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## Groupoids of OEIS A002378 and A016754 Numbers (oblong and odd square numbers)

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Here we discuss the binary operators of the sets made by the OEIS sequences of integers A002378 and A016754. A002378 are defined as oblong numbers.

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Let us use the definition of the first type of groupoid given in [1]: it is an algebraic structure on a set with a binary operator. The only restriction on the operator is closure. This properties means that, applying the binary operator to two elements of a given set S, we obtain a value which is itself a member of S. Here, we consider the groupoids of the sets of the numbers given by OEIS sequences A002378 and A016754 [2,3], which are told as oblong and odd squares (centered octagonal) numbers.

An A002378 number is also known as a promic, pronic, or heteromecic number (formerly M1581 N0616). It is an integer having the following form [2]:

$$O_n = n(n+1)$$

OEIS gives: 2, 6, 12, 20, 30, 42, 56, 72, 90, 110, 132, 156, 182, 210, 240, 272, 306, 342, 380, 420, 462, 506, 552, 600, 650, 702, 756, 812, 870, 930, 992, 1056, and so on.

Ref. 2 tells that  $4O_n+1$  are the odd squares A016754 numbers.

An A016754 number (odd square number) is defined as [3]:

$$o_n = (2n+1)^2$$

So we have [3]: 1, 9, 25, 49, 81, 121, 169, 225, 289, 361, 441, 529, 625, 729, 841, 961, 1089, 1225, 1369, 1521, and so on.

As we did in some previous discussions (see for instance [4,5]), we can find a binary operator, which satisfy the closure, of given sets of numbers. In [4], we considered the groupoids of Mersenne, Fermat, Cullen and Woodall numbers. Here, we follow the same approach as in [6], for Carol and Kynea numbers.

Here how to find the operator for A002378 numbers. Let us use:

$$(n(n+1)+1/4)^{1/2} = (n^2+n+1/4)^{1/2} = ((n+1/2)^2)^{1/2}$$

So we define:

$$(O_m+1/4)^{1/2}=(m+1/2)=A_m \quad ; \quad (O_n+1/4)^{1/2}=(n+1/2)=A_n \quad ;$$

$$(O_{m+n}+1/4)^{1/2}=(m+n+1/2)=A_{m+n}$$

We use numbers  $A_m$  to help us in the calculation. We have for them the binary operator:

$$A_{m+n}=A_m \oplus A_n = A_m + A_n - 1/2 = (m+1/2) + (n+1/2) - 1/2 = m+n+1/2$$

Therefore:  $(O_{m+n}+1/4)^{1/2} = A_{m+n} = A_m \oplus A_n = A_m + A_n - 1/2$

$$(O_{m+n}+1/4)^{1/2} = (O_m+1/4)^{1/2} + (O_n+1/4)^{1/2} - 1/2$$

We can find the binary operator for the Oblong numbers as:

$$O_{m+n}+1/4 = O_m + O_n + 3/4 + 2(O_m+1/4)^{1/2}(O_n+1/4)^{1/2} - (O_m+1/4)^{1/2} - (O_n+1/4)^{1/2}$$

$$O_{m+n} = O_m + O_n + 1/2 + 2(O_m+1/4)^{1/2}(O_n+1/4)^{1/2} - (O_m+1/4)^{1/2} - (O_n+1/4)^{1/2}$$

So we have the binary operator defined as:

$$O_m \oplus O_n = O_m + O_n + 1/2 + 2(O_m+1/4)^{1/2}(O_n+1/4)^{1/2} - (O_m+1/4)^{1/2} - (O_n+1/4)^{1/2}$$

Associativity:

$$O_m \oplus (O_n \oplus O_p) = O_m \oplus O_{n+p} = O_{m+n+p} \quad ; \quad (O_m \oplus O_n) \oplus O_p = O_{m+n} \oplus O_p = O_{m+n+p}$$

From this binary operation, we can have the recursive relation:  $O_{n+1} = O_n \oplus O_1$  .

From  $O_1=2$  , we have: 6, 12, 20, 30, 42, 56, 72, 90, 110, 132, 156, 182, 210, 240, 272, 306, 342, 380, 420, 462, and so on.

Let us consider the odd square numbers.

Here how to find the operator for A016754 numbers:  $o_n = (2n+1)^2$  . Let us use:

$$o_m^{1/2} = (2m+1) = A_m \quad ; \quad o_n^{1/2} = (2n+1) = A_n \quad ; \quad o_{m+n}^{1/2} = (2(m+n)+1) = A_{m+n}$$

We use numbers  $A_m$  to help us in the calculation. So we have the binary operator:

$$A_{m+n} = A_m \oplus A_n = A_m + A_n - 1 = (2m+1) + (2n+1) - 1 = 2(m+n) + 1$$

Therefore:  $o_{m+n}^{1/2} = A_{m+n} = A_m \oplus A_n = A_m + A_n - 1$

We can find the binary operator for the odd square numbers as:

$$o_{m+n} = o_m + o_n + 1 + 2o_m^{1/2} o_n^{1/2} - 2o_m^{1/2} - 2o_n^{1/2}$$

So we have the binary operator defined as:

$$o_m \oplus o_n = o_m + o_n + 1 + 2o_m^{1/2} o_n^{1/2} - 2o_m^{1/2} - 2o_n^{1/2}$$

Associativity:

$$o_m \oplus (o_n \oplus o_p) = o_m \oplus o_{n+p} = o_{m+n+p} \quad ; \quad (o_m \oplus o_n) \oplus o_p = o_{m+n} \oplus o_p = o_{m+n+p}$$

From this binary operation, we can have the recursive relation:  $o_{n+1} = o_n \oplus o_1$  .

From  $o_1 = 9$  , we have: 25, 49, 81, 121, 169, 225, 289, 361, 441, 529, 625, 729, 841, 961, 1089, 1225, 1369, 1521, 1681, 1849, and so on.

## References

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