Summary

The evolution of the aviation industry has always been driven by the achievement of better performances and the reduction of weight. One successful approach for saving weight is the design of aircraft structures able to work in *postbuckling* conditions. In addition, the ongoing trend of replacing metals with high-performance fibre composites has led to both lighter and more efficient airplanes.

However, the approaches employed for the design of isotropic material structures are not suitable for composites because of the different mechanical behaviour of these materials due to their physical and chemical properties.
For this reason, new methodologies have been created to properly model the behaviour of composite structures and the phenomena caused by their heterogeneous nature. Moreover, additional failure mechanisms have to be considered for composites, such as the intra-laminar damages (the fibres or the matrix failure) and the delaminations.

The new analytical techniques developed for the analyses of composite beams can be grouped in three main categories: the Higher-Order theories, the Layer-Wise approaches and the Zigzag theories. The Higher-Order theories have been proven to be not enough accurate for highly heterogeneous composite beams that are moderately thick, whereas the Layer-Wise approaches are generally accurate but inefficient for laminates made of many layers. The Zigzag theories are a sub-class of the general Layer-Wise theories but they have been developed in the way that the number of variables, thus the computational cost, is independent of the number of layers.

The Finite Element (FE) commercial codes are usually the preferred choice for the structural analysis because of their high accuracy and versatility to analyse even complex geometries and material laminations. However, highly-detailed FE models of composite beams are usually computationally inefficient in the commercial codes. Moreover, the computational effort further increases when nonlinear analyses are performed, as for the postbuckling analysis of composite beams, becoming unacceptable when elements like piezoelectric actuators are involved in nonlinear analyses.

Nevertheless, the finite element formulations based on the Refined Zigzag Theory (RZT), which is one of the Zigzag theories, have proven to be as accurate as the commercial codes and also more efficient. The RZT has already been assessed for the static and free-vibration analyses of beams with highly heterogeneous material laminations, demonstrating its superior performances.

In this context, the present work has the primary objective of creating a successful methodology for the buckling and postbuckling analyses of composite and sandwich beams with piezoelectric actuators, able to reach the same accuracy as highly-detailed FE commercial code models with lower computational cost.

A literature review was conducted about the methodologies employed for the structural analyses of composite and sandwich beams. The research showed that the analytical and numerical methods used for the buckling and postbuckling analyses of composites were either based on theories not suitable for both composite laminated and sandwich beams or not enough efficient. In addition, the RZT was never employed for nonlinear postbuckling analyses, despite its superior capabilities. Hence, a new method based on the Refined Zigzag Theory was created for more efficient buckling and postbuckling analyses of both composite laminated and sandwich beams with piezoelectric actuators bonded to the structure. A finite element formulation based on the new RZT model was developed to extend the method to the analysis of beams of any laminations, boundary and loading conditions, with either continuous piezoelectric layers or a discrete number of piezoelectric patches, and able to predict the local buckling behaviour in sandwich beams.

As a first step, the RZT was extended introducing both the geometric nonlinearities and the geometric imperfections of the beam and also the inverse piezoelectric behaviour in the theory formulation.
The Principle of Virtual Work for electro-mechanical fields was employed to obtain the nonlinear equilibrium equations based on the RZT for composite beams with piezoelectric actuator layers. The equilibrium equations were solved under specific assumptions on the beam properties and loading conditions.
A model based on the RZT-beam finite elements was then created to find approximate solutions of the equilibrium equations in general situations. The Newton-Raphson method was employed for solving the nonlinear FE equilibrium equation.

The possibility of having a variation of thickness and lamination along the beam length was also taken into account using a strategy based on the Lagrange Multipliers method. This modification allowed the modelling of discrete piezoelectric patches bonded to the external surfaces of the beam and it was also the basis for modelling the local buckling of pre-delaminated beams.

The new RZT finite element model was numerically assessed for the buckling and postbuckling analyses of sandwich beams and composite laminated beams with piezoelectric actuators through a comparison with highly-detailed FE commercial code models.
Firstly, the buckling and postbuckling analyses were performed for various sandwich beams with different geometrical and material properties and the RZT results were compared to those obtained by Abaqus, Nastran and a model based on Timoshenko beam finite elements.
Then, the RZT model was assessed for the analyses of composite beams with piezoelectric patches bonded to the external surfaces. The buckling and postbuckling analyses and the static response to the piezoelectric actuation were calculated using both the RZT and a highly-detailed Abaqus model.

The new model was validated experimentally. Foam core sandwich beams with carbon-fibre reinforced-polymer (CFRP) facesheets and monolithic beams with piezoelectric actuators bonded to their external surfaces were manufactured and tested at the RMIT University material testing laboratory. Specific support for each kind of beam were prepared to realise simply-supported boundary conditions. The sandwich beams were tested using the Instron compression-testing machine and the Southwell method was employed for calculating the critical buckling loads.

Two kind of tests were performed for the monolithic beams. Firstly, the static response increasing the voltage in the actuators was calculated measuring the transversal displacement of the beam. Subsequently, the beams were tested in compression for different values of voltage applied to the actuators to improve the beam postbuckling response. The Southwell method was used also in this case to calculate the effect of the piezoelectric actuation on the critical buckling load.

The tests demonstrated the excellent capabilities of the new RZT model for predicting the buckling and postbuckling behaviour of composite and sandwich beams, even including the effect of the piezoelectric actuation.

The new RZT-FE model was then employed for the numerical buckling analysis of a sandwich beam with a debonding between the core and the top facesheet. It was able to successfully predict the local buckling depending on the length of the debonding. Subsequently, the possibility of controlling the local instability was investigated considering piezoelectric actuators on the external surfaces of the beam and the postbuckling response was optimised through the application of a suitable voltage to the piezoelectric layers. The RZT model could identify the best voltage values and distribution along the beam length to avoid the local buckling of the sandwich beam.

The present work provides a new FE model which can be employed for the buckling and postbuckling analyses of composite laminated and sandwich beams (even highly heterogeneous laminations) with geometric imperfections and piezoelectric actuators. The major outcome of this effort is the possibility to perform complex nonlinear static analyses reaching the same accuracy of highly-detailed FE commercial code models but with a significantly lower computational cost. In addition, using the proposed method, the piezoelectric effect can be efficiently introduced in nonlinear postbuckling analyses, also for beams with delaminations.