choosing the reference model M which solves Problem 2, stated in Section 4.3.

The main idea of this contribution is given in the following result.

Result 4.4.1: Main Result

Assume that the LTI plant is stable and minimum phase. Consider the reference model $\tilde{M}$ given by,

$$\tilde{M} = \tilde{T} = \frac{\tilde{K}\tilde{G}}{1+\tilde{K}\tilde{G}} \tag{4.11}$$

where, $\tilde{K}$ is the controller obtained by solving the following H_∞ feasibility problem

$$\tilde{K} = \arg \min_{\tilde{K} \in \tilde{K}^{stab}} ||T_{WZ}||_\infty \tag{4.12}$$

and, with reference to the block diagram in Fig. 4.3, $\tilde{G}$ is called the *fictitious plant* and its transfer function is set, without loss of generality, to 1, i.e.

$$\tilde{G} = 1, \tag{4.13}$$

$\tilde{K}^{stab}$ is the class of all the controllers which provide internal stability of the nominal feedback system; and T_{WZ} is the closed loop transfer function between the input W and the output Z, i.e.

$$T_{WZ} = \begin{bmatrix} W_T\tilde{T} \\ W_S\tilde{S} \end{bmatrix} \tag{4.14}$$

where

$$\tilde{S} = \frac{1}{1+\tilde{L}}, \; \tilde{T} = 1-\tilde{S}, \; \tilde{L} = \tilde{K}\tilde{G}. \tag{4.15}$$

Then, the model reference $\tilde{M}$ solves problem 2 if

$$||T_{WZ}(\tilde{K})||_\infty \leq 1 \tag{4.16}$$

Proof. The proof follows straightforwardly from the fact that $||T_{WZ}(\tilde{K})||_\infty \leq 1$ is a sufficient condition for both $||W_T\tilde{T}||_\infty \leq 1$ and $||W_S\tilde{S}||_\infty \leq 1$.

□

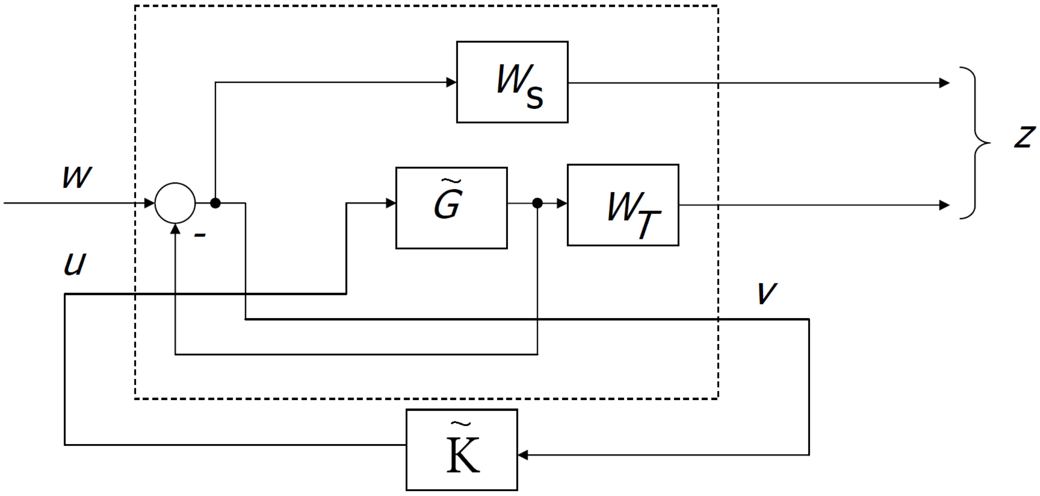

Figure 4.3 Generalized plant for nominal performance.

Remark 4.4.1

It is worth remarking that problem (4.12) is a fictitious control problem where the design of $\tilde{K}$ is instrumental to the computation of the complementary sensitivity function $\tilde{T}$ to be used as reference model $\tilde{M}$. Therefore, the fictitious controller $\tilde{K}$ does not solve the control problem described by constraints (4.7) and (4.8) for the actual plant G. The controller K that solves the actual control problem described by constraints (4.7) and (4.8) is to be designed by applying a DDDC approach on the basis of the computed reference model $\tilde{M}$.

As it is well known, one of the main difficulties of DDDC approaches regards the stability of the closed-loop plant. In model-based approaches, the model of the plant can be used to analyze whether the controller is suitable, before actual implementation. In a data-driven method, since no model is available, stability is typically not guaranteed before the implementation of the controller [152]. Therefore, in the following two sections, we present proper stability conditions such that the reference model designed by equation (4.11) is able to guarantees internal stability for stable and possibly NMP plants.

Remark 4.4.2

In papers [10] and [51] it is pointed out that, apart from the location of the NMP zeros, two additional pieces of information regarding the plant transfer function plays a crucial role in the proper selection of the reference model, namely: (i) an upper bound for the relative degree of the plant and (ii) ballpark values for the plant's dominant time constants. If such a-priori knowledge on the plant is available, it can be accounted for in the procedure proposed in this work, by proper selection of the relative degree and pole location of the fictitious plant. In case such a-priori information is not available, it can be retrieved by means of a suitable data-driven procedure as discussed in [10] and [51].

4.5 DDDC approach for NMP systems

There is no guarantee that a controller determined by model-reference DDDC approach actually stabilizes the plant. Instability can occur if the reference model is designed inappropriately or if the measurements are strongly affected by noise. Thus, in this section, we discussed stability conditions when the plant is a stable and possibly NMP system.

Direct data-driven control approaches do not rely on plant model identification since available input-output data experimentally collected from the plant are directly used to design the controller. The control specifications are usually given in terms of the desired closed-loop map, therefore the controller parameters are computed by formulating the problem in terms of model matching. However, if the unknown plant shows non-minimum phase (NMP) zeros that are not included in the desired reference model, the internal stability of the closed-loop system cannot be guaranteed because the designed controller provides perfect cancellation of the NMP zeros, see e.g., [9] and [146].

When dealing with DDDC for NMP MIMO systems the definition of zeros, or in this case transmission zeros, and their implications on the closed-loop have to be generalized to prevent perfect cancellation and to guarantee internal stability. More precisely, in DDDC MIMO linear time-invariant (LTI) systems, the closed-loop controllers are typically tuned to achieve specific performance and to eliminate process interactions between the output. However, if the system is stable and has NMP transmission zero, then the reference model must have the same transmission zeros of the system to prevent any pole-zero cancellation.

In recent years, several contributions have been proposed to deal with the controller design when the system includes an NMP zero. In Patete et al. [112], a self-tuning control of minimum and non-minimum phase auto-regressive systems with constant but unknown parameters is considered. For direct adaptive control, the solution of including the NMP zeros in the reference model is well documented in the literature (see e.g., [5] and [147]), but this solution requires knowing in advance the NMP zeros. While, for DDDC approach, Lecchini and Gevers in [93] propose a solution for Iterative Feedback Tuning (IFT) (see, e.g., [72]) to overcome the NMP zero problem by introducing what they call a *flexible reference model*. This model has the same poles as the desired reference model, while the parameters of its numerator polynomial are free; therefore, an optimization problem of the numerator for both the reference model and the controller is required. Inspired by the same idea, Campestrini et al. in [21] use the flexible reference model in the context of virtual reference feedback tuning (VRFT) approach (see, e.g., [66]). An extension of this method for multi-input-multi-output (MIMO) plants has been recently discussed in [53].

An alternative solution in the context of VRFT, when a-priori information on the location of the NMP zeros of the plant are available, was introduced in [126] and [52] for SISO and MIMO system respectively. As far as the Correlation based Tuning (CbT) approach proposed by Karimi [84] is concerned, the solution was obtained by imposing a suitable convex stability constraint in order to avoid unstable pole-zero cancellations, as discussed in [152]. The same strategy was also applied to the VRFT procedure in [120] where an interesting comparison is also proposed between different approaches.

In this Section, we present an approach to deal with NMP zero when no a-priori information about the plant and the presence of NMP zeros is available. First, by assuming that the available input-output data are corrupted by bounded noise, we formulate the problem of designing a controller in order to match the behaviour of an assigned reference model in terms of an equivalent set-membership errors-in-variables (SM-EIV) identification problem discussed in Chapter 3. Then, a two-stage procedure, able to detect the presence of NMP zeros in the plant, is proposed for the design of the controller.

4.5.1 DDDC design for SISO NMP plant

Pole-zero cancellation issues may arise when applying model matching design techniques to NMP systems. This section studies a data-driven design methodology to be used in the case of NMP SISO systems.

As stated in the introduction, when the plant has NMP zeros that are not included in the reference model, the model matching controller may lead to instability. In fact, in this case, the ideal controller $K^*(\rho)$ certainly includes in its denominator all the unstable zeros of G not included in the reference model M. However, unstable poles may arise in the ideal model matching controller $K^*(\rho)$ also in the case of minimum phase plant as illustrated in the following simple example.

Illustrative Example 4.1

In this illustrative example we want to highlight the fact that when a controller is designed to perfectly match an assigned input-output stable, minimum-phase reference model M, unstable poles may appear in the controller no matter if the plant is a non-minimum phase or not. Let us consider the following unstable controller,

$$K(s) = \frac{4}{s-3}. \tag{4.17}$$

As can be checked by means of elementary algebra, $K(s)$ in (4.17) leads to perfect matching of the reference model,

$$M_1(s) = \frac{4}{s+9} \tag{4.18}$$

when the plant transfer function is the following,

$$G_1(s) = \frac{(s-3)}{(s+5)} \tag{4.19}$$

giving rise to internal instability of the controlled system, due to perfect unstable pole-zero cancellation.

At the same time, the same controller $K(s)$ in (4.17) lead to perfect matching of the reference model

$$M_2(s) = \frac{8}{s+5} \tag{4.20}$$

when the plant transfer function is the following,

$$G_2(s) = 2. \tag{4.21}$$

This simple example shows that the presence of an unstable pole in the controller providing perfect algebraic matching of an assigned input-output model is not necessarily associated with an unstable pole-zero cancellation. Therefore, in the case the controller $K(\rho)$ designed by means of a direct data-driven approach shows an unstable pole, we must detect if the unstable pole in the controller is due to the presence of an NMP zero in the plant or not.

Now we are in the position to state, in general terms, the problem to be solved in this Section.

Problem 3. ***[Direct Data-driven design problem for NMP systems]*** *The problem addressed in this section is to find the fully parametrized LTI controller transfer function $K(\rho)$ which makes the output matching error signal $\epsilon(t)$ as close as possible to zero, and avoiding perfect cancellation of the unknown unstable zeros (if any).*

To solve problem 3, we introduce the following notion of the *prospective plant.*

Definition 4.5.1: Prospective plant G^*

Given a reference model with transfer function M and a controller with transfer functions K, the *prospective plant* is defined as the plant with transfer function given by,

$$G^* = \frac{M}{K(1-M)}. \tag{4.22}$$

Remark 4.5.1

Given a controller K, the *prospective plant* G^* is such that the closed-loop input-output transfer function of the controlled system depicted in Fig. 4.1 perfectly match the reference model M.

Based on the notion of *prospective plant*, we propose the following two-stage design procedure which allows the user to detect if the designed controller could lead to unstable pole-zero cancellation.

- **Stage 1:(Controller design and detection of unstable pole-zeros cancellation)** In the first stage, the reference model M is designed using H_∞ control method discussed in Section 4.4. Then, a data-driven controller $K(\rho)$ is designed using a set of input-output experimental data collected on the plant and according to the approach proposed in Chapter 3. If the transfer function of the designed controller is stable, then the controller can be implemented since unstable cancellation will not take place (the controller does not have any unstable poles). In the case the designed controller is unstable, the prospective plant G^* defined as,

  $$G^* = \frac{M}{K(1-M)}. \tag{4.23}$$

  is computed. If G^* is a minimum phase system or has an NMP zero which is not close to a controller pole, we know that the designed controller, although unstable, leads to a controlled system perfectly matching the reference model without giving rise to unstable pole-zero cancellations [26]. Therefore, the designed controller can be implemented. On the contrary, if the prospective plant G^* shows NMP zeros close to the unstable poles of $K(\rho)$, the designed controller has to be rejected and a new controller has to be designed according to stage 2.

- **Stage 2:** All the unstable poles of the designed controller $K(\rho)$ are added to the numerator of the fictitious plant $\tilde{G}$ and new reference model M is designed using equation (4.11). Then, the newly designed reference model is used to design a new controller $K(\rho)$ by using the originally collected I/O experimental data.

Remark 4.5.2

It is worth noting that, in case the prospective plant G^* shows the unstable poles of the controller as zeros this suggest that the unstable poles which appeared in the controller in order to match the selected reference model M. Therefore, the selected reference model has to be modified. On the contrary, if G^* doesn't show unstable zeros corresponding to the unstable poles in the designed controller K this suggests that the presence of the unstable poles in the controller was not forced by the selected reference model M. Therefore, we don't need to modify the selected reference model.

4.5.2 DDDC design for MIMO NMP plant

This section studies a data-driven design methodology to be used in the case of NMP MIMO systems.

As stated in the SISO case in Section 4.5.1, when the plant has NMP zeros that are not included in the reference model, the model matching controller may lead to instability. In fact, in this case, the ideal controller $\boldsymbol{K}^*(q^{-1})$ given by

$$\boldsymbol{K}^*(q^{-1}) = \left(\boldsymbol{G}(q^{-1})\left[\boldsymbol{I} - \boldsymbol{M}(q^{-1})\right]\right)^{-1}\boldsymbol{M} \tag{4.24}$$

will certainly include in its denominator all the transmission zeros of $\boldsymbol{G}$ that are not included in the reference model $\boldsymbol{M}$. However, unstable poles may arise in the ideal model matching controller $K^*(\rho)$ also in the case of minimum phase plant as shown in [42]. Moreover, the notion of NMP zero is much more complex for MIMO systems than for the SISO case. In particular in order to discuss internal stability of NMP MIMO systems we need to refer to the notion of *transmission zeros* and their direction.

Definition 4.5.2: Transmission zeros ([132])

z_i is a zero of $\boldsymbol{G}(s)$ if the rank of $\boldsymbol{G}(z_i)$ is less than the normal rank of $\boldsymbol{G}(s)$. The transmission zero polynomial is defined as $z(s) = \Pi_{i=1}^{n_z}(s - z_i)$ where n_z is the number of finite zeros of $G(s)$.

Definition 4.5.3: Transmission Zero direction ([132])

Let $\boldsymbol{G}(s)$ have a zero at $s = z$. Then $\boldsymbol{G}(s)$ loses rank at $s = z$, and there will exist non-zero vectors u_z and y_z such that $G(z)u_z = 0$. y_z. Where, u_z is defined as the input zero direction, and y_z is defined as the output zero direction.

In principle, we may obtain u_z and y_z from an SVD of $\boldsymbol{G}(z) = U\Sigma V^H$; and we have that u_z is the last column in V corresponding to the zero singular value of $\boldsymbol{G}(z)$ and y_z is the last column of U, for more details see [132].

Definition 4.5.4: Internal stability of MIMO feedback system ([68])

With the reference to the block diagram depicted in Fig. 4.1, if $\boldsymbol{G}$ is stable and has NMP transmission zero at z_i with output direction y_{z_i}, then the feedback system with controller K will be internally stable if the following interpolation constraint is satisfied:

$$y_{z_i}^H \boldsymbol{T}_{wr}(z_i) = 0 \tag{4.25}$$

Remark 4.5.3

In words, Definition 4.5.4 says that $\boldsymbol{M}$ must have the same transmission zeros of $\boldsymbol{G}$ in the same output direction. It is important to notice that the constraint is a function of the transmission zeros and has no direct relationship with the zeros of the elements of $\boldsymbol{G}$.

Remark 4.5.4

If the reference model $\boldsymbol{M}$ is diagonal ($M_{ij} = 0, \quad \forall i \neq j$), the NMP transmission zero will be present in each element of $\boldsymbol{M}$. Thus, for a diagonal structure of the reference model one does not need to be concerned with the zero output direction, since the constraint in Definition 4.5.4 will be satisfied because $\boldsymbol{T}_{wr}(z_i) = 0$. For more details see e.g, [52].

To detect and locate the transmission zeros and their output direction we introduce the notion of a prospective plant for MIMO system.

Definition 4.5.5: Prospective plant $\boldsymbol{G}^*(\rho)$ for MIMO systems

Given a reference model with transfer function $\boldsymbol{M}$ and a controller with transfer functions $\boldsymbol{K}(\rho)$, the *prospective plant* is defined as the plant with a transfer function given by

$$\boldsymbol{G}^*(\rho) = \boldsymbol{M}[\boldsymbol{K}(\rho)(I - \boldsymbol{M})]^{-1} \tag{4.26}$$

Based on the notion of the prospective plant, we propose the following two-stage design procedure which allows the user to detect if the designed controller could lead to unstable pole-zero cancellation (see Fig. 4.4 and 4.5).

- **Stage 1**:(Controller design and detection of unstable transmission zero) In the first stage, a data-driven controller $\boldsymbol{K}(\rho)$ is designed using a set of input-output experimental data collected on the plant. If the transfer function of the designed controller is stable, then the controller can be implemented since unstable cancellation will not take place (the controller does not have any unstable poles). In the case the designed controller is unstable, the prospective plant $\boldsymbol{G}^*(\rho)$ is computed. If $\boldsymbol{G}^*(\rho)$ is a minimum phase system (no transmission zeros), we know that the designed controller, although unstable, leads to a controlled system perfectly matching the reference model without giving rise to unstable pole-zero cancellations. Therefore the designed controller can be implemented. On the contrary, if the prospective plant $\boldsymbol{G}^*(\rho)$ shows NMP transmission zeros close to the unstable poles of $\boldsymbol{K}(\rho)$, the designed controller has to be rejected and a new controller has to be designed according to **Stage 2**.

- **Stage 2**:
  - **Diagonal reference model**: all the NMP transmission zeros of the perspective plant $\boldsymbol{G}^*(\rho)$ are added to the numerator of the reference model $\boldsymbol{M}$ as NMP transmission zeros (without the need of the zero output direction). Then, the modified reference model is used to design a new controller $\boldsymbol{K}(\rho)$ by using the originally collected I/O experimental data.

  - **Non-diagonal reference model**: all the NMP transmission zeros of the designed controller $\boldsymbol{K}(\rho)$ and their direction are added to the numerator of the reference model $\boldsymbol{M}$. However, in this case, the NMP transmission zero of the reference model will have an output direction equal to the process (see Definition 4.5.4), but its input direction will be different of the process zero input direction. Therefore, the effect of the NMP transmission zero should be moved to a specific output by using e.g., block triangular structure (for more details see [52]). Note that, the decision of how to move the NMP zero to the right output is out of the scope of this report.

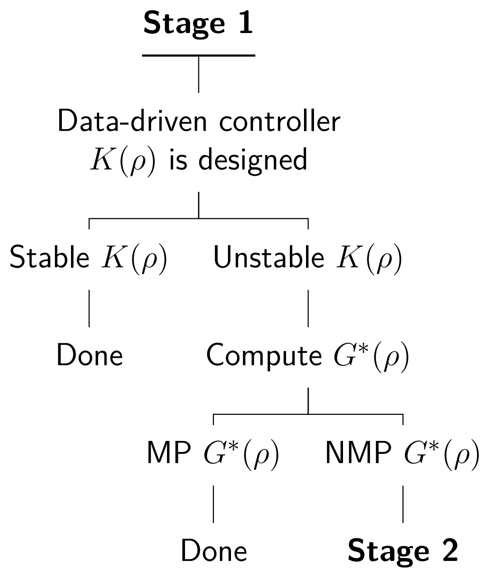

Figure 4.4 Stage 1 design for MIMO NMP systems.

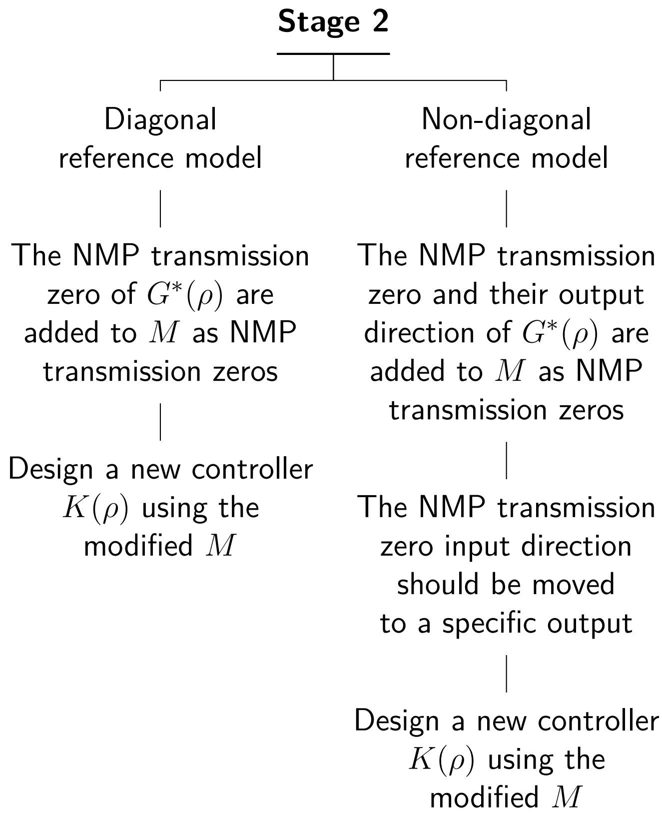

Figure 4.5 Stage 2 design for MIMO NMP systems.

4.6 Simulation examples

In this section, the effectiveness of the presented approach is shown by means of simulation examples.

4.6.1 Example 1. DDDC design for SISO NMP system

In this example, the proposed approach is employed to tune an NMP SISO controller to be compared with the standard NCbT introduced in [152] and the VRFT approach (see e.g., [60]) when suitable convex stability constraint is imposed (see e.g., [152]). The plant considered in this example is taken from [120] and has the following transfer function,

$$G(s) = \frac{s-0.5}{s^2+2s+1}$$

while the assigned reference model has a transfer function

$$M(s) = \frac{1}{s^2+1.1s+1}$$

The system is excited by a random input signal $r(t)$ uniformly distributed in the range $[-1,+1]$. The plant output signal $w(t)$ is corrupted by a random additive noise $\eta(t)$, uniformly distributed in the range $[-\Delta\eta,+\Delta\eta]$. The chosen error bound $\Delta\eta$ is such that the signal to noise ratio is $26dB$.

Input-output samples are collected with a sampling time $T_s = 0.1s$ and, discrete-time models of $G(s)$ and $M(s)$ are obtained, for simulation purposes, through ZOH discretization method, according to [120].

In this example the controller $K(\rho)$ is obtained by computing the central estimate $\rho_j^c = (\underline{\rho}_j^c + \overline{\rho}_j^c)/2$ through the convex relaxation approach proposed in [36] for a relaxation order $\delta = 2$. The general LTI controller structure in (3.39) is considered here where $n_a = n_b = n$ and the controller order $n = 2$ is selected by trial and error starting from $n = 1$ and by increasing n until the feasible set is not empty. The final selected controller structure is as follows,

$$K(q^{-1}) = \frac{b_0 + b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}}$$

where, by taking the central estimate of the controller parameters, the following controller transfer function is obtained

$$K(q^{-1}) = \frac{0.054807(1+0.9527q^{-1})(1-0.9189q^{-1})}{(1-1.0515q^{-1})(1-q^{-1})}$$

Since the obtained controller has an unstable pole, according to the two-stage procedure presented in Section 3, we have to compute the prospective plant G^* in equation (4.23),

$$G^*(q^{-1}) = \frac{0.087901(1+0.964q^{-1})(1-1.0515q^{-1})(1-0.2495q^{-1})}{(1+0.9527q^{-1})(1-0.9189q^{-1})(1-0.8912q^{-1})(1-0.2384q^{-1})}.$$

Since G^* has one NMP zero, we can conclude that the unknown plant G is an NMP system and the NMP zero of G^* (i.e. the unstable pole in K) is expected to be a good estimate of the NMP zero of G. Therefore the unstable pole of $K(\rho)$ is added to the reference model M as an NMP zero. Finally, the problem is solved by exploiting the modified reference model with a controller order $n = 3$. The final obtained controller transfer function (corresponding to the central estimate of the controller parameters) is the following

$$K(q^{-1}) = \frac{-1.0638(1+0.9633q^{-1})(1-1.809q^{-1}+0.8186q^{-2})}{(1-q^{-1})(1-0.6435q^{-1})(1-0.1474q^{-1})}$$

As for VRFT and NCbT, a PRBS signal of 255 samples with unity amplitude is used as input to the system. Four periods of this signal are used to design the controller, $N = 1020$. The periodic output is disturbed by a zero-mean white noise such that the signal-to-noise ratio is about $26dB$. The length of the instrumental variable l and the rectangular window l_2 (used for computing the H_∞ norm estimate for the stability constraint, for more details see e.g., [152]) are found to be 10 and 120 respectively by trial-and-error.

Fig. 4.6 displays the comparison between the reference model and the obtained closed-loop system in terms of step responses respectively for SM-EIV, VRFT and NCbT methods. From the comparison, we see that the controlled system obtained with the proposed method performs better than the NCbT and VRFT methods.

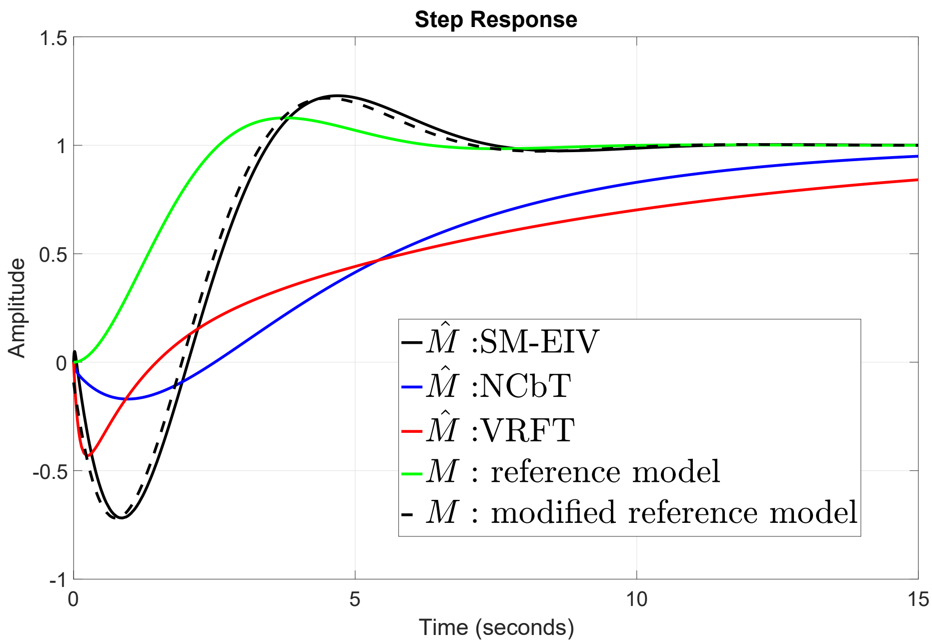

Figure 4.6 Step responses: designed feedback control system with the SM-EIV approach (black solid-line), the NCbT method (blue-line), the VRFT method (red-line), reference model (green-line) and the modified reference model obtained (black dashed-line).

4.6.2 Example 2. DDDC design for MIMO NMP system

In this simulation example, the DDDC approach for NMP system in the framework of SM-EIV is tested in simulation on the system proposed by [52].

The plant transfer function used for generating the data is given by

$$\boldsymbol{G}(q^{-1}) = \begin{bmatrix} \frac{1}{(1-0.9q^{-1})(1-0.8q^{-1})} & \frac{0.6q^{-1}}{(1-0.9q^{-1})} \\ \frac{q^{-1}}{(1-0.9q^{-1})} & \frac{0.2q^{-1}}{(1-0.9q^{-1})} \end{bmatrix}$$

while the assigned reference model is

$$\boldsymbol{M}(q^{-1}) = \begin{bmatrix} \frac{0.08q^{-1}}{1-1.4q^{-1}+0.48q^{-2}} & 0 \\ 0 & \frac{0.08q^{-1}}{1-1.4q^{-1}+0.48q^{-2}} \end{bmatrix}$$

The plant has a transmission zero: $z_i = 1.2$ and its output direction: $y_{z_i} = [-0.316 \quad 0.948]^T$. It is worth noting that, no a priori information about the plant is considered for this example for the SM-EIV method.

The system has been excited by two inputs defined as $\boldsymbol{r}^{[1]}(t) = [s(t), 0]^T$ and $\boldsymbol{r}^{[2]}(t) = [0, s(t)]^T$, where $s(t)$ is a random signal uniformly distributed in the range $[-1, +1]$ with length $N = 80$. The plant output sequences $\boldsymbol{w}(t)^{[1]}$ and $\boldsymbol{w}(t)^{[2]}$ are corrupted by random additive noise $\boldsymbol{\eta}(t)$, uniformly distributed in the range $[-\Delta\eta, +\Delta\eta]$. The chosen error bound $\Delta\eta$ is such that the signal to noise ratio is given by

$$SNR_w = 10log\frac{\sum_{t=1}^{N} w_t^2}{\sum_{t=1}^{N} \eta_t^2} \cong 24dB$$

Input-output samples are collected with a sampling time $T_s = 0.1s$. The parameters are estimated by solving problem (2.4) according to the method presented in *Chapter 2*. The software SparsePop ([159]) with a relaxation order $\delta = 2$ has been used to convert the identification problem (2.4) into a corresponding SDP relaxed problem, solved numerically by the solver MOSEK ([1]).

The general LTI controller structure in (3.39) is considered here where $n_a = n_b = n$ and the controller order $n = 3$ is selected by trial and error starting from

$n = 1$ and by increasing n until the feasible set is not empty. The final selected controller structure is as follows,

$$\boldsymbol{K}(q^{-1}) = \begin{bmatrix} \dfrac{b_0^{11} + b_1^{11}q^{-1} + b_2^{11}q^{-2} + b_3^{11}q^{-3}}{1 + a_1q^{-1} + a_2q^{-2} + a_3^{11}q^{-3}} & \dfrac{b_0^{12} + b_1^{12}q^{-1} + b_2^{12}q^{-2} + b_3^{12}q^{-3}}{1 + a_1q^{-1} + a_2q^{-2} + a_3q^{-3}} \\ \\ \dfrac{b_0^{21} + b_1^{21}q^{-1} + b_2^{21}q^{-2} + b_3^{21}q^{-3}}{1 + a_1q^{-1} + a_2q^{-2} + a_3q^{-3}} & \dfrac{b_0^{22} + b_1^{22}q^{-1} + b_2^{22}q^{-2} + b_3^{22}q^{-3}}{1 + a_1q^{-1} + a_2q^{-2} + a_3q^{-3}} \end{bmatrix}$$

where by taking the central estimate of the controller parameters the following controller transfer function is obtained

$$\boldsymbol{K}(q^{-1}) = \begin{bmatrix} \dfrac{0.2353 - 0.6753q^{-1} + 0.6374q^{-2} - 0.1982q^{-3}}{1 - 2.129q^{-1} + 1.045q^{-2} + 0.08471q^{-3}} & \dfrac{-0.7059 + 2.026q^{-1} - 1.912q^{-2} + 0.5946q^{-3}}{1 - 2.129q^{-1} + 1.045q^{-2} + 0.08471q^{-3}} \\ \\ \dfrac{-1.176 + 3.376q^{-1} - 3.187q^{-2} + 0.9911q^{-3}}{1 - 2.129q^{-1} + 1.045q^{-2} + 0.08471q^{-3}} & \dfrac{1.176 - 2.435q^{-1} + 1.239q^{-2} + 1.82e - 09q^{-3}}{1 - 2.129q^{-1} + 1.045q^{-2} + 0.08471q^{-3}} \end{bmatrix}$$

Since the obtained controller has unstable poles, according to the two-stage procedure presented in Section 3, we have to compute the prospective plant G^* in equation (4.26)

$$\boldsymbol{G}^*(q^{-1}) = \begin{bmatrix} \dfrac{q^{-1}}{1 - 1.71q^{-1} + 0.73q^{-2}} & \dfrac{q^{-1}}{1 - 0.92q^{-1}} \\ \dfrac{1.02q^{-1}}{(1 - 0.92q^{-1})} & \dfrac{0.23q^{-1}}{(1 - 9.1q^{-1})} \end{bmatrix}$$

Since $\boldsymbol{G}^*$ has NMP transmission zero ($z_i = 1.205$), we can conclude that the unknown plant $\boldsymbol{G}$ is an NMP system and the NMP transmission zero of $\boldsymbol{G}^*$ (i.e. the unstable pole in $\boldsymbol{K}$) is expected to be a good estimate of the NMP transformation zero of $\boldsymbol{G}$. Therefore the unstable poles of $\boldsymbol{K}(\rho)$ are added to the reference model $\boldsymbol{M}$ as an NMP transmission zero. Finally, the problem is solved by exploiting the modified reference model with a controller order $n = 2$. The final obtained controller transfer function (corresponding to the central estimate of the controller parameters) is the following,

$$\boldsymbol{K}(q^{-1}) = \begin{bmatrix} \dfrac{0.2279 - 0.3855q^{-1} + 0.1627q^{-2}}{1 - 0.9262q^{-1} - 0.07376q^{-2}} & \dfrac{-0.695 + 1.172q^{-1} - 0.4935q^{-2}}{1 - 0.9262q^{-1} - 0.07376q^{-2}} \\ \\ \dfrac{-1.157 + 1.951q^{-1} - 0.8214q^{-2}}{1 - 0.9262q^{-1} - 0.07376q^{-2}} & \dfrac{1.149 - 1.021q^{-1} + 0.004273q^{-2}}{1 - 0.9262q^{-1} - 0.07376q^{-2}} \end{bmatrix}$$

A comparison between the obtained closed-loop system for the SM-EIV method proposed in this chapter and the VRFT method proposed in [52], in terms of the step response is presented in Fig. 4.7.

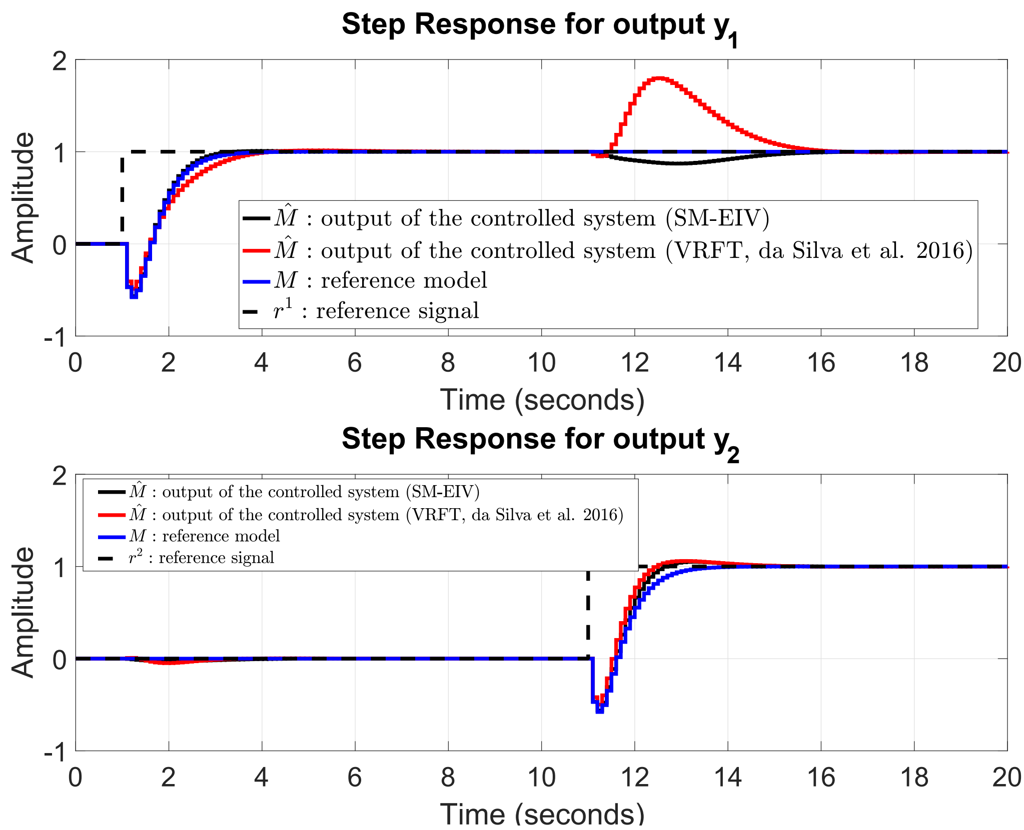

Figure 4.7 Step responses: designed feedback control system with the SM-EIV approach (black solid-line), the VRFT method (red-line), reference model (blue-line) and reference signals (black dashed-line).

As can be seen from Fig. 4.7, the proposed approach (SM-EIV) provides a good decoupling (when no a priori information on the NMP transmission zero is available), while the VRFT method (proposed in [52]) for MIMO NMP system presents a high coupling, even by using the actual zero location in the reference model.

4.6.3 Example 3. Reference model design (MP)

In this simulation example, a plant described by the following LTI stable minimum phase transfer function is considered:

$$G(s) = \frac{12.5}{(1+s/0.8)(1+s/2.5)}. \tag{4.27}$$

Additive output and sensor disturbances are defined, respectively, as

$$\begin{aligned} d_p(t) &= a_p \sin(\omega_p t),\ |a_p| \leq 2 \cdot 10^{-2}, \omega_p \leq 0.02 \text{ rad s}^{-1} \\ d_s(t) &= a_s \sin(\omega_s t),\ |a_s| 10^{-1}, \omega_s \geq 40 \text{ rad s}^{-1} \end{aligned} \tag{4.28}$$

The objective is to design a controller K, through the set-membership direct-data driven control approach, such that the designed control system meets the following requirements: (i) steady-state output error when the reference is a unity ramp $|e_r^\infty| \leq 1.5 \cdot\ 10^{-1}$, (ii) steady-state output error in the presence of d_p $|e_{d_p}^\infty| \leq 5 \cdot\ 10^{-4}$, (iii) steady-state output error in the presence of d_s $|e_{d_s}^\infty| \leq 5 \cdot\ 10^{-4}$, (iv) step response overshoot $\hat{s} \leq 10\%$, (v) rise time $t_r \leq 3s$, (vi) settling time $t_{s,5\%} \leq 12s$.

In the controller design procedure, the plant transfer function G is assumed to be unknown, although input-output data can be collected by performing suitable experiments on the plant.

In this example, input-output data are collected by simulating the transfer function (4.27) with a sampling time $T_s = 0.5$s. The system is excited by a random input signal $r(t)$ uniformly distributed in the range $[-1, +1]$. The plant output signal $w(t)$ is corrupted by a random additive noise $\eta(t)$, uniformly distributed in the range $[-\Delta\eta, +\Delta\eta]$. The chosen error bound $\Delta\eta$ is such that the signal to noise ratio is given by,

$$SNR_w = 10log\frac{\sum_{t=1}^{N} w_t^2}{\sum_{t=1}^{N} \eta_t^2} \cong 28dB.$$

The time-domain performance specifications are converted into corresponding frequency domain constraints (4.7) and (4.8) (see, e.g., [175]) by choosing, for example:

$$W_T(s) = \frac{810.2s^2 + 3889s + 4667}{s^2 + 140s + 4900} \tag{4.29}$$

and

$$W_S(s) = \frac{0.7353s^2 + 0.2002s + 0.02727}{s^2 + 0.015s} \tag{4.30}$$

First, according to Result 4.4.1, an H_∞ problem with $\tilde{G} = 1$ is solved and the following reference model M is obtained:

$$M(s) = \frac{4.59s + 0.2527}{s^3 + 5.481s^2 + 4.601s + 0.2527}$$

Then, based on a discretized version $M(z)$ of the reference model $M(s)$, a data-driven controller $K(\rho)$ is obtained by applying the SM-DDDC approach. The obtained discrete-time data driven controller is described by the following transfer function:

$$K(q^{-1}) = \frac{0.1963 - 0.3693q^{-1} + 0.204q^{-2} - 0.0297q^{-3}}{1 - 1.439q^{-1} - 0.1211q^{-2} + 0.5597q^{-3}}.$$

Comparison between the continuous time reference model $M(s)$, the discretized one $M(z)$ and the output of the closed-loop system are presented in Fig. 4.8. While, the steady state error output are shown in Fig. 4.9. From Fig. 4.8 and Fig. 4.9, we see the obtained performance of the designed control system: (i) $\hat{s} = 5.52\% \leq 10\%$, (ii) $t_r = 1.85s \leq 3s$, (iii) $t_{s,5\%} = 8.87s \leq 12s$, (iv) $|e_r^\infty| = 0.053 \leq 0.15$, (v) $|e_{d_p}^\infty| = 1.775 \cdot 10^{-4} \leq 5 \cdot 10^{-4}$, (vi) $|e_{d_s}^\infty| = 2.116 \cdot 10^{-4} \leq 5 \cdot 10^{-4}$.

From the previous results we can conclude that, the closed loop control system obtained with the reference model designed by means of the proposed H_∞ method, fulfill both steady-state and transient-time response requirements.

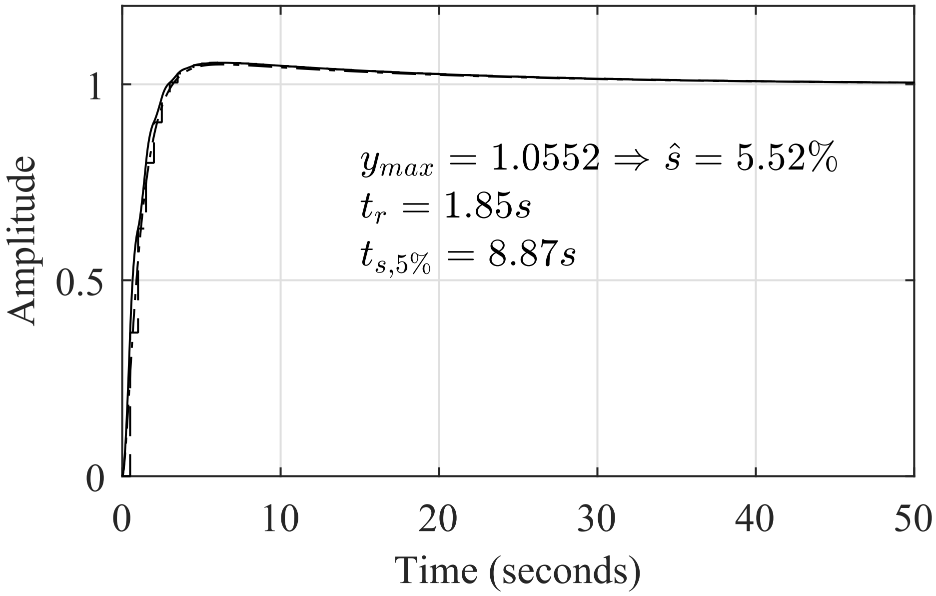

Figure 4.8 Comparison of step responses: designed feedback control system (solid), the continuous time reference model (dashed-dotted) and the discrete reference model (dashed).

4.6.4 Example 4. Reference model design (NMP)

In this example, the proposed approach is employed to tune a controller for a SISO NMP plant. A comparison with the standard approach proposed in [41] is also presented, in order to highlight the advantages of the reference model selection proposed in this work.

We consider the feedback system shown in Fig. 4.2, where the plant is described by

$$G(s) = \frac{(s+9.925)(s-1.818)}{(s+12.04)(s+2.231)} \tag{4.31}$$

and the output and sensors disturbances are

$$\begin{aligned} d_p(t) &= a_p \sin(w_p t),\ |a_p| \leq 2\cdot\ 10^{-2},\ w_p \leq 0.02 \text{ rad s}^{-1} \\ d_s(t) &= a_s \sin(w_s t),\ |a_s| \leq 10^{-1},\ w_s \geq 40 \text{ rad s}^{-1} \end{aligned} \tag{4.32}$$

In this example we define the following time requirements: (i) steady-state output error when the reference is a ramp $|e_r^\infty| \leq 0.5$, (ii) steady-state output

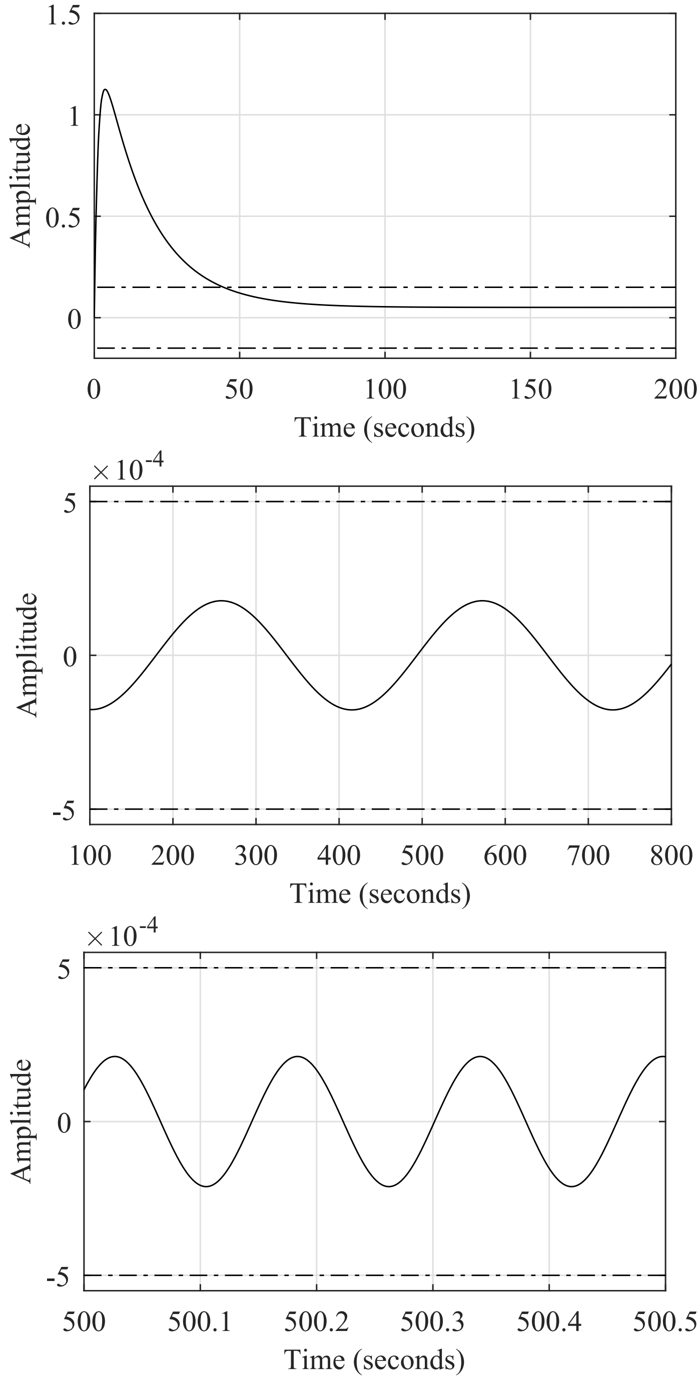

Figure 4.9 (Top) Output error when the reference is a ramp, (Middle) Steady-state output error in the presence of the sinusoidal disturbance d_p on the output, (Bottom) Steady-state output error in the presence of the sinusoidal disturbance d_s on the sensor.

error in the presence of d_p $|e^{\infty}_{d_p}| \leq 6 \cdot\ 10^{-4}$, (iii) steady-state output error in the presence of d_s $|e^{\infty}_{d_s}| \leq 8 \cdot\ 10^{-3}$, (iv) step response overshoot $\hat{s} \leq 11\%$, (v) rise time $t_r \leq 2s$, (vi) settling time $t_{s,5\%} \leq 8s$.

It is worth noting that the plant (4.31) has a NMP zero at $s = 1.818$. However, due to the considered control design technique, no a-priori information on the plant is exploited.

Input-output samples are collected by simulating (4.31) with a sampling time $T_s = 0.1$s. The system is excited by a random input signal $r(t)$ uniformly distributed in the range $[-1, +1]$. The plant output signal $w(t)$ is corrupted by a random additive noise $\eta(t)$, uniformly distributed in the range $[-\Delta\eta, +\Delta\eta]$. The chosen error bound $\Delta\eta$ is such that the signal to noise ratio is given by

$$SNR_w = 10log\frac{\sum_{t=1}^{N} w_t^2}{\sum_{t=1}^{N} \eta_t^2} \cong 28dB.$$

The time-domain performance specifications are converted into corresponding frequency domain constraints (4.7) and (4.8) by choosing, e.g.:

$$W_T(s) = \frac{1.51e05s^2 + 7.246e05s + 8.696e05}{s^2 + 2000s + 1e06} \tag{4.33}$$

and

$$W_S(s) = \frac{0.6899s^2 + 0.3715s + 0.1}{s^2 + 0.02001s} \tag{4.34}$$

First, a reference model $M(s)$ is obtained by applying Result 4.4.1 (i.e. by solving a fictitious H_∞ problem with the fictitious plant $\tilde{G}(s) = 1$). The obtained reference model transfer function $M(s)$ is the following,

$$M(s) = \frac{0.06612(s+97.93)(s+0.2087)}{(s+3.938)(s+1.317)(s+0.2606)}$$

Then, based on a discretized version $M(z)$ of the the computed reference model $M(s)$, a data-driven controller $K(\rho)$ is obtained by applying the SM-DDDC approach. The obtained discrete-time data driven controller is described by the following transfer function:

$$K(q^{-1}) = \frac{0.036784(1-0.7684q^{-1})(1+0.2625q^{-1})(1+0.1462q^{-1})}{(1-1.209q^{-1})(1-q^{-1})(1-0.5849q^{-1})}.$$

Since the obtained controller has an unstable pole we have to compute the prospective plant G^* according to equation (4.26).

$$G^*(z) = \frac{0.87808q^{-1}(1+0.5486q^{-1})(1-1.209q^{-1})}{(1-0.7684q^{-1})(1+0.2625q^{-1})(1+0.1462q^{-1})}.$$

Since G^* has one NMP zero, the model reference M has to be modified in order to account for the NMP nature of the plant. In this example, we will compare two different approaches as far as the modification of the reference model is concerned. The first approach is based on the standard modification proposed in reference [42], while the second approach is based on the methodology presented in this work and, more precisely, the two-stage procedure presented in Section 4.5.1.

Approach (1) - In this approach, referred to as Standard Reference Model Choice (SRMC), the reference model is chosen through standard techniques in DDDC methods for NMP system, where the NMP zeros of the plant (which are assumed to be known) are simply added to the original reference model M as NMP zeros (see e.g., [41], [93] and [21]). In this way we get the following modified reference model:

$$M_{1_{mod}}(q^{-1}) = \frac{0.006612(1+6.554q^{-1})(1-1.209q^{-1})}{(1-0.8766q^{-1})(1-0.6745q^{-1})}$$

Approach (2) - In this case, referred to as H_∞ Reference Model Choice (H_∞RMC), the modified reference model is obtained by solving the H_∞ control problem of Result 4.4.1 with the following fictitious plant

$$\tilde{G}(s) = \frac{(s-1.821)}{s} \tag{4.35}$$

which, according to the two-stage procedure presented in Section 4.4, includes the NMP zero of the prospective plant G^* and one pole at the origin. The obtained modified reference model is

$$M_{2_{mod}}(s) = \frac{-12.098(s+96.17)(s-1.821)(s+0.1141)}{(s+1.509)(s+0.1322)(s^2+9.376s+23.81)}.$$

Then, the controllers are designed by means of the SM-DDDC approach for both the reference models $M_{1_{mod}}$ and $M_{2_{mod}}(s)$.

A comparison between the obtained closed-loop system for the SM-DDDC method using SRMC and H_∞RMC is presented in Fig. 4.10, from which we see that the step responses for the control system designed by means of the two different approaches are almost the same. However, by extending the comparison to all the assigned performance specifications, we get the results reported in Fig. 4.11 and Table 4.1.

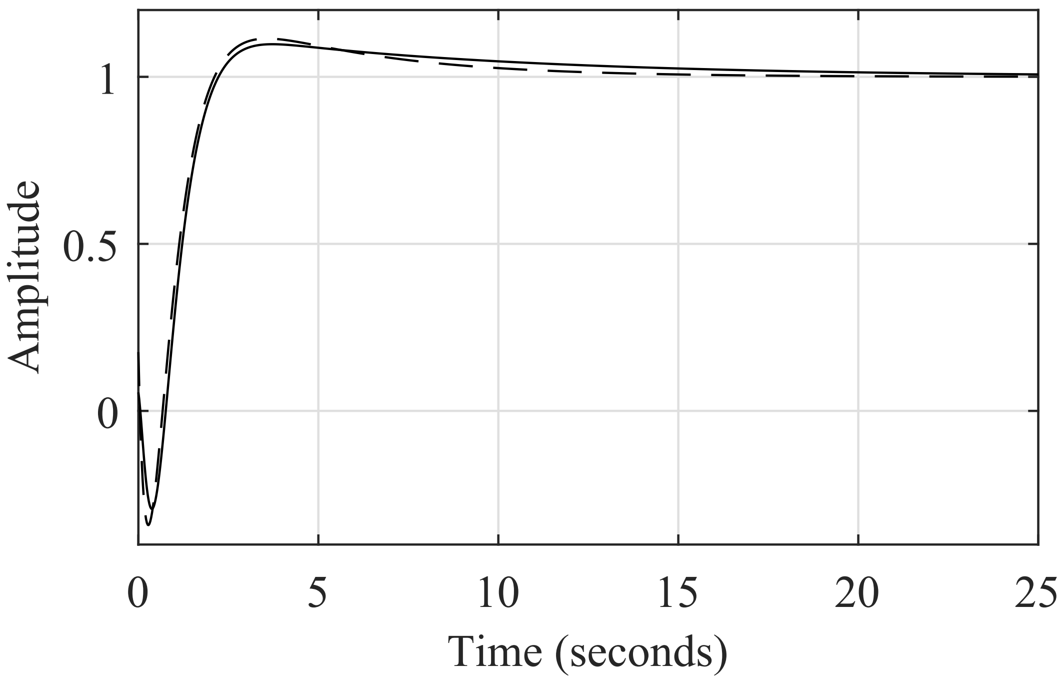

Figure 4.10 Comparison of step responses: designed feedback control system using SRMC approach (dashed) and the designed feedback control system using H_∞RMC approach (solid).

Table 4.1 Comparison of performance specifications obtained using the two approaches and the upper bound

Performance Specifications	Upper-limit Value	SRMC	H_∞RMC
Steady-state output error when the reference is a ramp	0.5	**0.7**	0.495
Steady-state output error in the presence of d_p	$6 \, . \, 10^{-4}$	$2.84 \, . \, 10^{-4}$	$2.22 \, . \, 10^{-4}$
Steady-state output error in the presence of d_s	$8 \, . \, 10^{-3}$	**0.0196**	$6.83 \, . \, 10^{-3}$
Step response overshoot	11%	**11.5%**	9.77%
Rise time	2 (s)	0.939 (s)	0.988 (s)
Settling time	8 (s)	6.36 (s)	7.33 (s)

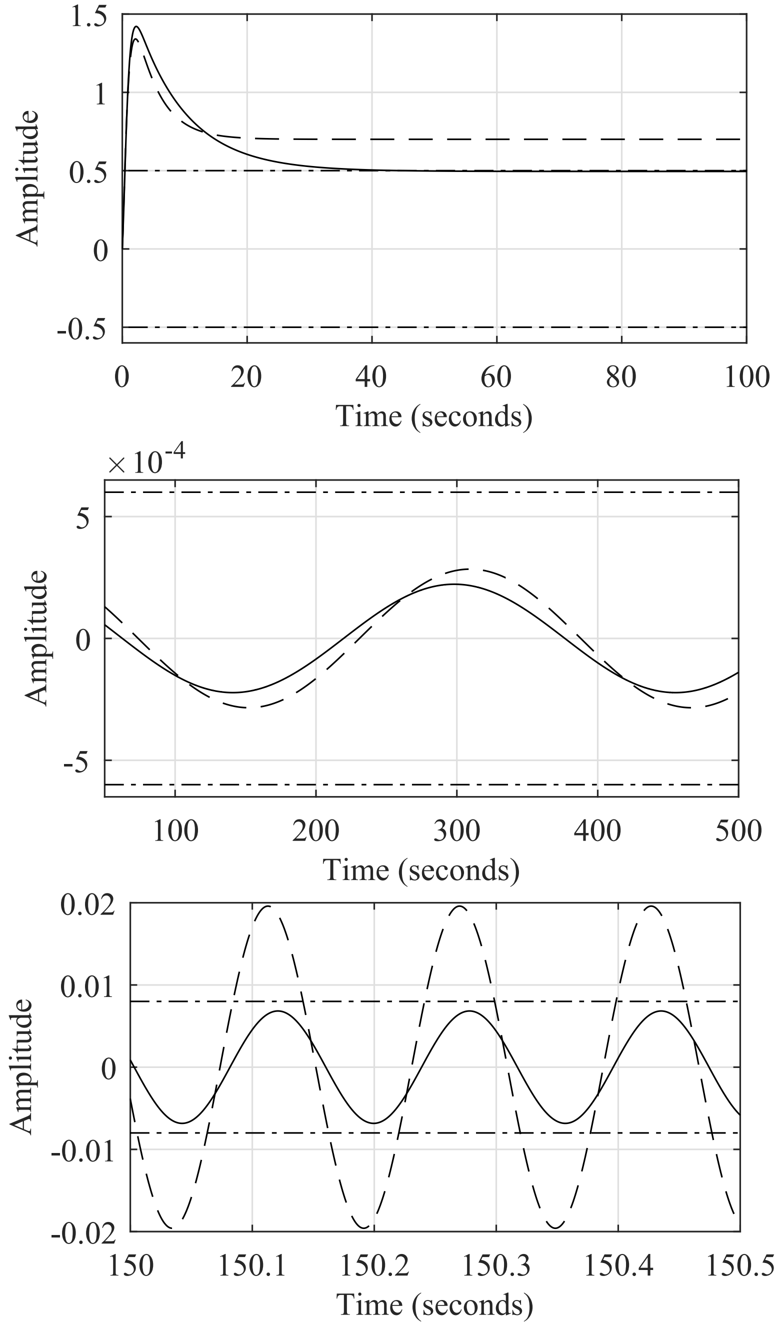

Figure 4.11 (Top) Output error when the reference is a ramp, (Middle) Steady-state output error in the presence of the sinusoidal disturbance d_p on the output, (Bottom) Steady-state output error in the presence of the sinusoidal disturbance d_s on the sensor.

4.7 Experimental results

The algorithm presented in Section 4.5.1 has also been tested on the experimental input-output data collected on a benchmark NMP SISO electronic filter proposed by [145].

Schematic diagram of the NMP SISO circuit is reported in Fig. 4.12, where the nominal values of parameters are $R_1 = 150\Omega$, $R_2 = 330\Omega$, $R_3 = 820\Omega$, $C_1 = 330\mu F$ and $C_2 = 200\mu F$. The transfer function of this NMP circuit is obtained as follows,

$$G(s) = \frac{V_o(s)}{V_I(s)} = \frac{(C_1R_1s-1)}{-(1+C_1R_1s)(1+C_2R_3s)}$$

The system has an RHP zero at $s = 1/(R_1C_1)$. We point out that, in this example, we do not assume any a-priori information on the plant G.

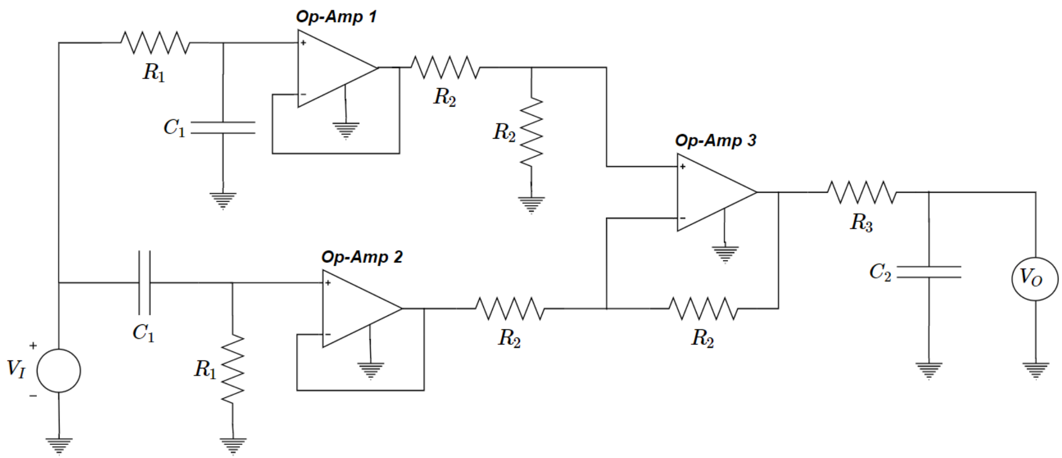

Figure 4.12 Schematic diagram of the non-minimum phase SISO circuit, proposed by [145].

The system has been excited with a random input signal $r(t)$ of 300 samples, uniformly distributed within the range $[-1,+1]$V. A National Instruments PXI equipped with a NI–6221 DAQ board has been used to generate the input signal $r(t)$ and to collect the output signal $\boldsymbol{y}(t)$ at a sample rate of 2kHz. The upper bound on the measurement errors is derived from the precision of the measurement equipment which is given by $\Delta\eta = 0.005$V. The software

SparsePop and MOSEK have been used to solve the underlying optimization problems. The central estimate of the controller parameters for the SM-EIV method (ρ^c_{SM-EIV}) has been obtained by exploiting the convex relaxation approach proposed in [36] for a relaxation order $\delta = 2$.

The reference model, which has been chosen with no assumptions on the presence of an NMP zero, has the following transfer function

$$M(z) = \frac{0.01392z - 0.01272}{(z^2 - 1.935z + 0.9362)}$$

Application of the SM-EIV approach discussed in Section 2, leads to the computation of the following controller (corresponding to the central estimate of the controller parameters)

$$K(z) = \frac{-0.53718(1-0.9742z^{-1})(1-0.9138z^{-1})(1-0.8848z^{-1})}{(1-1.114z^{-1})(1-z^{-1})(1-0.9489z^{-1})}$$

As for the previous example, the obtained controller shows an unstable pole; therefore, a second stage is required in order to detect the presence of a possible NMP zero, modify the reference model and design a new controller. The final obtained central-estimate controller transfer function is the following

$$K(z) = \frac{4.7121(1-0.9742z^{-1})(1-0.9138z^{-1})(1-0.8848z^{-1})}{(1-z^{-1})(1-0.9445z^{-1})(1+0.1316z^{-1})}$$

A comparison between the reference model and the obtained closed-loop system for the SM-EIV method (by setting the value of the parameter to the central estimate $\boldsymbol{\rho}^c_{SM-EIV}$) and the modified reference model M in terms of the step response is presented in Fig. 4.13, from which we see that the output of the controlled response for the SM-EIV approach overlaps the modified reference model.

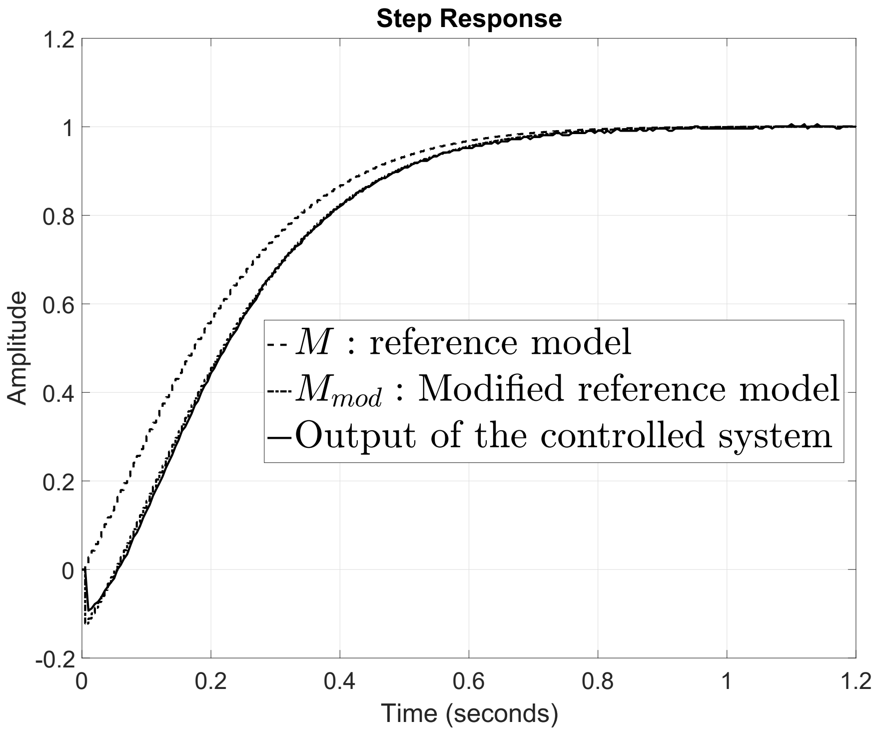

Figure 4.13 Step responses: designed feedback control system with the SM-EIV approach (solid), reference model (dashed) and the modified reference model obtained (dashed-dotted). Notice that, dashed-dotted and solid lines are overlapped.

4.8 Discussion and conclusion

In this chapter, the problem of designing a suitable reference model M for non-iterative direct data-driven control techniques is investigated using the H_∞ method. First, an H_∞ controller $\tilde{K}$ is designed to meet different performance requirements, by using what is called a fictitious plant $\tilde{G}(s)$. Then, the reference model M is taken as the fictitious complementary sensitivity function $\tilde{T}$ obtained by using both $\tilde{K}$ and $\tilde{G}$. On the basis of the designed reference model, a DDDC controller that satisfies the performance specifications is designed using collected I/O data.

Stability conditions have been also discussed for stable and possibly NMP system to guarantee the internal stability of the closed-loop system. In particular, no a-priori information on the location of possible NMP transmission zeros is needed and a two-stage design procedure able to avoid unstable pole-zero cancellations is proposed. First, the controller is designed, on the basis of the assigned reference model and a set of input-output data, by exploiting the set-membership based approach proposed in Chapter 3. Then, on the basis of the designed controller, the presence of possible unstable pole-zero cancellations is detected and the reference model is modified in order to avoid such cancellations. On the basis of the modified reference model, a new controller that guarantees internal stability is designed. At each stage, the problem of computing the controller parameters is formulated in terms of set-membership errors-in-variables identification and solved by means of suitable convex relaxations discussed in Chapter 2.

The main advantages of the proposed method, with respect to previously proposed non-iterative direct data-driven design algorithms for NMP systems, are that (i) the reference model M is designed such that the closed-loop system is able to fulfil performance specifications; (ii) no a-priori information on the NMP zeros location is needed; (iii) the proposed strategy guarantees internal stability without the need of additional constraints on the problem formulation when the plant is stable and possibly NMP system. These distinctive features make the approach considered here significantly more flexible than the previously available ones. The effectiveness of the proposed approach is shown by means of simulation examples and through the application to a laboratory test bench.

Chapter 5

Nonparametric approach to DDDC design

5.1 Introduction

Significant research efforts have been devoted to the direct data-driven control (DDDC) theory in recent years, where experimental data are directly used to design the controller. The DDDC approach is of particular interest in real-world applications, where an accurate model of the plant to be controlled is not available.

DDDC approaches do not rely on plant model identification since available input-output data experimentally collected from the plant are directly used to design the controller. The control specifications are usually given in this context in terms of a desired closed-loop reference model; then, the controller parameters are computed by formulating the problem in terms of model matching design. DDDC approaches using a single set of input-output data, also known as non-iterative data-driven control, have been already studied in the linear time-invariant (LTI) framework. Well established approaches, like Virtual Reference Feedback Tuning (VRFT) proposed by [66] and Non-iterative Correlation-based Tuning (NCbT) proposed by [84], have been widely discussed in the literature, see, e.g., [22, 23, 125, 168, 20, 153].

An alternative data-based approach for controller design, called Iterative feedback tuning (IFT), has been proposed by [72]. IFT is a data-driven control

DDC scheme involving iterative optimization of a fixed structure controller, whose parameters are tuned according to an estimated gradient of a control performance criterion. Furthermore, the Iterative Correlation-based Tuning method (ICbT), proposed by [82], is a data-driven control method in which the controller parameters are tuned iteratively to decorrelate the closed-loop output error, between the designed and achieved closed-loop system, from an external reference signal. Therefore, at each iteration, in general, several experiments are needed to estimate the gradient.

In this chapter, we present a novel non-iterative approach to direct data-driven nonparametric controller design. The approach is based on the method proposed in Chapter 3, where a novel set-membership based direct data-driven controller design technique is presented. By exploiting the results given in [30], where an original kernel-based set-membership nonparametric approach for LTI identification is proposed, DDDC problem is then formulated in the robust Reproducing kernel Hilbert space (RKHS) framework. First, by assuming that the available input-output data are corrupted by bounded noise, we formulate the problem of designing a controller in order to match the behaviour of an assigned reference model. Then, the controller is designed by means of a non-parametric approach, inspired by the results in [30].

The main distinctive features of the proposed approach with respect to those already available in the literature are as follows: (i) the noise corrupting the data is assumed to be bounded and no statistical information is assumed to be a-priori available; (ii) in contrast to IFT and ICbT approaches, where an iterative procedure is exploited, the proposed approach leads to a non-iterative algorithm to design the controller; (iii) to the best of the authors' knowledge, this is the first attempt to design a direct data-driven controller for LTI systems where the controller structure/order is not imposed a-priori. As a matter of fact, all the previously proposed approaches to direct data-driven design look for the controller of a given structure/order that minimizes a certain function of the model matching error. In the case the obtained performance is not satisfactory, the controller structure/order is updated and the design is repeated. As a consequence, the search of a satisfactory controller order/structure could lead to a number of time-consuming iterations in case a high order controller is needed to adequately match the behaviour of the considered reference model. On the contrary, in this work the controller structure required to optimally solve the problem is directly learned from the available experimental data by means of the robust RKHS-based identification framework proposed in [30]. Although through this approach, a high order controller could be obtained, the designer knows that this is the solution which provides the best possible achievable performance.

The chapter is organized as follows. In Section 5.2 we give the problem formulation, while in Section 5.3 we present the proposed approach. In Section 5.4 we provide a robust optimization based algorithm to compute the solution to the formulated problem. In Section 5.5 we show the effectiveness of the presented method by means of two simulation examples. Concluding remarks end the chapter.

5.2 Problem formulation

In this section, the problem statement discussed in Chapter 3.3.1 will be briefly reviewed for self consistency of this chapter.

Let us consider the discrete-time linear-time invariant (LTI) single-input single-output (SISO) feedback control scheme depicted in Fig. 5.1, where q^{-1} denotes the standard backward shift operator, $G(q^{-1})$ is a stable plant transfer function, $K(\rho,q^{-1})$ is the controller transfer function, ρ is the vector of controller parameters, and $M(q^{-1})$ is the transfer function of a suitable given reference model describing the desired behavior the controlled plant.

The objective of the contribution is to propose an algorithm to design an LTI controller $K(\rho,q^{-1})$ such that the closed loop transfer function $T_{\tilde{w}r}(q^{-1})$ given by

$$T_{\tilde{w}r}(q^{-1}) = \frac{K(\rho,q^{-1})G(q^{-1})}{1+K(\rho,q^{-1})G(q^{-1})} \tag{5.1}$$

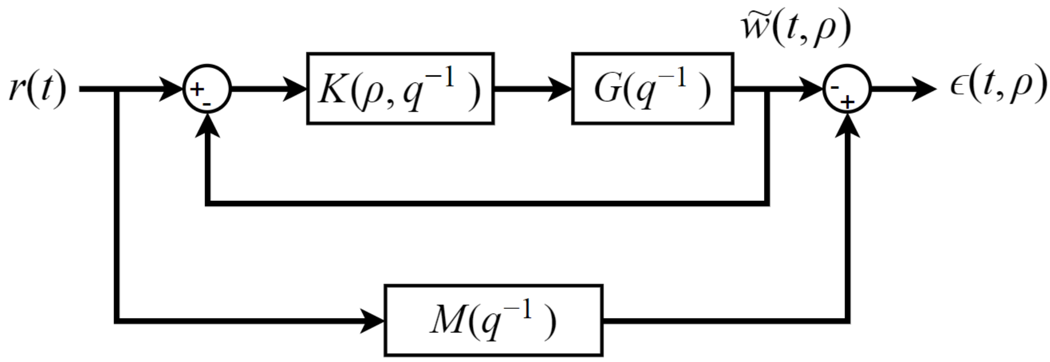

Figure 5.1 Feedback control system to be designed compared with the reference model $M(q^{-1})$.

matches, as close as possible, in some sense, $M(q^{-1})$. We assume that the plant transfer function $G(q^{-1})$ is unknown, and only a set of input-output data, collected through suitable experiments on the plant, is available.

Let us now introduce the following definitions.

Definition 5.2.1: Model matching error transfer function

The model matching error transfer function $E(q^{-1})$ is defined as the difference between the reference model and the achieved closed-loop transfer function, i.e.

$$E(\rho, q^{-1}) = M(q^{-1}) - \frac{G(q^{-1})K(\rho, q^{-1})}{1 + G(q^{-1})K(\rho, q^{-1})} \tag{5.2}$$

Definition 5.2.2: Output matching error

The output matching error $\epsilon(\rho, t)$ is defined as the signal obtained by multiplying both sides of equation (5.2) by a reference signal $r(t)$, i.e.

$$\epsilon(t, \rho) = M(q^{-1})r(t) - \frac{K(\rho, q^{-1})w(t)}{1 + G(q^{-1})K(\rho, q^{-1})} \tag{5.3}$$

where $w(t) = G(q^{-1})r(t)$ is the output of the plant obtained by applying the reference signal $r(t)$ as input, while the signal $\epsilon(t, \rho) = E(q^{-1})r(t)$ is the *output matching error* corresponding to the reference signal $r(t)$ (see Fig. 5.2 for a block diagram description of this error).

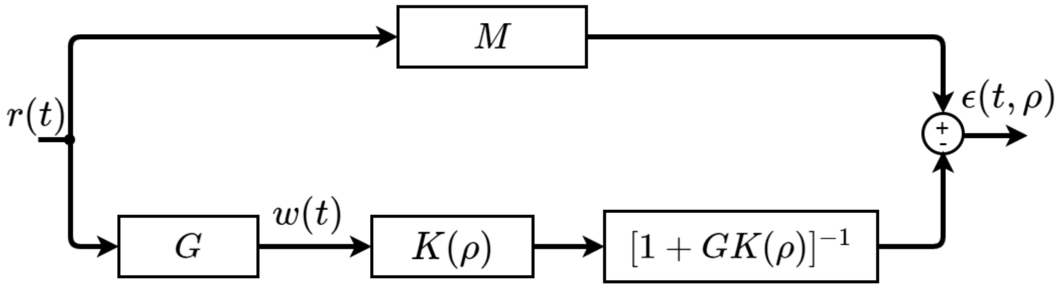

Figure 5.2 A block diagram description of the output matching error $\epsilon(t,\rho)$

To simplify notation, in the rest of the chapter we drop the backward shift operator q^{-1} from equations and corresponding block diagrams.

Since the output matching error $\epsilon(t,\rho)$ in equation (5.3) still depends on the unknown plant G, we derive an alternative way of designing the controller $K(\rho)$. For this purpose, we introduce the following result.

Result 5.2.1

The following three conditions are equivalent

$$\textbf{(i)} \quad E(\rho) = 0 \tag{5.4}$$

$$\textbf{(ii)} \quad \epsilon(\rho,t) = 0, \quad \forall r(t) \tag{5.5}$$

$$\textbf{(iii)} \quad M(1-M)^{-1}r(t) = K(\rho)w(t), \ \forall r(t) \tag{5.6}$$

Remark 5.2.1

Result 5.2.1 plays a crucial role here since it suggests a way for turning the condition on the model matching error $E = 0$, which depends on the unknown plant transfer function G, into a condition on the output matching error $\epsilon(t) = 0$ which, on the contrary, depends only on the output sequence $w(t)$ collected by applying the signal $r(t)$ to the plant (see condition (iii)).

Remark 5.2.2

Condition (ii) considered in Result 5.2.1 can be approximated in practice by the condition $\epsilon(\rho, t) = 0$ for a reference signal $r(t)$ which is persistently exciting in the sense that its spectrum is rich enough to properly excite the dynamics of both M and G. For more details see Section 3.3.1.

Based on equation (5.6) in Result 5.2.1, we can finally formulate the problem of designing the controller $K(\rho)$ such that

$$M(1-M)^{-1}r(t) = K(\rho)w(t) \tag{5.7}$$

Equation (5.7) shows that the problem of direct data-driven controller design considered in this chapter is equivalent to a standard identification problem, where the input $w(t)$ of the controller to be identified $K(\rho)$ is affected by bounded additive noise while the output signal given by $M(1-M)^{-1}r(t)$ is noiseless.

Remark 5.2.3

Selection of the controller class is one of the most critical aspects of direct data-driven controller design. The typical approach is to arbitrarily select the model class and then look for the best possible controller in such a class, which may possibly lead to degraded performance. In order to overcome such a limitation, in this chapter, we only assume that the controller to be designed is a linear time-invariant discrete-time system without any a-priori information either on the controller transfer function structure or its order. Such a choice leads to the formulation of a suitable nonparametric DDDC problem as discussed in the next sections.

5.3 Direct data-driven nonparametric control tuning

The LTI controller K shown in equation (5.7) can be alternatively described by the following equation:

$$K(\rho)w(t) = \sum_{s=0}^{\infty} w(t-s)k(s) \tag{5.8}$$

where $k(s), s = 0, ..., \infty$, are samples of the impulse response of the controller to be identified, while $w(t)$ is the controller input signal.

Here we assume that a set of N input-output plant data are collected experimentally by applying a suitable (persistently exciting) signal $r(t)$ to the plant. The output plant measurements $y(t)$ are assumed to be corrupted by bounded additive noise according to

$$y(t) = w(t) + \eta(t) \tag{5.9}$$

where

$$w(t) = Gr(t) \tag{5.10}$$

is the noiseless output of the system, while the error $\eta(t)$ is a bounded signal assumed to belong to the following ball

$$\eta \in \mathcal{B}_{\Delta\eta} = \{\eta : \ ||\eta||_2^2 \leq \ \Delta\eta\}, \tag{5.11}$$

where $\Delta\eta$ is a known real constants.

In this chapter, the controller to be identified is only known to be linear and time-invariant (LTI). No structural information (e.g. model order) is assumed to be a-priori available and the control structure to be identified is not constrained to belong to a finite-dimensional model class (e.g. FIR, ARX, etc.). To this aim, we review and summarize some basic definitions related to the notion of Reproducing Kernel Hilbert Space (RKHS) (for more details see e.g., [123]).

Definition 5.3.1: Hilbert Space

A Hilbert Space H is a vector space equipped with an inner (scalar) product, such that the norm induced by the inner product turns H into a complete metric space.

In the following we will consider only Hilbert Spaces which elements are functions $g : \mathcal{T} \subset \mathbb{R} \to \mathbb{R}$ and we will use the symbol $\langle \cdot, \cdot \rangle_H$ to denote the inner product and the symbol $||g||_H$ to denote the norm induced by such an inner product.

Definition 5.3.2: Reproducing Kernel Hilbert Space (RKHS)

A Hilbert Space H is a Reproducing Kernel Hilbert Space (RKHS) iff the point evaluation operator is a continuous linear functional i.e.

$$\forall \; \exists \; S(t) : |g(t)| \leq S(t)||g(t)||_H \quad \forall g \in H \tag{5.12}$$

Since all the RKHSs are Hilbert spaces, the Rietz representation theorem holds.

***Theorem 1:* Rietz Representation Theorem**

Let H be a Hilbert space, and let H^* denote its dual space (i.e. the space of all continuous linear functionals $\mathcal{F}[\cdot] : H \to \mathbb{R}$). For each $\mathcal{F} \in H^*$ there exists a unique element $f \in H$ such that

$$\mathcal{F}[g] = \langle f, g \rangle_H \; \forall g \in H. \tag{5.13}$$

This theorem states that every continuous linear functional can be uniquely written by using the inner product.

Based on the previous notation about the RKHS, the following nonparametric LTI DDDC problem is considered,

$$\hat{k} = \arg\min_{k \in H} \left\{ \sum_{t=1}^{N} \left(M(1-M)^{-1} r(t) - \sum_{s=0}^{\infty} w(t-s)k(s) \right)^2 + \tilde{\lambda} J(k) \right\} \tag{5.14}$$

where $k = [k_1, ...k_\infty]$, $\hat{k} = [\hat{k}_1, ...\hat{k}_\infty]$, H is a (generic) reproducing kernel Hilbert space, $J(k)$ is a regularization term and $\tilde{\lambda}$ is a real constant to suitably tune the weight of the regularization term.

Equation (5.14) can be equivalently rewritten as follows,

$$\hat{k} = \arg\min_{k \in H} \left\{ \sum_{t=1}^{N} \left(l(t) - \hat{l}(t,k) \right)^2 + \tilde{\lambda} J(k) \right\} \tag{5.15}$$

where

$$l(t) = M(1-M)^{-1} r(t), \tag{5.16}$$

$$\hat{l}(t,k) = \sum_{s=0}^{\infty} w(t-s) k(s), \tag{5.17}$$

By means of equations (5.8) and (5.9), the estimation problem (5.15) can be equivalently rewritten as

$$\hat{k} = \arg\min_{k \in H} \left\{ \sum_{t=1}^{N} \left(l(t) - \sum_{s=0}^{\infty} (y(t-s) - \eta(t-s)) k(s) \right)^2 + \tilde{\lambda} J(k) \right\} \tag{5.18}$$

Since $\eta(t)$ are uncertain variables only known to belong to the ball in equation (5.9), here we choose to compute the controller estimate by solving the following robust estimation problem, where we look for the minimum of the worst-case error between the true unknown output of the system and the output of the estimated model:

$$\hat{k}_r = \arg\min_{k \in H} \max_{\eta(t) \in \tilde{\mathcal{B}}_\rho} \left\{ \sum_{t=1}^{N} \left(l(t) - \sum_{s=0}^{\infty} (y(t-s) - \eta(t-s)) k(s) \right)^2 + \tilde{\lambda} J(k) \right\}. \tag{5.19}$$

The symbol $\hat{k}_r$ is used to represent the robust version of the estimate in (5.18). In the rest of the chapter, we use the shorthand notation $\max_\eta$ for $\max_{\eta(t) \in \tilde{\mathcal{B}}_{\Delta\eta}}$.

Since we assume that the data are collected on the plant starting from time $t = 0$, both $w(t)$ and $\eta(t)$ are identically zero for $t < 0$. Furthermore, for the sake of simplicity and without loss of generality, we do not consider the regularization term in the presentation of the algorithm proposed for the solution of the considered nonparametric identification problem. It is worth noting that all the presented technical results, discussed in details for the case without regularization term, can be straightforwardly extended to the problem in the general form given by (5.19) for some of the most common regularization strategies (see [30] for details).

Therefore, the problem to be solved simplifies as follows:

$$\hat{k}_r = \arg\min_{k \in H} \max_{\eta(t)} \sum_{t=1}^{N} \left(l(t) - \sum_{s=0}^{\infty} (y(t-s) - \eta(t-s))k(s) \right)^2. \tag{5.20}$$

5.4 A robust optimization approach

In this section, an original robust convex optimization approach is proposed to solve problem (5.20). The first step to derive the proposed algorithm is the following result, derived on the basis of the properties of the RKHS.

Result 5.4.1

The minimum of optimization problem (5.20) can be computed as follows:

$$\min_{\alpha \in \mathbb{R}^N} \max_{\eta(t)} \sum_{t=1}^{N} \left(l(t) - \sum_{s=0}^{t} (y(t-s) - \eta(t-s)) \sum_{i=0}^{N} \alpha_i F(s)^i \right)^2 \tag{5.21}$$

where $F(s)^i = F(t_i, s))$ and $F(t_i)$ is the kernel function of the reproducing kernel Hilbert space to which the controller impulse response is assumed to belong (see [30] for details). Furthermore, has discussed in [30], the minimizer of (5.21) has the following form for each t:

$$k(t) = \sum_{i=0}^{N} \alpha_i F(t)^i \tag{5.22}$$

Proof First we note that the argument inside the arg min operator depends only on the point evaluation of function k. Therefore, by applying Corollary 5 of [30] we can prove that the minimizer to problem (5.20) is given by (5.22).

Equation (5.21) is obtained by direct substitution of (5.22) into the functional of problem (5.20).

Thanks to Result 5.4.1 and the fact that $(y_{t-s} - \eta_{t-s})$ does not depend on i, the robust optimization to be solved can be rewritten as follows:

$$\begin{aligned}\hat{\alpha} &= \arg \min_{\alpha \in \mathbb{R}^N} \max_{\eta(t)} \mathcal{J} \\ &= \arg \min_{\alpha \in \mathbb{R}^N} \max_{\eta(t)} \sum_{t=1}^{N} \left(l(t) - \sum_{s=0}^{t} \sum_{i=0}^{N} \alpha_i (y(t-s) - \eta(t-s)) F(s)^i \right)^2. \end{aligned} \tag{5.23}$$

Before stating the main results of the chapter, we rewrite the functional in (5.23) in a suitable equivalent form. By properly grouping the terms for different t into suitable matrices and arrays, the following equivalent description is obtained

$$\mathcal{J} = \left\| l - E\alpha \right\|_2^2 \tag{5.24}$$

where

$$l = \begin{bmatrix} l_1 \\ . \\ . \\ . \\ l_N \end{bmatrix} \tag{5.25}$$

and $E \in \mathbb{R}^{N \times N}$ has the following form

$$E = \begin{bmatrix} w_0 F_0^0 & \dots & w_0 F_0^N \\ w_0 F_1^0 + w_1 F_0^0 & \dots & w_1 F_0^N + w_0 F_1^N \\ \dots & \dots & \dots \\ \sum_{i=1}^{N} w_{N-i} F_i^0 & \dots & \sum_{i=1}^{N} w_{N-i} F_i^N \end{bmatrix}, \tag{5.26}$$

with $w(t) = y(t) - \eta(t)$.

Now by defining the following matrices

$$A_0 = \begin{bmatrix} F_0^0 & \dots & F_0^N \\ F_1^0 & \dots & F_1^N \\ \dots & \dots & \dots \\ F_N^0 & \dots & F_N^N \end{bmatrix}, \dots, A_N = \begin{bmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & 0 \\ F_0^0 & \dots & F_0^N \end{bmatrix}, \tag{5.27}$$

matrix E can be rewritten as

$$E = w_0 A_0 + w_1 A_1 + \cdots + w_N A_N. \tag{5.28}$$

By exploiting the definition of $w(t)$, we get

$$E = y_0 A_0 + \cdots + y_N A_N - \eta_0 A_0 - \cdots - \eta_N A_N, \tag{5.29}$$

and by defining $E_0 = y_0 A_0 \alpha + y_1 A_1 + \cdots + y_N A_N$ we obtain

$$E = E_0 - \eta_0 A_0 - \cdots - \eta_N A_N. \tag{5.30}$$

Thanks to the introduction of a slack variable $\lambda \in \mathbb{R}$, we can rewrite the optimization problem to be solved in its final form:

$$\begin{aligned} &\hat{\alpha} = \arg \min_{\alpha \in \mathbb{R}^N, \lambda \in \mathbb{R}} \lambda \\ &\text{s.t.} \\ &\left\| y - (E_0 - \eta_0 A_0 - \cdots - \eta_N A_N)\alpha \right\|_2^2 \leq \lambda \\ &\forall \ \eta \ \in \ \mathcal{B}_{\Delta\eta} \end{aligned} \tag{5.31}$$

Finally, we are in the position to state our main result.

Result 5.4.2: Main Result

The solution to problem (5.31) is given by the solution to the following convex semidefinite optimization problem (SDP):

$$\hat{\alpha} = \arg \min_{\alpha\in\mathbb{R}^N, \lambda\in\mathbb{R}} \quad \lambda$$

$$\text{s. t.}$$

$$\left[\begin{array}{c|c|c} \tau I & 0 & R(\alpha)^T \\ \hline 0 & \lambda - \tau\rho & (y - E_0\alpha)^T \\ \hline R(\alpha) & (y - E_0\alpha) & I \end{array}\right] \geq 0, \tag{5.32}$$

where

$$R(\alpha) = \begin{bmatrix} \vdots & \vdots & \vdots \\ A_0\alpha & \vdots & A_N\alpha \\ \vdots & \vdots & \vdots \end{bmatrix}. \tag{5.33}$$

Sketch of proof of Result 5.4.2 By proper application of the so-called Schur complement and some algebraic manipulations, constraint (5.31) can be equivalently rewritten as follow

$$\begin{bmatrix} \eta_0 \\ \vdots \\ \eta_N \end{bmatrix}^T \begin{bmatrix} R(\alpha)^T R(\alpha) \end{bmatrix} \begin{bmatrix} \eta_0 \\ \vdots \\ \eta_N \end{bmatrix} + 2 \begin{bmatrix} \vdots \\ R(\alpha)^T (l - E_0\alpha) \\ \vdots \end{bmatrix}^T \begin{bmatrix} \eta_0 \\ \vdots \\ \eta_N \end{bmatrix} \tag{5.34}$$
$$+ ||l - E_0\alpha||^2 \leq \lambda, \quad \forall\, \eta \in \mathcal{B}_{\Delta\eta}$$

Thanks to the $\mathcal{S}$-procedure, Eq. (5.34) is equivalent to

$$\left[\begin{array}{c|c} -R(\alpha)^T R(\alpha) & -R(\alpha)^T (l - E_0\alpha) \\ \hline -(l - E_0\alpha)^T R(\alpha) & \lambda - ||l - E_0\alpha||^2 \end{array}\right] + \tag{5.35}$$

$$-\tau \left[\begin{array}{c|c} -I & 0 \\ \hline 0 & \rho \end{array}\right] \geq 0 \tag{5.36}$$

for some $\tau > 0$. Then, by splitting the linear and the non linear terms we get

$$\left[\begin{array}{c|c} \tau I & 0 \\ \hline 0 & \lambda - \rho \end{array}\right] + \left[\begin{array}{c|c} R(\alpha)^T R(\alpha) & R(\alpha)^T(l - E_0\alpha) \\ \hline (l - E_0\alpha)^T R(\alpha) & -|l - E_0\alpha||^2 \end{array}\right] \geq 0$$

and we can write the second term of 5.37 as

$$\begin{bmatrix} R(\alpha)^T(l - E_0\alpha) \\ \\ (l - E_0\alpha) \end{bmatrix} I \begin{bmatrix} R(\alpha)^T(l - E_0\alpha) \\ \\ (l - E_0\alpha) \end{bmatrix}^T \tag{5.37}$$

Then, by using the Schur complement, we get the result.

Remark 5.4.1

In the above calculations we have not considered regularization terms in the objective function in order not to increase the complexity of the notation. However, both Lasso and Ridge regularization can be incorporated.

If the Lasso regularization term is added, the problem becomes

$$\begin{aligned} &\arg\min_{\alpha \in \mathbb{R}^N} \quad \lambda + \tilde{\lambda} \sum_i |\alpha_i| \\ &\text{s. t.} \\ &\left[\begin{array}{c|c|c} \tau I & 0 & R(\alpha)^T \\ \hline 0 & \lambda - \tau\rho & (y - E_0\alpha)^T \\ \hline R(\alpha) & (y - E_0\alpha) & I \end{array}\right] \geq 0, \end{aligned} \tag{5.38}$$

The regularization term can be taken into account by adding some linear constraints to the optimization problem derived from the case without regularization. The obtained optimization problem is still a standard SDP problem and all the presented results and related discussion perfectly apply.

In the case a Ridge regularization term is added, then we have to perform some more transformations. In this case, the problem is given by

$$\arg\min_{\alpha\in\mathbb{R}^N} \quad \lambda+\tilde{\lambda}||\alpha||^2, \tag{5.39}$$

subject to (5.38).

In order to move the additional regularization term into the constraints, we have just to introduce a new slack variable λ_1, such that

$$||\alpha||^2_{\mathcal{H}} \leq \lambda_1. \tag{5.40}$$

Thus, the objective function can be rewritten as $\lambda+\lambda_1$ and, by observing that $||\alpha||^2_{\mathcal{H}} = \alpha^T V\alpha$, where V is the Gram matrix (i.e., V is symmetric positive definite) we can transform the quadratic constraint into the following SDP constraint:

$$\left[\begin{array}{c|c} I & V^{1/2}\alpha \\ \hline \alpha^T(V^{1/2})^T & \lambda_1 \end{array}\right] \geq 0. \tag{5.41}$$

5.5 Simulation examples

In this section we show the effectiveness of the presented approach by means of simulation examples.

5.5.1 Example 1. Nonparametric DDDC design (A)

In this example, the plant has the following transfer function

$$G(q^{-1}) = \frac{32.24q^{-1} + 21.41q^{-2}}{1 - 0.7647q^{-1} + 0.3012q^{-2}},$$

while the chosen reference model is given by

$$M_1(q^{-1}) = \frac{0.04472q^{-1} + 0.07442q^{-2} + 0.0297q^{-3}}{1 - 1.72q^{-1} + 1.14q^{-2} - 0.2715q^{-3}}$$

The system is excited by a random input signal $r(t)$ uniformly distributed in the range $[-1, +1]$. The plant output signal $w(t)$ is corrupted by a random additive noise $\eta(t)$, uniformly distributed in the range $[-\Delta\eta, +\Delta\eta]$ and length $N = 100$. The chosen error bound $\Delta\eta$ is such that the signal to noise ratio is given by

$$SNR_w = 10\log \frac{\sum_{t=1}^{N} w(t)^2}{\sum_{t=1}^{N} \eta(t)^2} \cong 28dB.$$

We compute the solution to problem (5.32) by using the optimization software Sedumi (see [141]). The RKHS considered is the one described in [57] with $\beta = 0.1$. As a regularization term we consider the Lasso one, i.e. $J(k) = ||k||_1$ and we set $\tilde{\lambda} = 10$.

A comparison between the reference model and the obtained closed-loop system for the proposed method, in terms of the step response, is presented in Fig. 5.3, which shows that, although no a-priori information on the controller parameters is considered (e.g., model order), the controller appears to be effective in terms of good matching of the desired closed-loop behaviour. Note that, in order to improve the readability of the step responses comparison, the output of both the controlled system and of the reference model are depicted in Fig. 5.3 by assuming that a zero-order-hold (ZOH) filter is connected to the outputs of the two systems.

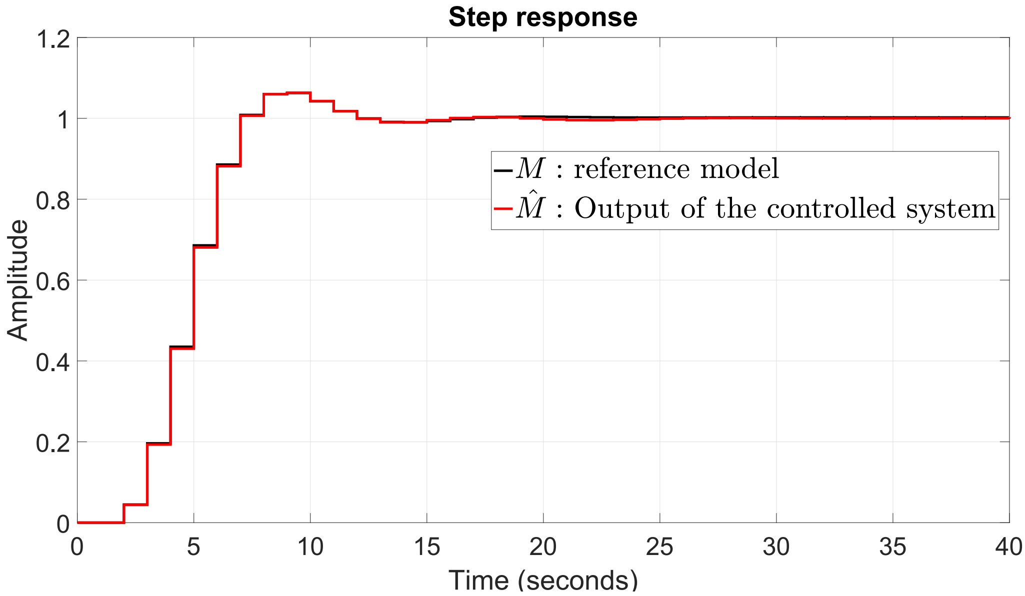

Figure 5.3 Comparison of step responses: Output of the controlled system (black-line), reference model (red-line).

5.5.2 Example 2. Nonparametric DDDC design (B)

In this example, the proposed approach is employed to tune a SISO controller to be compared with the standard NCbT approach (see e.g., [84]). The plant considered in this example is a *Piezoelectric* Drive system proposed by Richard & Astrom in 2010 [4], and has the following transfer function

$$G(s) = \frac{k w_2^2 w_3^2 w_5^2 (s^2 + 2\zeta_1 w_1 s + w_1^2)(s^2 + 2\zeta_4 w_4 s + w_4^2)}{w_1^2 w_4^2 (s^2 + 2\zeta_2 w_2 s + w_2^2)(s^2 + 2\zeta_3 w_3 s + w_3^2)(s^2 + 2\zeta_5 w_5 s + w_5^2)}$$

where, $k = 5$; $w_1 = 2420$; $w_2 = 2550$; $w_3 = 6450$; $w_4 = 8250$; $w_5 = 9300$; $\zeta_1 = 0.03$; $\zeta_2 = 0.03$; $\zeta_3 = 0.042$; $\zeta_4 = 0.025$; $\zeta_5 = 0.032$.

The reference Model is given by:

$$M(q^{-1}) = \frac{0.01144q^{-1} + 0.009164q^{-2}}{1 - 1.79q^{-1} + 0.8106q^{-2}}.$$

The system is excited by a random input signal $r(t)$ uniformly distributed in the range $[-1, +1]$. The plant output signal $w(t)$ is corrupted by a random additive noise $\eta(t)$, uniformly distributed in the range $[-\Delta\eta, +\Delta\eta]$. 100

input-output samples are collected with a sampling time $Ts = 0.01s$ and, discrete-time model of $G(s)$ is obtained, for simulation purposes, through ZOH discretization method. The chosen error bound $\Delta\eta$ is such that the signal to noise ratio is $30dB$.

We have also designed a controller by using the NCbT method proposed by [84]. For the NCbT approach, the output data have been collected by applying the signal $u(t)$ to the plant input, where $u(t)$ is given as a random signal with amplitude within the range $[-1,+1]$ with the length $N = 5000$. The output is corrupted by a zero-mean white noise such that the signal-to-noise ratio is about $30dB$. The controller structure for the NCbT is chosen as follows,

$$K(q^{-1}) = \frac{\sum_{j=0}^{n_b} b_j q^{-j}}{1-q^{-1}}$$

where n_b is the controller order. In this example, three different controller orders ($n_b = 3,6,9$) have been considered. Note that, the controller structure for the NCbT method is chosen to be linearly parameterized with a fixed known denominator according to [60].

For the NCbT method, the length of the instrumental variable l is found to be 20 by trial-and-error. Fig. 5.4 displays the comparison between the reference model and the obtained closed-loop system in terms of step responses for the proposed approach (DDDC-RKHS) and NCbT methods.

The results obtained by the DDDC-RKHS method show that, although no a-priori information on controller parameters is used, the designed controller achieves a good performance to match the given reference model. It is worth noting that, through the DDDC-RKHS approach no integral action has been directly enforced in the parametrization of the controller. On the other hand, by analyzing the results obtained by the NCbT controllers (Fig. 5.4), it can be seen that the a-priori selection of the controller order n_b affects the achieved matching performance, showing that a-priori selection of the controller structure is a critical step, avoided by the proposed DDDC-RKHS method.

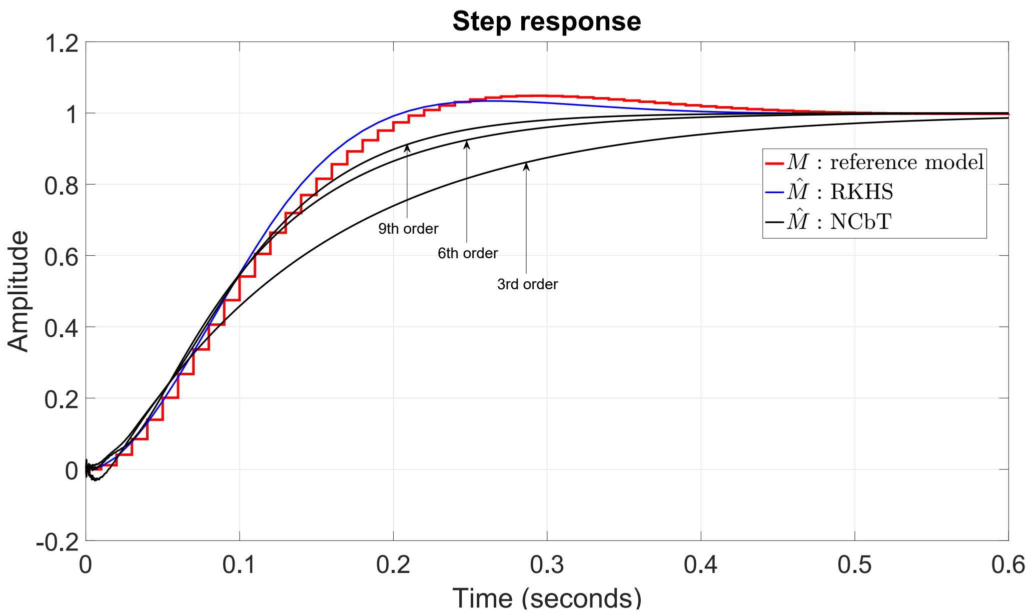

Figure 5.4 Step responses: designed feedback control system with the DDDC-RKHS approach (blue-line), the NCbT method (black-lines), reference model (red-line).

5.6 Discussion and conclusion

In this chapter, we present a novel non-iterative approach to direct data-driven **nonparametric** controller design. The approach is inspired by the method described in Chapter 3, where a novel set-membership based direct data-driven controller design technique is presented. By exploiting the results given in [30], where an original kernel-based set-membership nonparametric approach for LTI identification is proposed, DDDC problem is then formulated in the robust Reproducing kernel Hilbert space (RKHS) framework. First, by assuming that the available input-output data are corrupted by bounded noise, we formulate the problem of designing a controller in order to match the behaviour of an assigned reference model. Then, the controller is designed by means of a non-parametric approach, inspired by results in [30].

The main distinctive features of the proposed approach with respect to those already available in the literature are as follows: (i) the noise corrupting the data is assumed to be bounded and no statistical information is assumed to be a-priori available; (ii) in contrast to IFT and ICbT approaches, where an iterative procedure is exploited, the proposed approach leads to a non-iterative algorithm to design the controller; (iii) to the best of the authors' knowledge, this is the first attempt to design a direct data-driven controller for LTI systems where the controller structure/order is not imposed a-priori.

Finally, we have shown the effectiveness of the presented technique by means of simulation examples.

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